Universidad de los Andes Colombia

Solving Hard Shortest Path Problems with the Pulse Framework

Andrés Medaglia, Ph.D.
(amedagli@uniandes.edu.co)

Joint work with:
L. Lozano, Ph.D. (U. of Cincinnati); D. Duque, Ph.D.(c) (Northwestern U.); N. Cabrera, M.Sc. & D. Yamín (U. de los Andes); M. Bolívar, M.Sc. (Glovo)

Departamento de Ingeniería Industrial
Centro para la Optimización y Probabilidad Aplicada
Universidad de los Andes (Colombia)

Universidad de la República; Montevideo (Uruguay), Marzo 9-13

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Agenda

- Part I: fundamentals
- Part II: intuition
- Part III: extensions
- Part IV: applications
- Part V: perspectives

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Lecture 1: Fundamentals

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Andrés Medaglia

- Profesor titular del Departamento de Ingeniería Industrial
- Co-fundador y actual director del Centro para la Optimización y Probabilidad Aplicada (COPA)
- Ph.D. en Investigación de Operaciones (IO) de North Carolina State University (NC State U.) (Raleigh, EEUU; 2001).
- De 1999 a 2002 tuvo vínculos con SAS (Cary, EEUU)
- Su investigación se concentra en el desarrollo y la aplicación de técnicas de optimización, con especial interés en las áreas de logística y transporte; agro-sistemas; ciudades saludables y sostenibles; selección y programación de proyectos; y diseño en ingeniería.
- Tiene más de 50 publicaciones arbitradas en el área de IO en revistas arbitradas.
- Hace parte del cuerpo editorial de TOP (SEIO), Computers and Operations Research y European Journal of Industrial Engineering.
- Se ha desempeñado como Secretario y Vicepresidente de la Asociación Latino-Iberoamericana de IO (ALIO); como Vicepresidente de Centroamérica / Sudamérica por el Instituto de Ingenieros Industriales (IIE); y como el Vicepresidente de la Sociedad de Investigación Operativa de Colombia (ASOCIO).
- En INFORMS, se ha desempeñado en el Publications Committee, Transportation Science & Logistics (TSL) Society como el de Liason para las Américas y en el Best Paper Award Committee (2015-2017).
- IFORS Prize for Operational Research in Development (Quebec City, Canada, 2017), INFORMS Railway Application Section Problem Solving Competition (2011), EURO Award for the Best EJOR (Review) Paper (2015) y AIMMS/MOPTA Modeling Competition (2016).
- En 2018, fue invitado a EURO 2018 (Valencia, España) como el IFORS Invited Tutorial Lecturer.
- Medallas de oro (2018), plata (2019) y bronce (2017) en los Campeonatos Nacionales de Ciclismo (categoría máster).
- Mas información en: http://wwwprof.uniandes.edu.co/~amedagli
- Correo: amedagli@uniandes.edu.co

Agenda

- Part I: fundamentals
  - Dijkstra
  - A*
  - Labeling algorithms
  - Branch and bound
- Part II: intuition
- Part III: extensions
- Part IV: applications
- Part V: perspectives

Agenda

- Part I: fundamentals
  - **Dijkstra**
  - A*
  - Labeling algorithms
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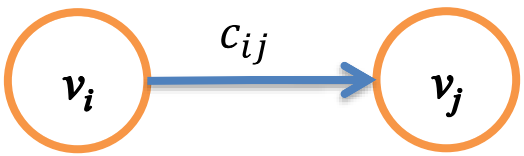
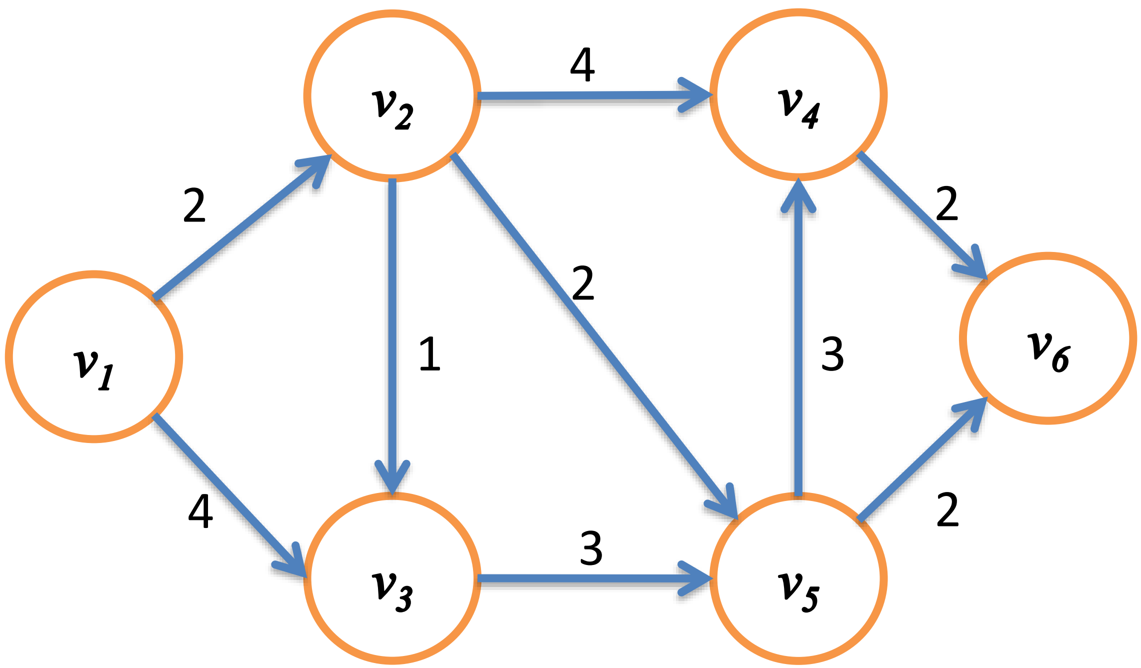
- Part II: intuition
- Part III: extensions
- Part IV: applications
- Part V: perspectives

Definición del Problema

Ruta más corta (uno a uno)





Definición del Problema

Ruta más corta (uno a uno)

Conjuntos:
$G(\mathcal{N}, \mathcal{A})$: grafo dirigido
$\mathcal{N}$: conjunto de nodos
$\mathcal{A}$: conjunto de arcos

Parámetros:
c_{ij}: costo de enviar una unidad por el arco $(i,j) \in \mathcal{A}$
s: nodo inicial
t: nodo final

Variables:
x_{ij}: si el arco $(i,j) \in \mathcal{A}$ está en la ruta más corta entre s y t

Definición del Problema

Ruta más corta (uno a uno)

$$\min \sum_{(i,j)\in\mathcal{A}} c_{ij}x_{ij}$$

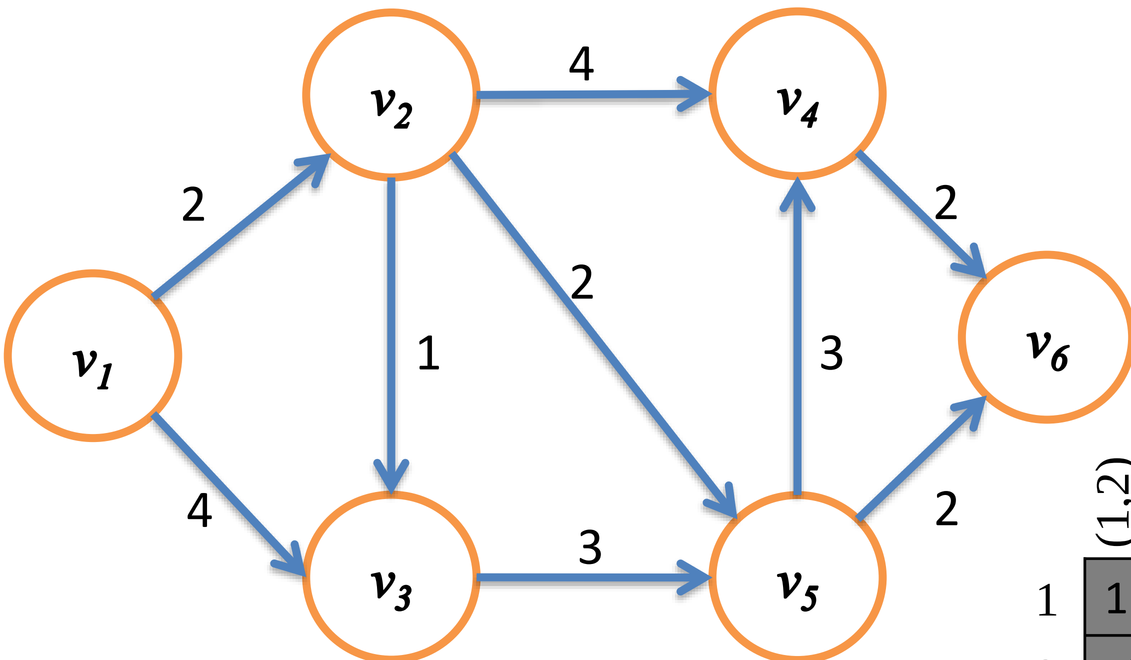
s.a.

$$\sum_{\{j|(i,j)\in\mathcal{A}\}} x_{ij} - \sum_{\{j|(j,i)\in\mathcal{A}\}} x_{ji} = \begin{cases} 1, i = s \\ 0, i \neq s,t \\ -1, i = t \end{cases} \quad \forall i \in \mathcal{N}$$

