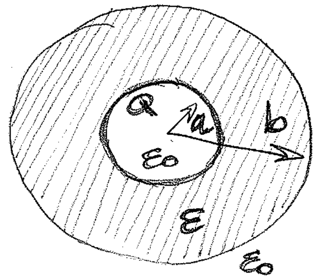
(1)



Diferencia de potencial entre $r=0$ y $r=\infty$:

$$V = V(0) - \underbrace{V(\infty)}_{0} = -\int_{\infty}^{0} \vec{E}\cdot d\vec{l}$$

Gauss :

$$\begin{cases} r>a \,;\ \vec{D} = \dfrac{Q}{4\pi r^2}\,\hat{e}_r \longrightarrow \begin{cases} (a<r<b) & \vec{E}(r) = \dfrac{Q}{4\pi\varepsilon r^2}\,\hat{e}_r \\ (r>b) & \vec{E}(r) = \dfrac{Q}{4\pi\varepsilon_0 r^2}\,\hat{e}_r \end{cases} \\ r<a \,;\ \vec{D} = 0 \longrightarrow \vec{E} = 0 \end{cases}$$

$$\Rightarrow V = -\int_{\infty}^{b}\left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)dr - \int_{b}^{a}\left(\frac{Q}{4\pi\varepsilon r^2}\right)dr - \int_{a}^{0}(0)\,dr$$

$$= -\frac{Q}{4\pi}\left(-\frac{1}{r}\Big|_{\infty}^{b}\frac{1}{\varepsilon_0} - \frac{1}{r}\Big|_{b}^{a}\frac{1}{\varepsilon}\right) = \frac{Q}{4\pi}\left[\frac{1}{b\varepsilon_0} + \frac{1}{\varepsilon}\left(\frac{1}{a} - \frac{1}{b}\right)\right] =$$

$$\boxed{V = \frac{Q}{4\pi}\left(\frac{1}{\varepsilon_0 b} + \frac{1}{\varepsilon a} - \frac{1}{\varepsilon b}\right)}$$

—— x ——

(2) Energía almacenada en la configuración :

$$U = \frac{1}{2}\int_V \vec{D}\cdot\vec{E}\,\underbrace{dV}_{(4\pi r^2)dr} = \frac{1}{2}\frac{Q^2}{(4\pi)^{\cancel{2}}}\cancel{(4\pi)}\left\{\frac{1}{\varepsilon}\int_a^b\left(\frac{1}{r^2}\right)^{\cancel{2}}\cancel{r^2}\,dr + \frac{1}{\varepsilon_0}\int_b^{\infty}\left(\frac{1}{r^2}\right)^{\cancel{2}}\cancel{r^2}\,dr\right\}$$

$$= \frac{1}{2}\frac{Q^2}{4\pi}\left\{\frac{1}{\varepsilon}\left(-\frac{1}{r}\Big|_a^b\right) + \frac{1}{\varepsilon_0}\left(-\frac{1}{r}\Big|_b^{\infty}\right)\right\} = \frac{1}{2}\frac{Q^2}{4\pi}\left\{\frac{1}{\varepsilon}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{1}{\varepsilon_0}\left(\frac{1}{b}\right)\right\}$$

$$\boxed{U = \frac{Q^2}{8\pi}\left[\frac{1}{\varepsilon a} + \frac{1}{b}\left(\frac{1}{\varepsilon_0} - \frac{1}{\varepsilon}\right)\right]}$$

Obs: $U = \frac{1}{2}QV$ ✓
La configuración se puede pensar como un capacitor cuya placa con carga $-Q$ se encuentra en el infinito.

—— x ——

SOLUCION EJERCICIO 3

Potencial entre placas: $\phi(\theta) = \left(\frac{V_0}{\theta_0}\right)\theta$

campo eléctrico: $\vec{E} = -\left(\partial\phi/\partial r + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\right) = -\left(0, \frac{1}{r}\frac{V_0}{\theta_0}\right)$

carga sobre placa horizontal: $\varepsilon_0 \vec{E}\cdot\hat{n} = \sigma$ donde $\hat{n} = (0,1)$

$$\Rightarrow \boxed{\sigma = -\varepsilon_0 \frac{V_0}{\theta_0 x}}$$

SOLUCION EJERCICIO 4

W (ENERGIA ELECTROST.) $= \frac{\varepsilon_0}{2}\iiint |\vec{E}|^2 dV$ donde $|\vec{E}|^2 = \frac{1}{r^2}\left(\frac{V_0}{\theta_0}\right)^2$

$$= \frac{\varepsilon_0 L}{2}\left(\frac{V_0}{\theta_0}\right)^2 \int_d^{L+d}\left[\int_0^{\theta_0} d\theta\right]\frac{\cancel{r}}{r^{\cancel{2}}}dr$$

$$= \frac{\varepsilon_0 L}{2}\frac{V_0^2}{\theta_0} Ln\left(\frac{L+d}{d}\right)$$

τ (TORQUE) $= -\frac{\partial W}{\partial \theta_0} \Rightarrow \boxed{|\tau| = \left(\frac{\varepsilon_0 L}{2}\right)\left(\frac{V_0}{\theta_0}\right)^2 Ln\left(\frac{L+d}{d}\right)}$

SOLUCION EJERCICIO 5

Carga placa izquierda $Q_I(t) = Q_0 e^{-g_2 t/\varepsilon_2}$

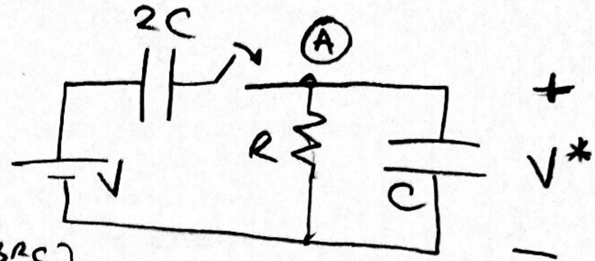
Carga placa derecha $Q_D(t) = -Q_0 e^{-g_1 t/\varepsilon_1}$