$$x_{ij} \geq 0 \ \ \forall (i,j) \in \mathcal{A}$$

Definición del Problema

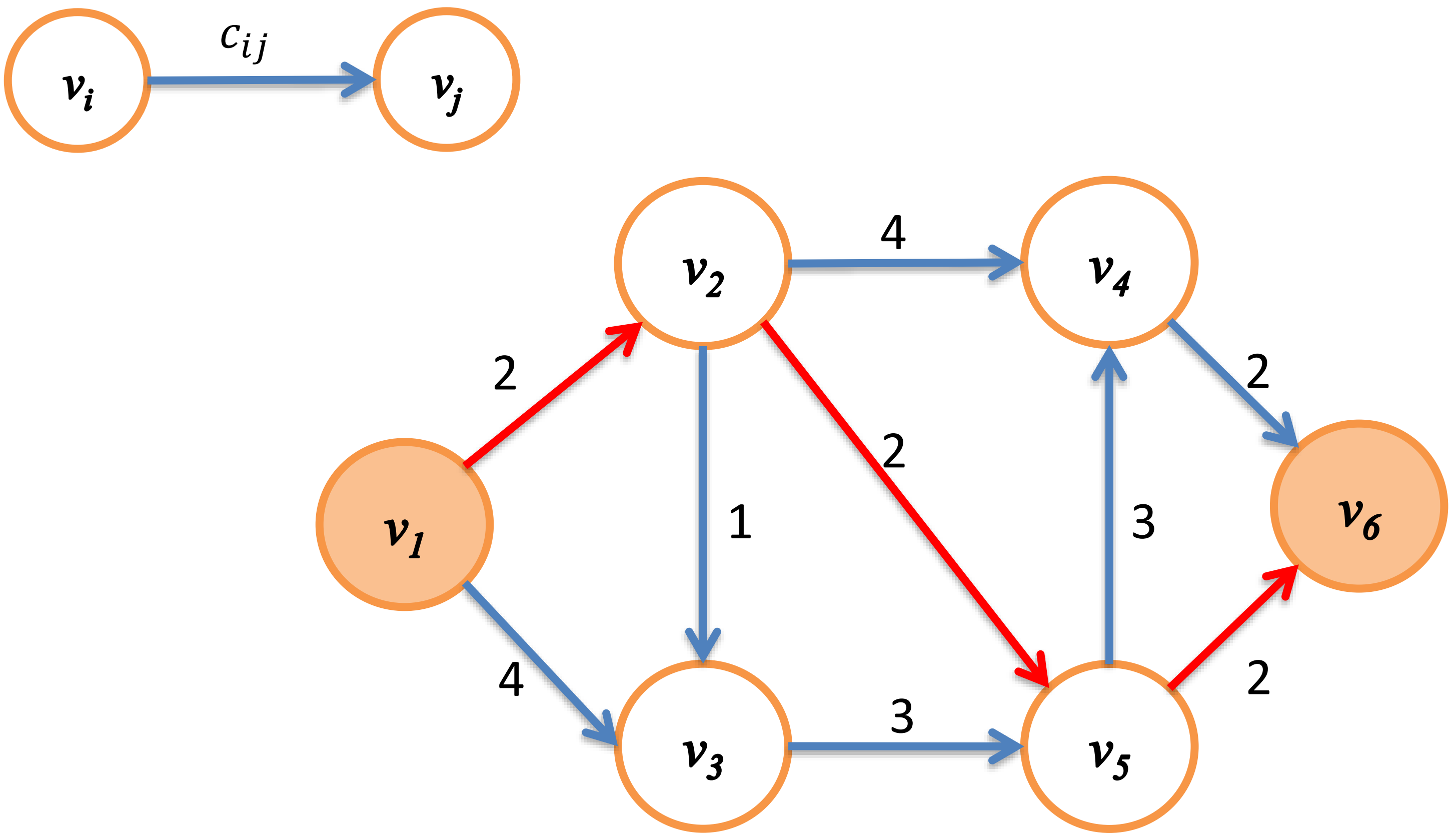
Ruta más corta (uno a uno)



	(1,2)	(1,3)	(2,3)	(2,4)	(2,5)	(3,5)	(4,6)	(5,4)	(5,6)
1	1	1							
2	-1		1	1	1				
3		-1	-1			1			
4				-1			1	-1	
5					-1	-1		1	1
6							-1		-1

Definición del Problema

Ruta más corta (uno a uno)



Definición del Problema

Ruta más corta (uno a todos)

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Definición del Problema
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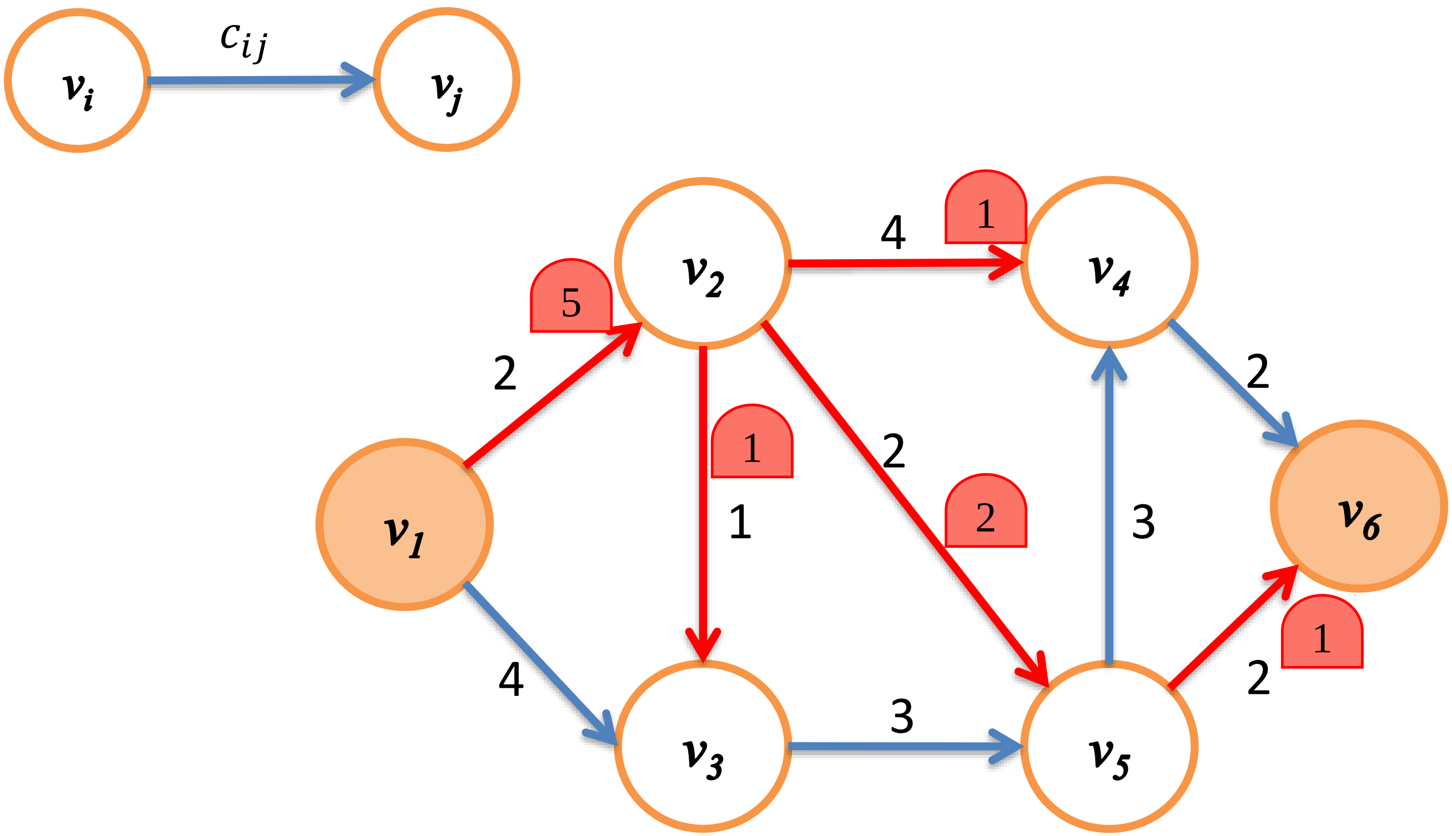
Definición del Problema

Ruta más corta (uno a todos)



Condiciones de optimalidad KKT

- Factibilidad primal
- Factibilidad dual
- Holgura complementaria

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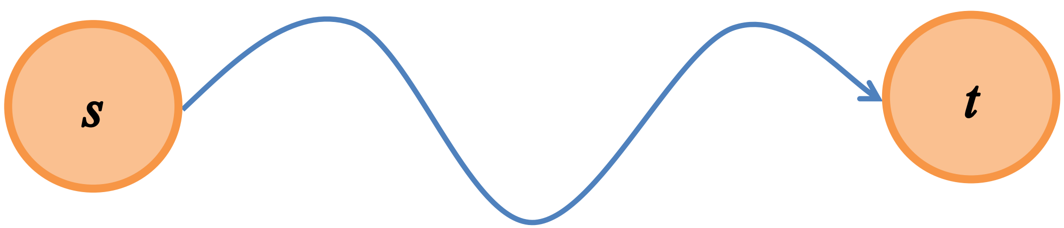

Condiciones de optimalidad KKT

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Dual del Problema de Ruta más Corta

$$\min \sum_{(i,j)\in\mathcal{A}} c_{ij}x_{ij}$$

s.a.

$$\sum_{\{j|(i,j)\in\mathcal{A}\}} x_{ij} - \sum_{\{j|(j,i)\in\mathcal{A}\}} x_{ji} = \begin{cases} |\mathcal{N}| - 1, i = s \\ -1, i \neq s \end{cases} \quad \forall i \in \mathcal{N} \qquad \boxed{[\pi_i]}$$

$$x_{ij} \geq 0 \ \ \forall (i,j) \in \mathcal{A}$$

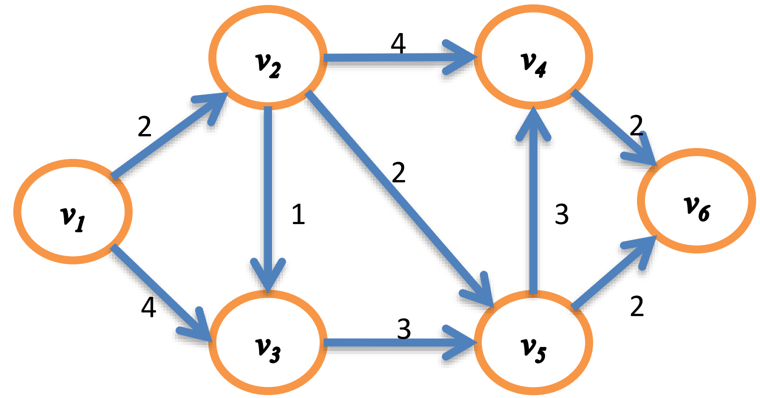
Dual del Problema de Ruta más Corta

$$\max \sum_{i \in \mathcal{N}} b_i \pi_i$$

s.a.

$$\pi_i - \pi_j \leq c_{ij} \;\forall (i,j) \in \mathcal{A}$$

$$\pi_i \; libre \;\; \forall\, i \in \mathcal{N}$$



	(1,2)	(1,3)	(2,3)	(2,4)	(2,5)	(3,5)	(4,6)	(5,4)	(5,6)
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4				-1			1	-1	
5					-1	-1		1	1
6							-1		-1

Dual del Problema de Ruta más Corta

$$\max \sum_{i \in \mathcal{N} \setminus \{s\}} -\pi_i + (|\mathcal{N}| - 1)\pi_s$$

s.a.

$$\pi_i - \pi_j \leq c_{ij} \;\forall (i,j) \in \mathcal{A}$$

$$\pi_i \; libre \;\; \forall \; i \in \mathcal{N}$$

Dual del Problema de Ruta más Corta

$$\max \sum_{i \in \mathcal{N} \setminus \{s\}} (\pi_s - \pi_i)$$

s.a.

$$\pi_i - \pi_j \leq c_{ij} \; \forall (i,j) \in \mathcal{A}$$

$$\pi_i \; libre \;\; \forall \, i \in \mathcal{N}$$

¿si π_i es un solución, entonces $\pi_i + c$ es una solución?

Dual del Problema de Ruta más Corta

- Si, con $\pi_s = 0$

$$\max \sum_{i \in \mathcal{N} \setminus \{s\}} -\pi_i$$

s.a.

$$\pi_i - \pi_j \leq c_{ij} \ \forall (i,j) \in \mathcal{A}$$

$$\pi_i \ libre \ \ \forall \ i \in \mathcal{N}$$

Dual del Problema de Ruta más Corta

- Reformulando, asuma $d(i) = -\pi_i \ \forall i \in \mathcal{N}$

$$\max \sum_{i \in \mathcal{N} \setminus \{s\}} -\pi_i$$

s.a.

$$\pi_i - \pi_j \leq c_{ij} \ \forall (i,j) \in \mathcal{A}$$

$$\pi_i \ libre \ \ \forall \ i \in \mathcal{N}$$

Dual del Problema de Ruta más Corta

- Reformulando, asuma $d(i) = -\pi_i \ \ \forall i \in \mathcal{N}$

$$\max \sum_{i \in \mathcal{N} \setminus \{s\}} d(i)$$

s.a.

$$d(j) - d(i) \leq c_{ij} \ \forall (i,j) \in \mathcal{A} \qquad \text{y } d(s) = 0$$

$$d(i) \ \ libre \ \ \forall \ i \in \mathcal{N}$$

Condiciones de optimalidad KKT

- Factibilidad primal
- Factibilidad dual
- Holgura complementaria

$$d(j) - d(i) \leq c_{ij} \; \forall (i,j) \in \mathcal{A}$$

Condiciones de optimalidad KKT

- Factibilidad primal
- Factibilidad dual
- Holgura complementaria

$$\mathbf{w}^T(\mathbf{Ax} - \mathbf{b}) = 0$$

$$(\mathbf{c}^T - \mathbf{w}^T\mathbf{A})\mathbf{x} = 0$$

Condiciones de optimalidad KKT

- Factibilidad primal
- Factibilidad dual
- Holgura complementaria

$$\mathbf{w}^T(\mathbf{A}\mathbf{x}-\mathbf{b}) = 0$$

0

$$(\mathbf{c}^T - \mathbf{w}^T\mathbf{A})\mathbf{x} = 0$$

?

Dual del Problema de Ruta más Corta

Volviendo al dual …

$$\max \sum_{i \in \mathcal{N} \setminus \{s\}} d(i)$$

s.a.

$$d(j) - d(i) \leq c_{ij} \; \forall (i,j) \in \mathcal{A}$$

$$d(i) \;\; libre \;\; \forall \; i \in \mathcal{N}$$

$$\max \sum_{i \in \mathcal{N} \setminus \{s\}} d(i)$$

s.a.

$$\bar{c}_{ij} = c_{ij} + d(i) - d(j) \; \forall (i,j) \in \mathcal{A}$$

$$d(i) \;\; libre \;\; \forall \; i \in \mathcal{N}$$

Condiciones de optimalidad KKT

- Factibilidad primal
- Factibilidad dual
- Holgura complementaria

$$\boxed{(\mathbf{c}^T - \mathbf{w}^T\mathbf{A})}\,\mathbf{x} = 0$$

$$\boxed{\bar{c}_{ij} = c_{ij} + d(i) - d(j) \qquad \forall (i,j) \in \mathcal{A}}$$

- Si $x_{ij} > 0 \ \rightarrow \bar{c}_{ij} = 0$ (costo reducido de una variable básica es cero)
- Si $x_{ij} = 0 \ \rightarrow \bar{c}_{ij} \geq 0$ (costo reducido de una variable no básica es no negativo)

Condiciones de optimalidad

Teorema 5.1 (Condiciones de optimalidad en ruta más corta):
Asuma que $d(j)$ representa la longitud de <u>algún</u> camino dirigido desde el nodo raíz hasta el nodo $j \in \mathcal{N}$. Entonces, los valores $d(j)$ representan la ruta más corta si y solo si se satisface que:

$$d(j) \leq d(i) + c_{ij} \qquad \forall (i,j) \in \mathcal{A}$$

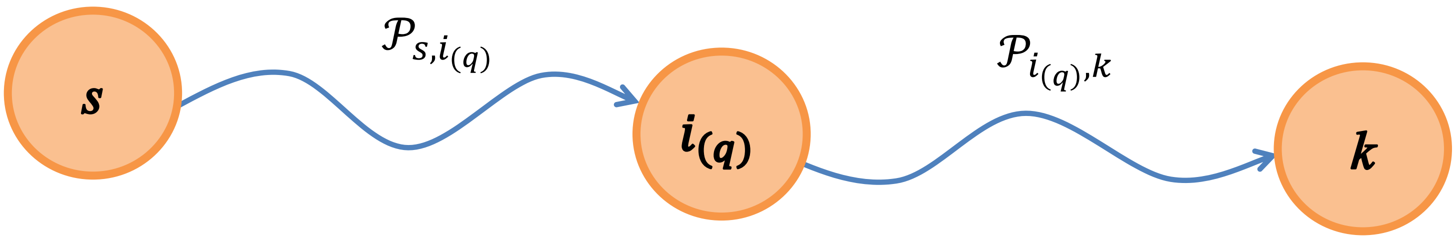
Ahuja et al. (1993), p. 136

Propiedades de RMC

Propiedad 1:

Si el camino $\{s = i_{(1)}, i_{(2)}, \dots, i_{(h)} = k\}$ es un camino de mínimo costo, entonces para todo $q = 2, \dots, h-1$, el subcamino $\{s = i_{(1)}, \dots, i_{(q)}\}$ es la ruta más corta de s a $i_{(q)}$

$$\mathcal{P}^* = \mathcal{P}_{s,i_{(q)}} \cup \mathcal{P}_{i_{(q)},k}$$



Propiedades de RMC

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Prueba:

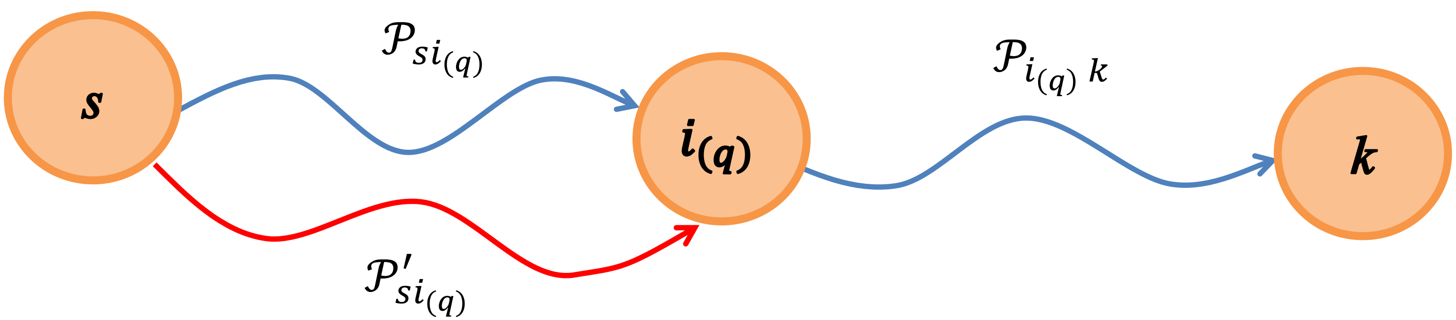
- Asuma que $\mathcal{P}^* = \mathcal{P}_{si_{(q)}} \cup \mathcal{P}_{i_{(q)}k}$ es la ruta más corta de s a k con costo $c(\mathcal{P}^*)$.
- Asuma además que existe un subcamino $\mathcal{P}'_{si_{(q)}}$ de s a $i_{(q)}$ tal que $c\left(\mathcal{P}'_{si_{(q)}}\right) \leq c\left(\mathcal{P}_{si_{(q)}}\right)$

Entonces, $\mathcal{P}^*$ no es la ruta más corta dado que es posible construir un camino $\mathcal{P}' = \mathcal{P}'_{si_{(q)}} \cup \mathcal{P}_{i_{(q)}k}$ tal que $c(\mathcal{P}') \leq c(\mathcal{P}^*)$, lo cual contradice el supuesto inicial.

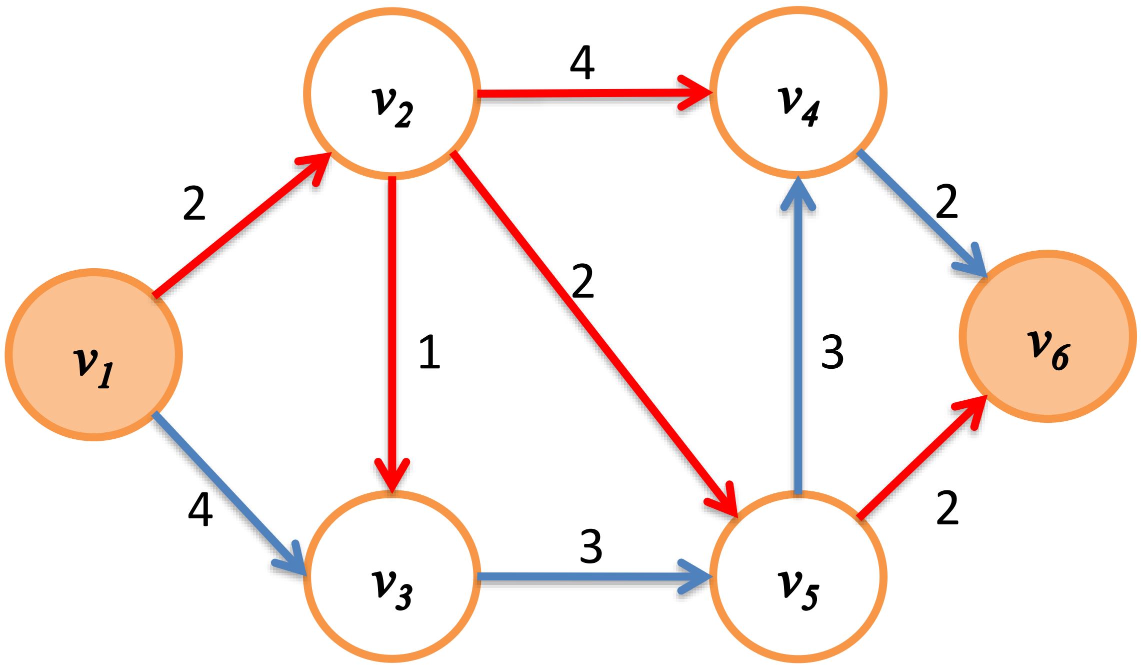
Propiedades de RMC

Propiedad 1:

Si el camino $\{s = i_{(1)}, i_{(2)}, \dots, i_{(h)} = k\}$ es un camino de mínimo costo, entonces para todo $q = 2, \dots, h-1$, el subcamino $\{s = i_{(1)}, \dots, i_{(q)}\}$ es la ruta más corta de s a $i_{(q)}$



Propiedades de RMC



Ruta más corta de 1 a **6** → 1,2,5, **6**
Ruta más corta de 1 a **2** → 1, **2**
Ruta más corta de 1 a **5** → 1,2, **5**

Propiedades de RMC

Propiedad 2:

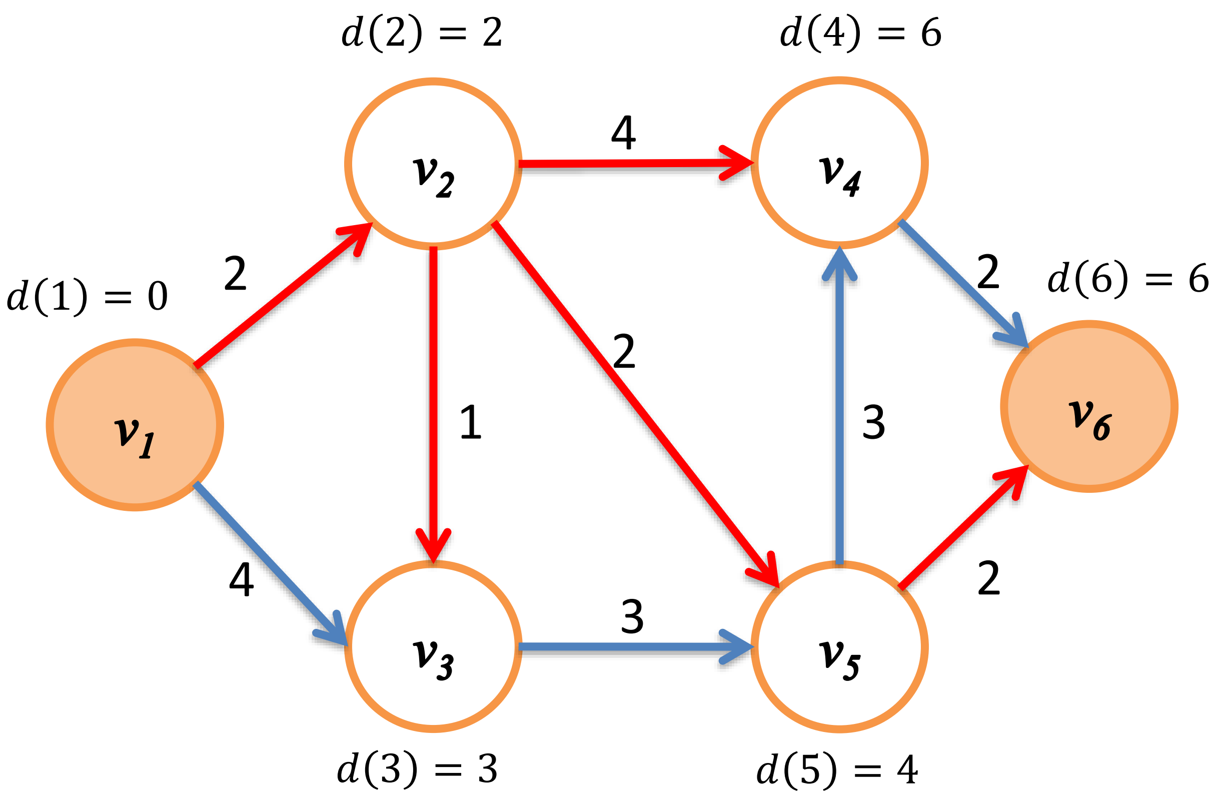
Si $\mathbf{d}$ es un vector de distancias mínimas (i.e., $d(i)\ \ \forall i \in \mathcal{N}$), entonces un camino de $s \in \mathcal{N}$ a $k \in \mathcal{N}$ es un camino de mínima distancia si y sólo si:

$$d(j) = d(i) + c_{ij} \quad \forall\ (i,j) \in \mathcal{P}_{sk}$$

$$\Downarrow$$

$$x_{ij} > 0 \rightarrow \bar{c}_{ij} = 0$$

Propiedades de RMC



$d(1) = 0$

$d(2) = d(1) + c_{12}$

$d(3) = d(2) + c_{23}$

$d(4) = d(2) + c_{24}$

$d(5) = d(2) + c_{25}$

$d(6) = d(5) + c_{56}$

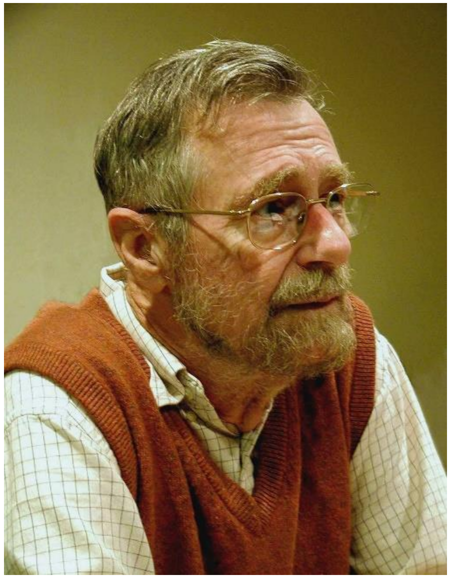
$d(3) \leq d(1) + c_{13}$

$d(4) \leq d(5) + c_{54}$

$d(5) \leq d(3) + c_{35}$

$d(6) \leq d(4) + c_{46}$

Edsger W. Dijkstra



Dijkstra was the 1972 recipient of the ACM Turing Award, often viewed as the Nobel Prize for computing. He was a member of the Netherlands Royal Academy of Arts and Sciences, a member of the American Academy of Arts and Sciences, and a Distinguished Fellow of the British Computer Society. He received the 1974 AFIPS Harry Goode Award, the 1982 IEEE Computer Pioneer Award, and the 1989 ACM SIGCSE Award for Outstanding Contributions to Computer Science Education. Athens University of Economics awarded him an honorary doctorate in 2001. In 2002, the C&C Foundation of Japan recognized Dijkstra "for his pioneering contributions to the establishment of the scientific basis for computer software through creative research in basic software theory, algorithm theory, structured programming, and semaphores".

He is well known for his amazingly efficient shortest path algorithm and for having designed and coded the first Algol 60 compiler. He was famously the leader in the abolition of the GOTO statement from programming.

fuente: http://www.cs.utexas.edu/users/EWD/

Algoritmo de Dijkstra

- $G(\mathcal{N}, \mathcal{A})$: grafo dirigido
- $\mathcal{N}$: conjunto de nodos
- $\mathcal{A}$: conjunto de arcos
- c_{ij}: costo del arco $(i, j) \in \mathcal{A}$
- $\mathcal{S}$: conjunto de nodos etiquetados permanentemente
- $\bar{\mathcal{S}}$: conjunto de nodos etiquetados temporalmente
- s: nodo inicial
- $d(i)$: etiqueta del nodo i
- $p(i)$: nodo que precede al nodo i
- $\mathcal{A}(i)$: conjunto de arcos de la estrella delantera para el nodo i

Algoritmo de Dijkstra

- Supuesto:

$$\boxed{c_{ij} \geq 0 \ \forall (i,j) \in \mathcal{A}}$$

- Ruta más corta de uno a todos

Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$

$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$

$d(s) \leftarrow 0,\ p(s) \leftarrow 0$

while $|\mathcal{S}| < |\mathcal{N}|$ **do**

  $i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}} \{d(j)\}$

  $d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}} \{d(j)\}$

  $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$

  $\bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}$

  for $(i, j) \in \mathcal{A}(i)$ **do**

    if $d(j) > d(i) + c_{ij}$ **then**

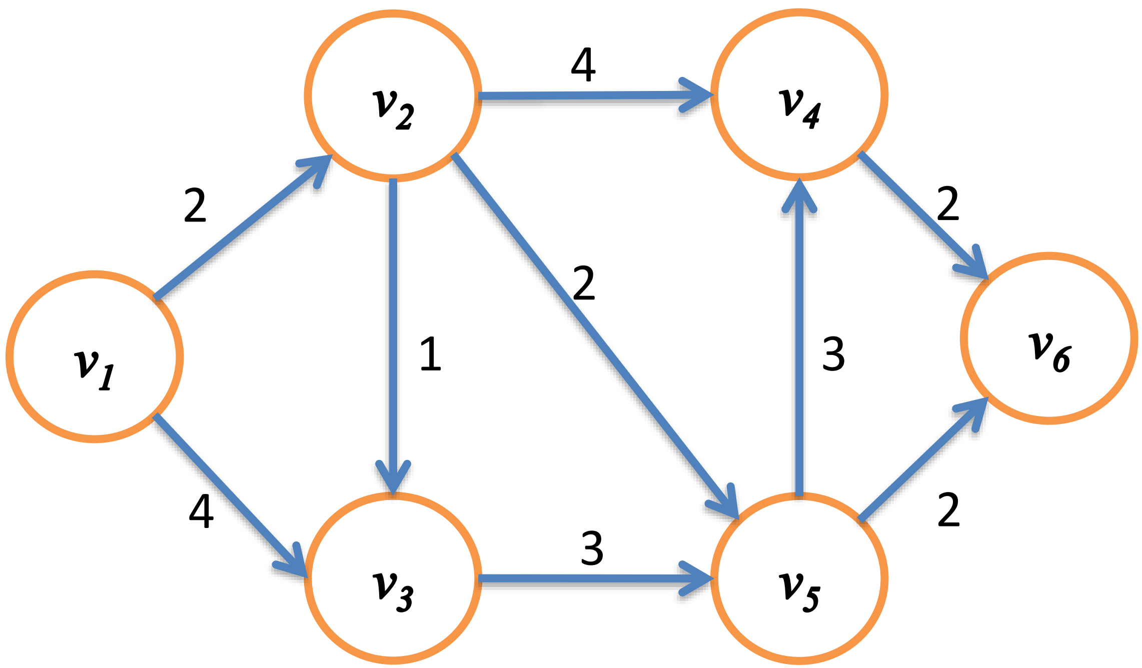
      $d(j) \leftarrow d(i) + c_{ij}$

      $p(j) \leftarrow i$

    end if

  end for

end while



Algoritmo de Dijkstra

Ejercicio en clase

PARE

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ \ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ \ p(s) \leftarrow 0$
while $|\mathcal{S}| < |\mathcal{N}|$ **do**
  $i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
  $d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
  $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
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  for $(i, j) \in \mathcal{A}(i)$ **do**
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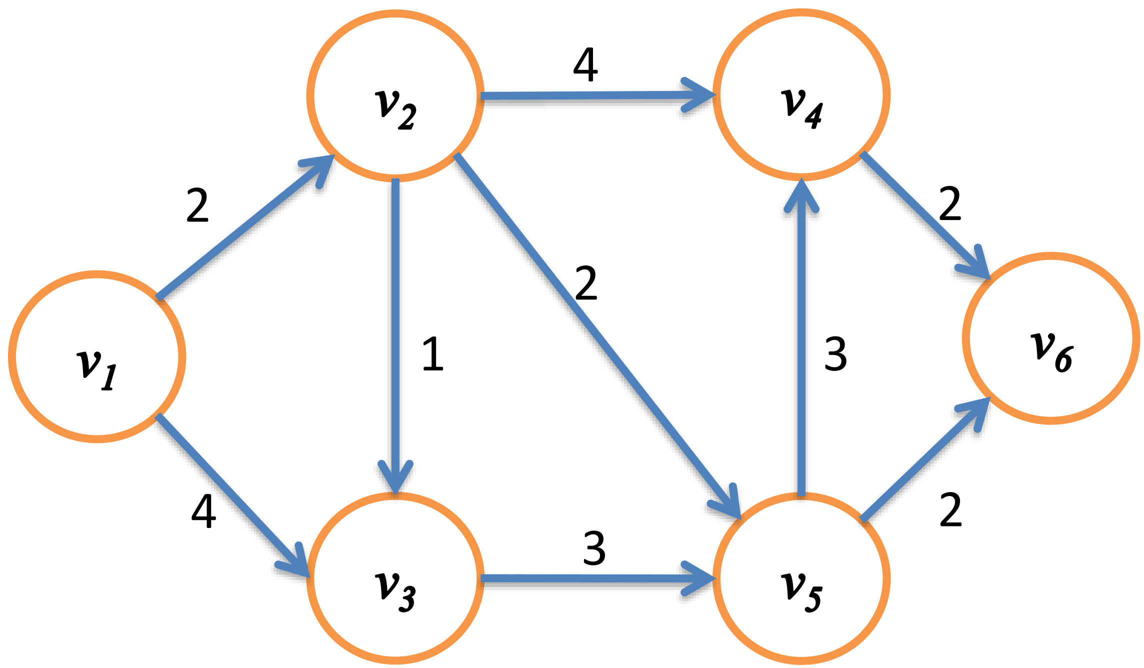