Conservación de la carga;

$$Q_I + Q_D + Q_{INTERFASE} = 0$$

$$\Rightarrow \boxed{Q_{INTERFASE} = Q_0\left[e^{-g_2 t/\varepsilon_2} - e^{-g_1 t/\varepsilon_1}\right]}$$

* Solución ejercicio 6



_Nodo A: $2C\frac{d}{dt}(V-V^*) = \frac{V^*}{R} + C\frac{dV^*}{dt}$

$$\Rightarrow -3C\frac{dV^*}{dt} = \frac{V^*}{R} \Rightarrow \left.\begin{array}{l} V^*(t) = Ae^{-t/3RC} \\ V^*(0) = \frac{q_0}{C} \end{array}\right\} \Rightarrow V^*(t) = \frac{q_0}{C}e^{-t/3RC}$$

$$\_P = \frac{V^{*2}}{R} \Rightarrow \boxed{E_{dis} = \int_0^\infty P\,dt = \int_0^\infty \frac{q_0^2}{RC^2}e^{-2t/3RC}\,dt = \frac{3RC}{2}\frac{q_0^2}{RC^2} = \frac{3q_0^2}{2C}}$$

* Solución ejercicio 7

Campos $\vec{P_0}$, $\vec{E}$ y $\vec{D}$ radiales.

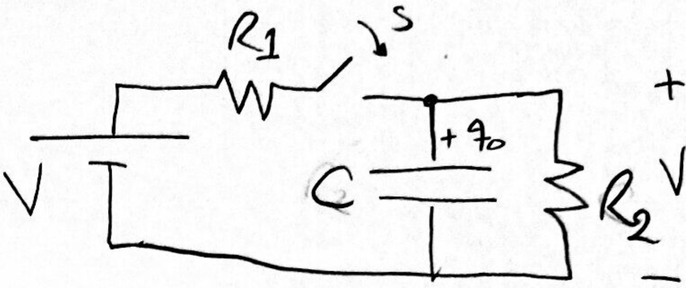
_Condiciones de borde:

$$\left.\begin{array}{l} \sigma_L = D = \varepsilon_0 E + P_0 \\ -\dot{\sigma}_L = J = gE \end{array}\right\} \Rightarrow \dot{\sigma}_L = -\frac{g}{\varepsilon_0}\sigma_L + \frac{g}{\varepsilon_0}P_0$$

$$\Rightarrow \sigma_L(t) = Ae^{-gt/\varepsilon_0} + P_0, \quad A \in \mathbb{R}$$

$$\Rightarrow \sigma_L(0) = \frac{Q}{2\pi aL} = A + P_0 \Rightarrow \boxed{\sigma_L(t) = \frac{Q}{2\pi aL}e^{-gt/\varepsilon_0} + P_0\left(1 - e^{-gt/\varepsilon_0}\right)}$$

* Solución ejercicio 8



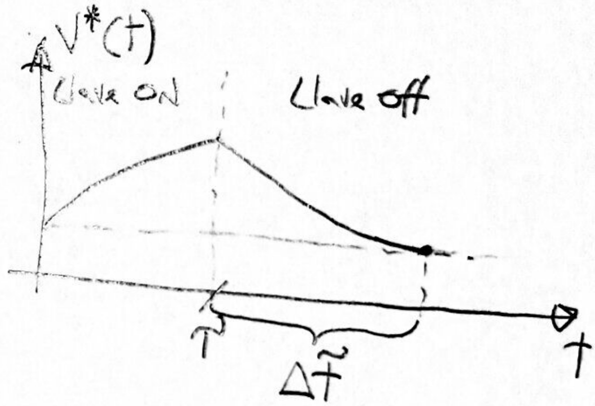
\- Llave S on: $\frac{V-V^*}{R_1} = C\dot{V}^* + \frac{V^*}{R_2}$

$$\Rightarrow \frac{dV^*}{dt} + \frac{R_1+R_2}{R_1R_2C}V^* = \frac{V}{R_1C}$$

$$\Rightarrow V^*(t) = Ae^{-t/\tau} + \frac{R_2V}{R_1+R_2}, \text{ siendo } \tau = \frac{R_1R_2C}{R_1+R_2}$$

$$V^*(0) = \frac{q_0}{C} = A + \frac{R_2V}{R_1+R_2} \Rightarrow \boxed{V^*(t) = \frac{q_0}{C}e^{-t/\tau} + \frac{R_2V}{R_1+R_2}\left(1-e^{-t/\tau}\right)} \quad 0<t<\tau$$

$$V^*(\tau) = \frac{q_0}{C}e^{-1} + \frac{R_2V}{R_1+R_2}\left(1-e^{-1}\right)$$



\- Llave S off:

$t' = t - \tau$

$V^*(t') = Be^{-t'/R_2C}$

$$V^*(t'=0) = \frac{q_0}{C}e^{-1} + \frac{R_2V}{R_1+R_2}\left(1-e^{-1}\right) = B$$

$$\Rightarrow V^*(t'=\Delta\tilde{t}) = \frac{q_0}{C} = \left[\frac{q_0}{C}e^{-1} + \frac{R_2V}{R_1+R_2}\left(1-e^{-1}\right)\right]e^{-\Delta\tilde{t}/R_2C}$$

$$\Rightarrow \boxed{\Delta\tilde{t} = R_2C\ln\left[e^{-1} + \frac{R_2V}{R_1+R_2}\cdot\frac{C}{q_0}\left(1-e^{-1}\right)\right]}$$