ALGORITMO DE DIJKSTRA

EJERCICIO EN CLASE

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$

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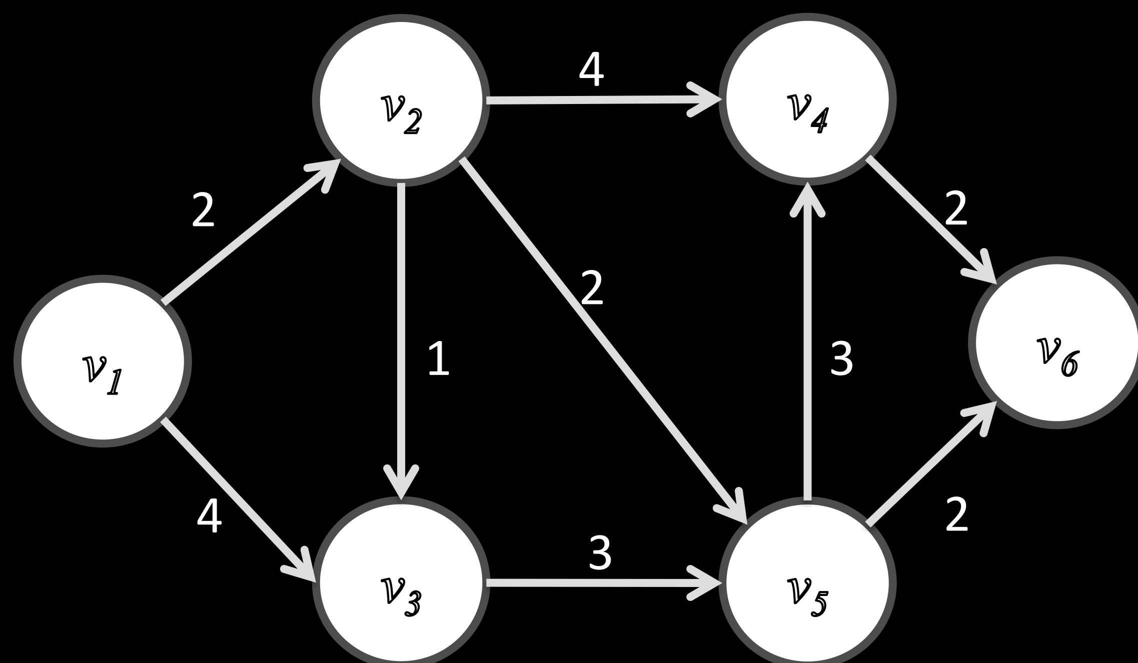

Algoritmo de Dijkstra

$\mathcal{S} = \{\quad\}$ $\qquad\qquad$ $\bar{\mathcal{S}} = \{1,2,3,4,5,6\}$

$\boxed{\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
while $|\mathcal{S}| < |\mathcal{N}|$ **do**
  $i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
  $d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
  $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
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  for $(i,j) \in \mathcal{A}(i)$ **do**
    if $d(j) > d(i) + c_{ij}$ **then**
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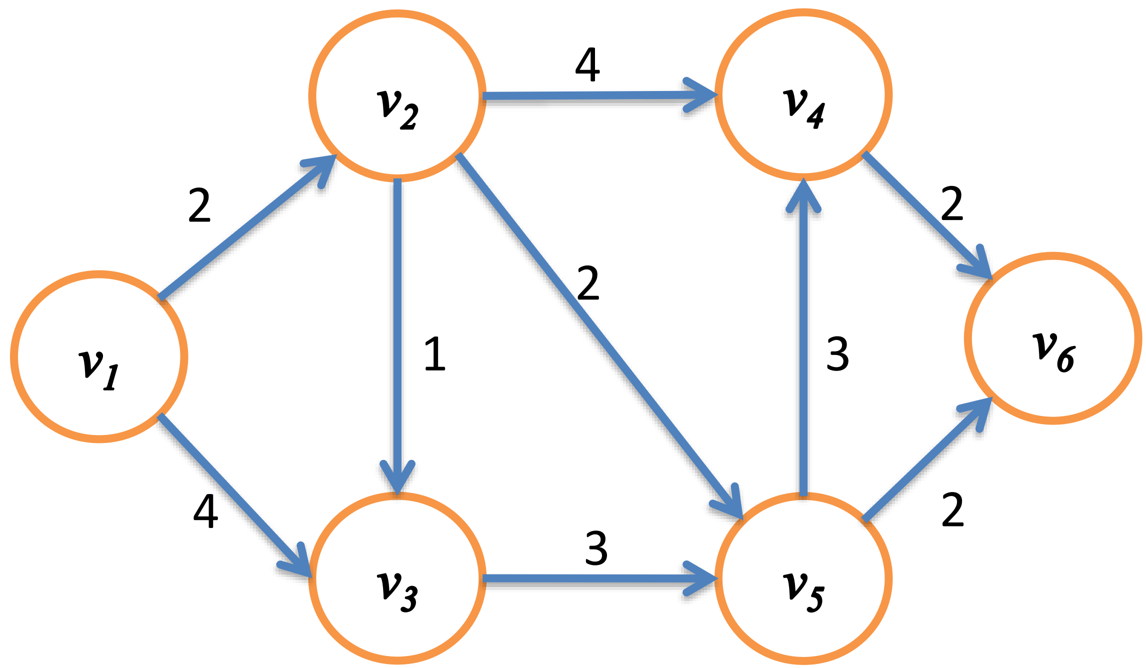
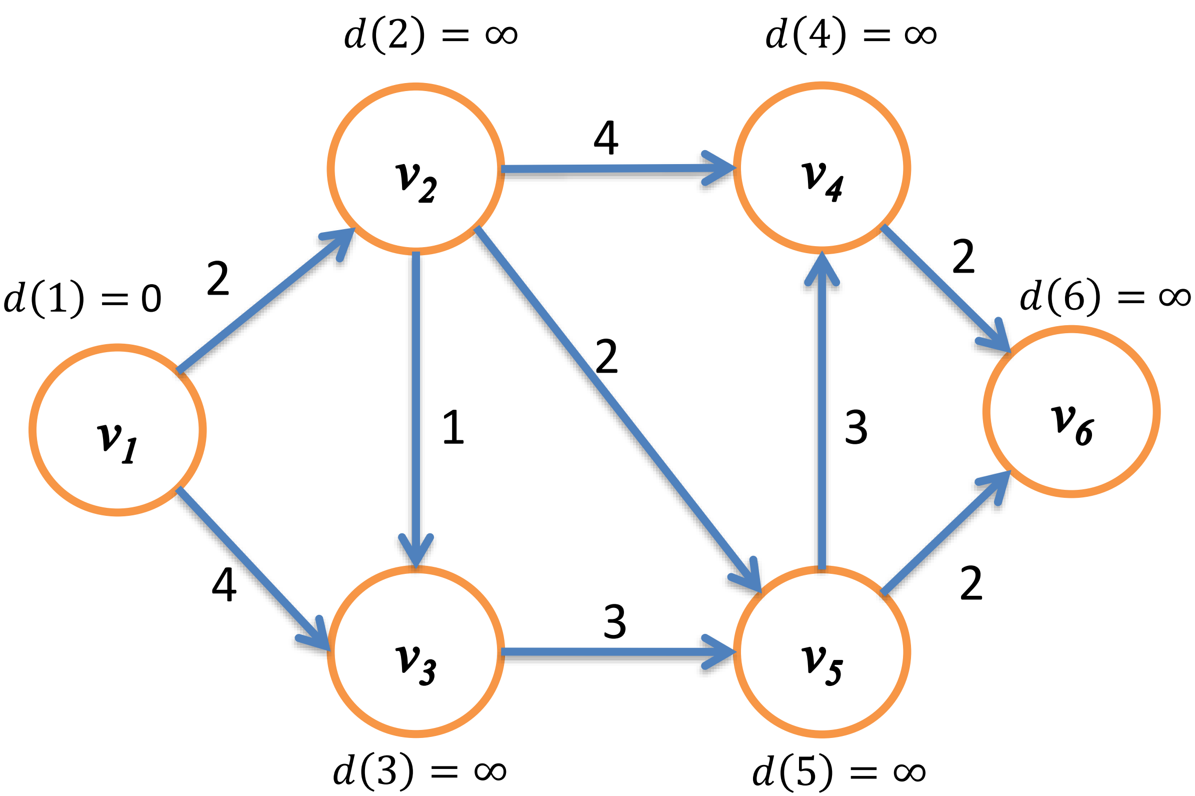
  end for
end while



Algoritmo de Dijkstra

$\mathcal{S} = \{\quad\}$ $\qquad$ $\bar{\mathcal{S}} = \{1,2,3,4,5,6\}$

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
while $|\mathcal{S}| < |\mathcal{N}|$ **do**
    $i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
    $d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
    $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
    $\bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}$
    for $(i,j) \in \mathcal{A}(i)$ **do**
        if $d(j) > d(i) + c_{ij}$ **then**
            $d(j) \leftarrow d(i) + c_{ij}$
            $p(j) \leftarrow i$
        end if
    end for
end while



Algoritmo de Dijkstra

$\mathcal{S} = \{\quad\}$ $\qquad$ $\bar{\mathcal{S}} = \{1,2,3,4,5,6\}$

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
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$\boxed{\textbf{while } |\mathcal{S}| < |\mathcal{N}| \textbf{ do}}$ $\qquad$ ¿0 < 6?
$\quad i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
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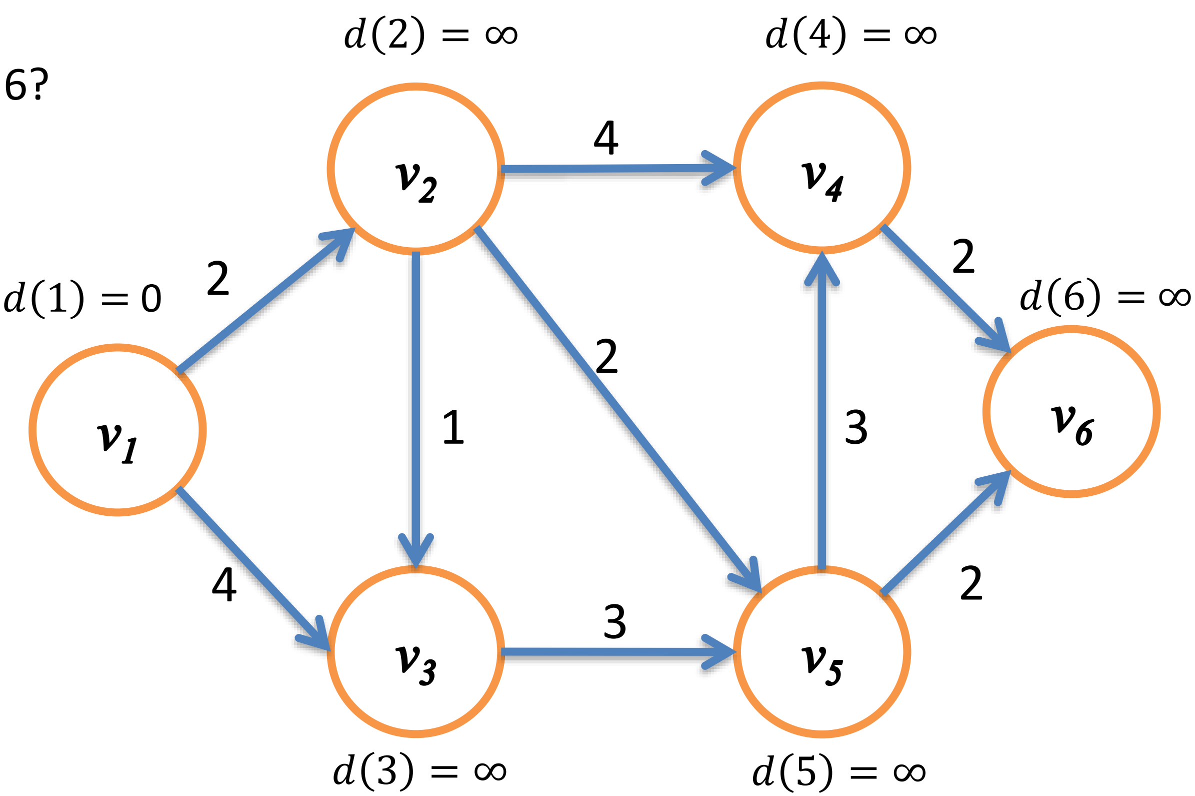
$\quad\quad\quad p(j) \leftarrow i$
$\quad\quad$ **end if**
$\quad$ **end for**
end while



Algoritmo de Dijkstra

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end while

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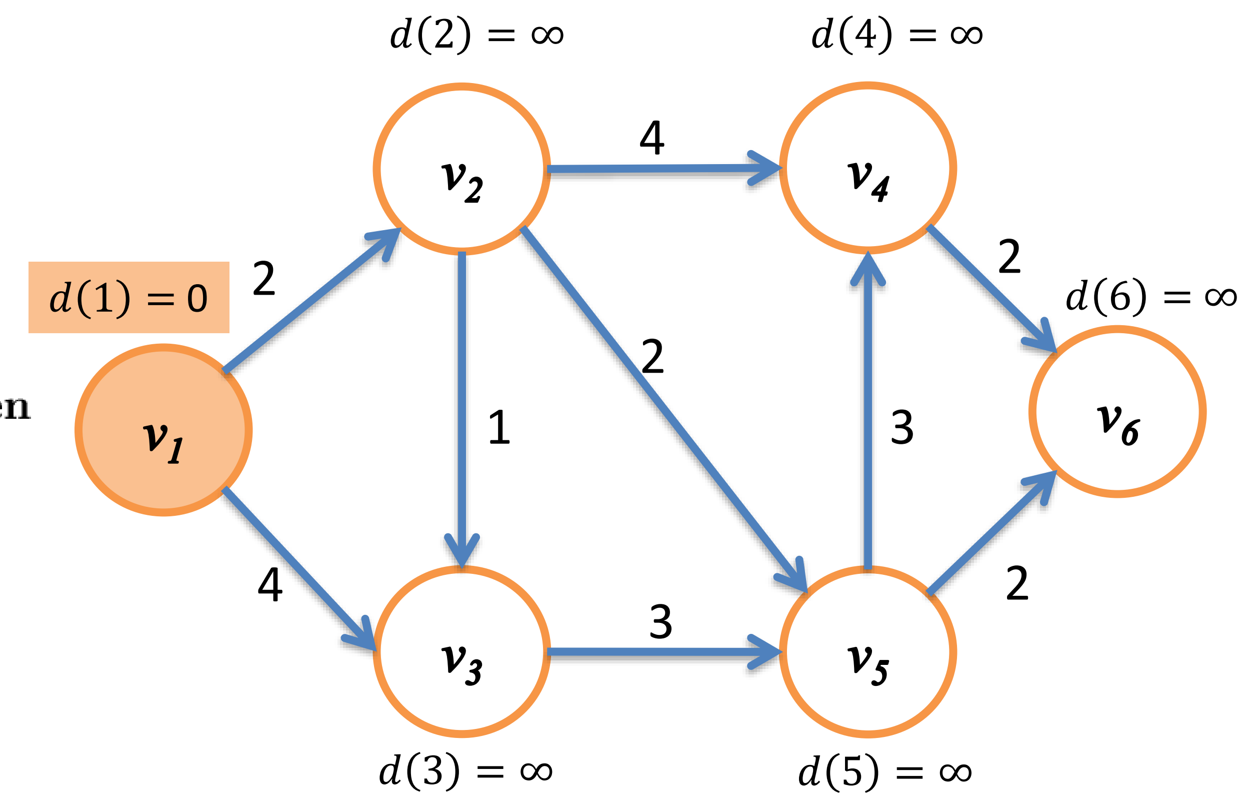

Algoritmo de Dijkstra

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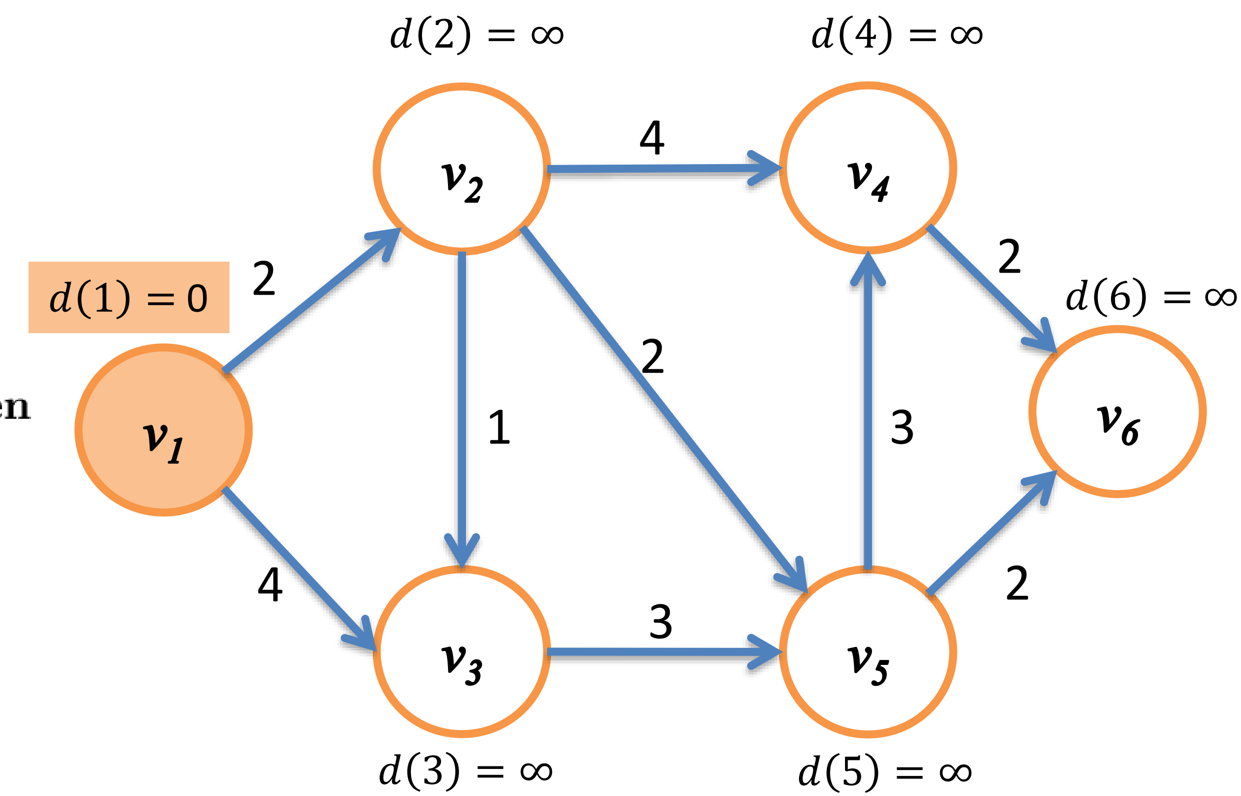
$\boxed{\mathcal{S} = \{1\}}$ $\boxed{\bar{\mathcal{S}} = \{2,3,4,5,6\}}$



Algoritmo de Dijkstra

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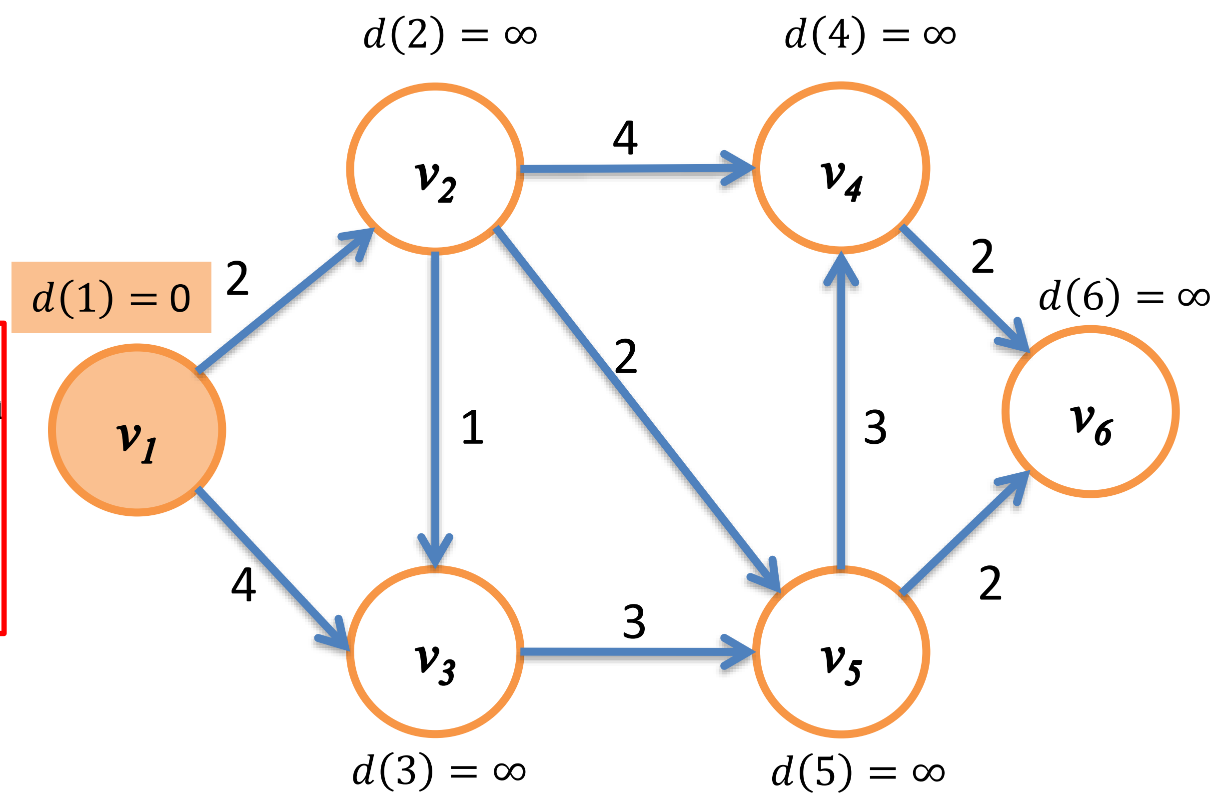

Algoritmo de Dijkstra

Universidad de los Andes Colombia

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
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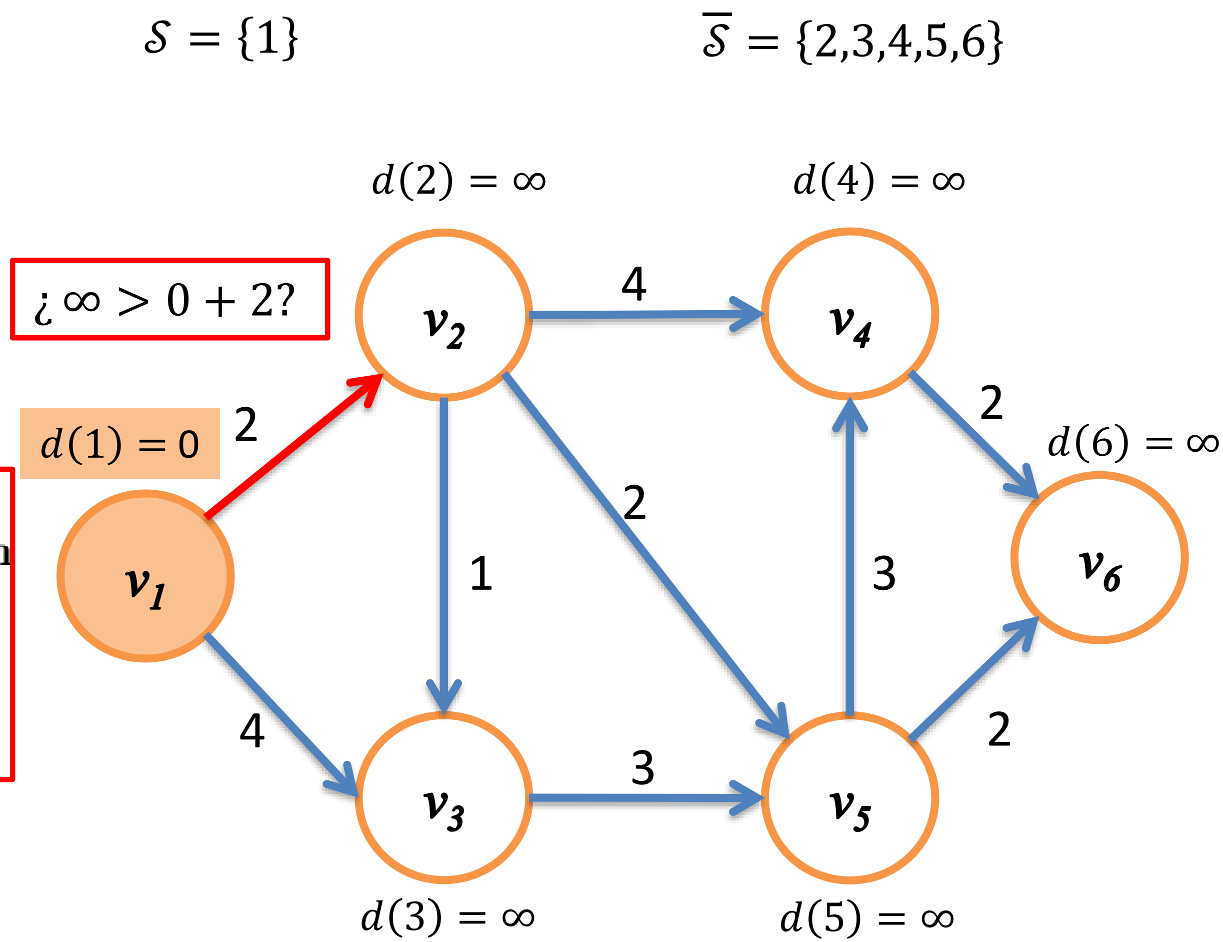

Algoritmo de Dijkstra

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$$
\begin{array}{l}
\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N} \\
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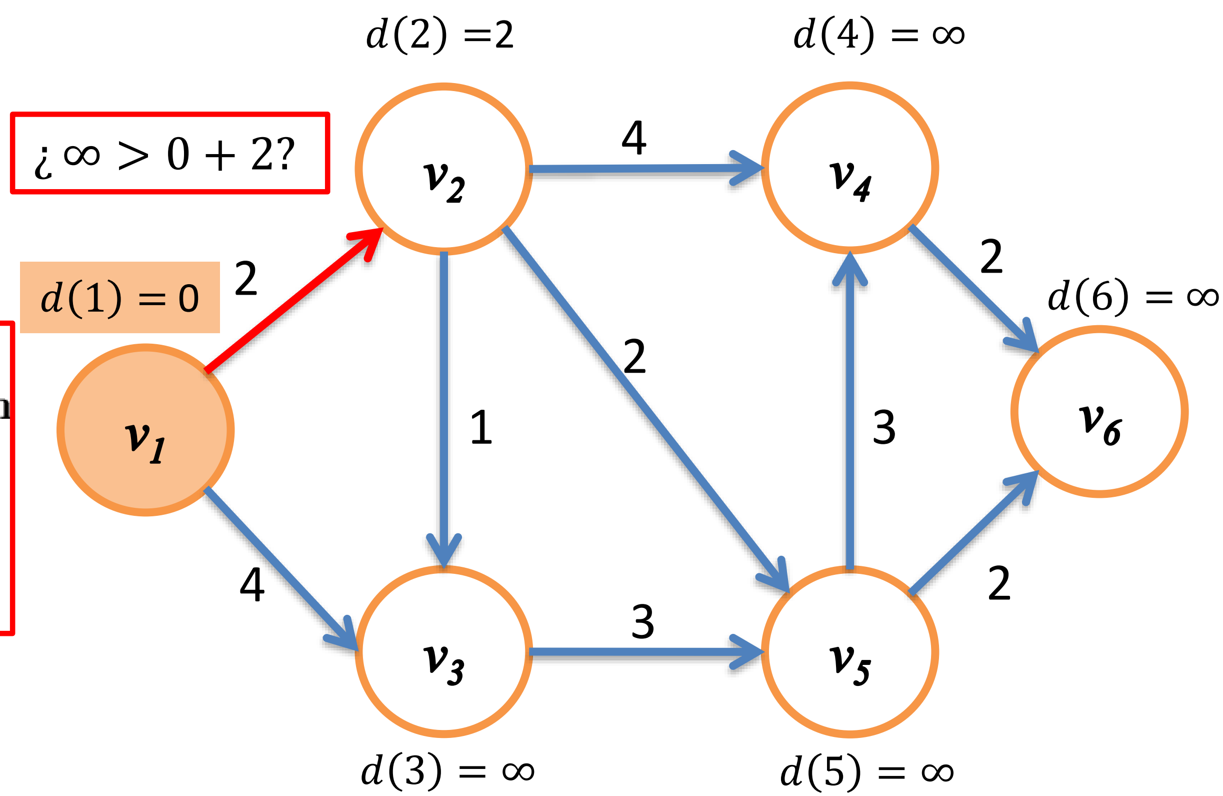
\quad\quad \textbf{end if} \\
\quad \textbf{end for} \\
\textbf{end while}
\end{array}
$$



Algoritmo de Dijkstra

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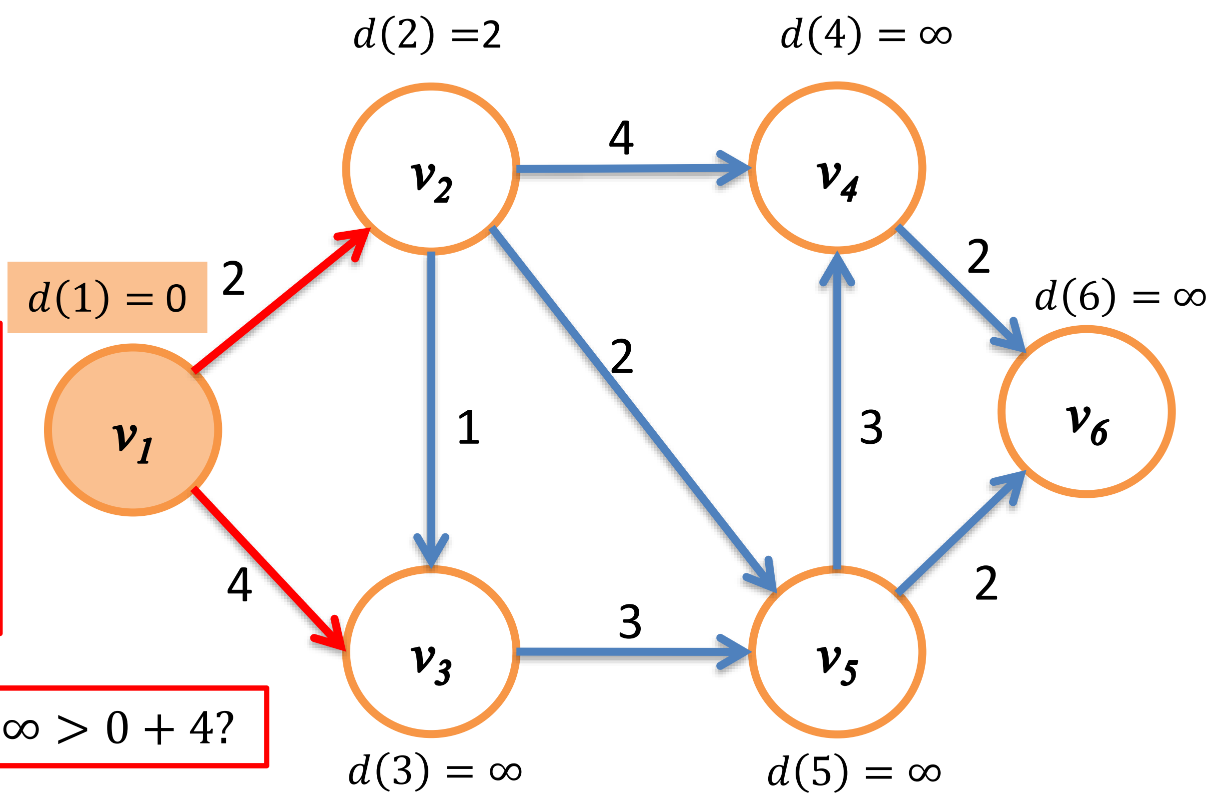
  end for
end while



$¿\infty > 0 + 4?$

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Universidad de los Andes Colombia

Algoritmo de Dijkstra

$\mathcal{S} = \{1\}$ $\quad\quad$ $\bar{\mathcal{S}} = \{2,3,4,5,6\}$

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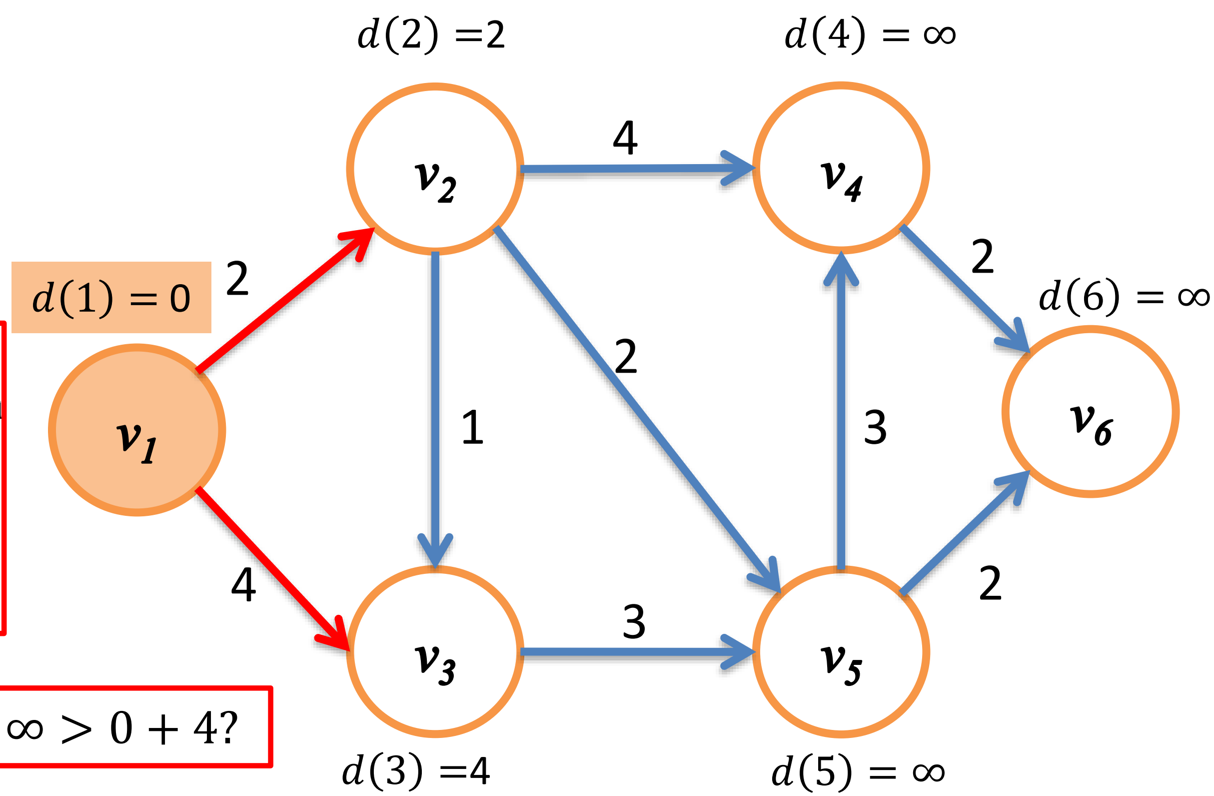
$\quad\quad$ **end if**
$\quad$ **end for**
end while



¿∞ > 0 + 4?

Algoritmo de Dijkstra

$\mathcal{S} = \{1\}$ $\qquad$ $\bar{\mathcal{S}} = \{2,3,4,5,6\}$

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
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$\boxed{\textbf{while } |\mathcal{S}| < |\mathcal{N}| \textbf{ do}}$
$\quad i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
$\quad d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
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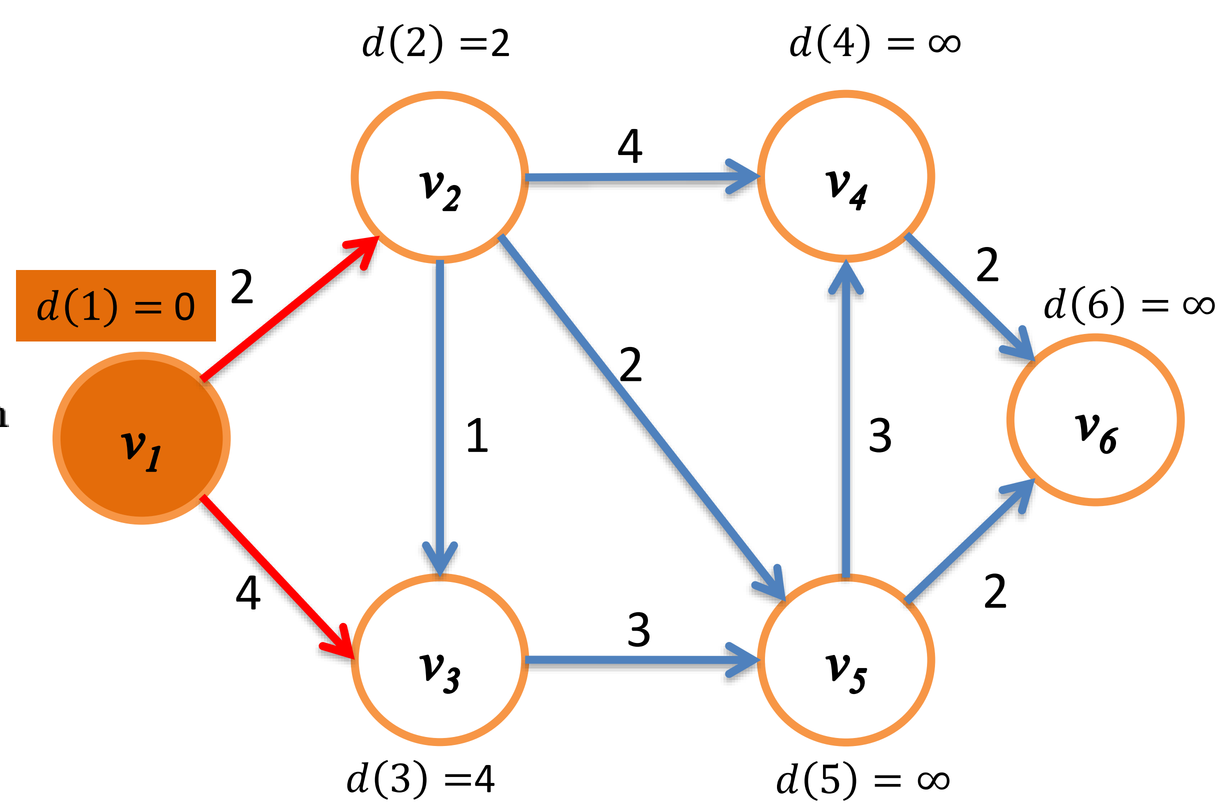
$\qquad \textbf{end if}$
$\quad \textbf{end for}$
$\textbf{end while}$



Algoritmo de Dijkstra

$\mathcal{S} = \{1\}$ $\qquad$ $\bar{\mathcal{S}} = \{2,3,4,5,6\}$

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
while $|\mathcal{S}| < |\mathcal{N}|$ **do**
$\boxed{\begin{array}{l} i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}}\{d(j)\} \\ d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}}\{d(j)\} \end{array}}$
  $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
  $\bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}$
  for $(i,j) \in \mathcal{A}(i)$ **do**
    if $d(j) > d(i) + c_{ij}$ **then**
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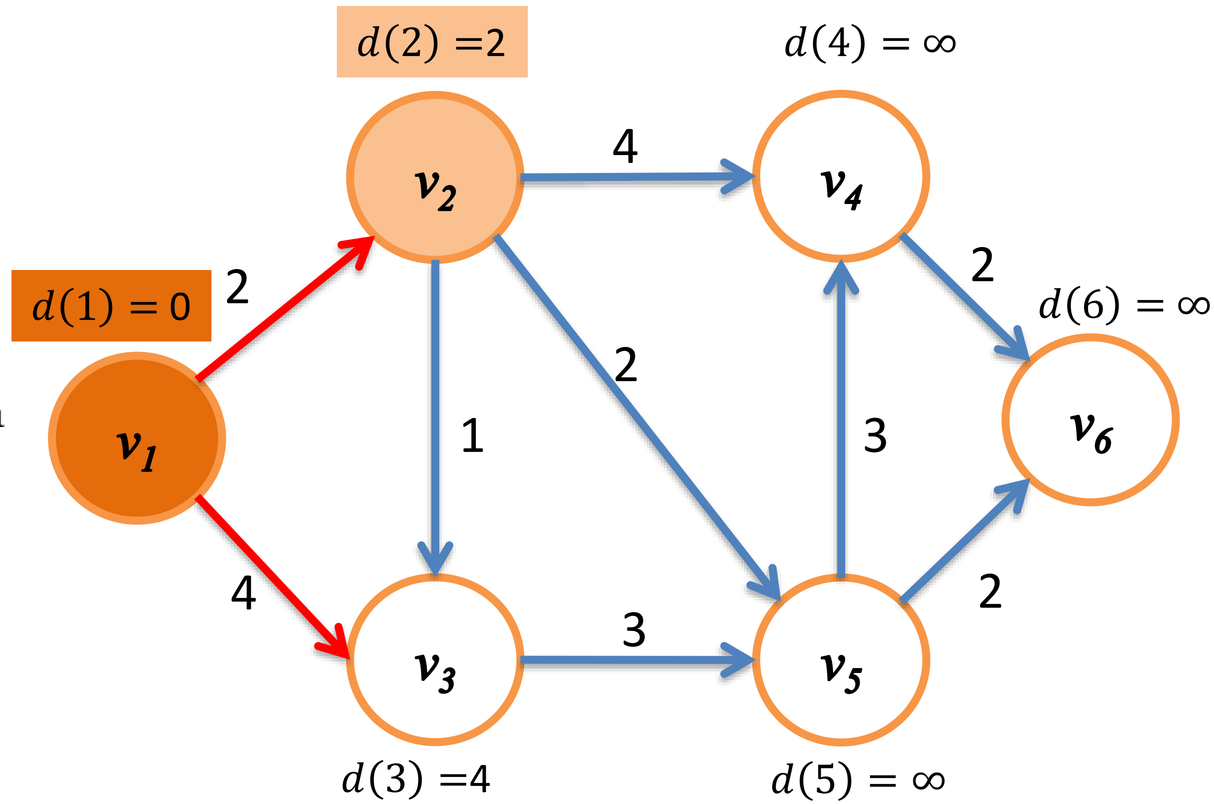
      $p(j) \leftarrow i$
    end if
  end for
end while



Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
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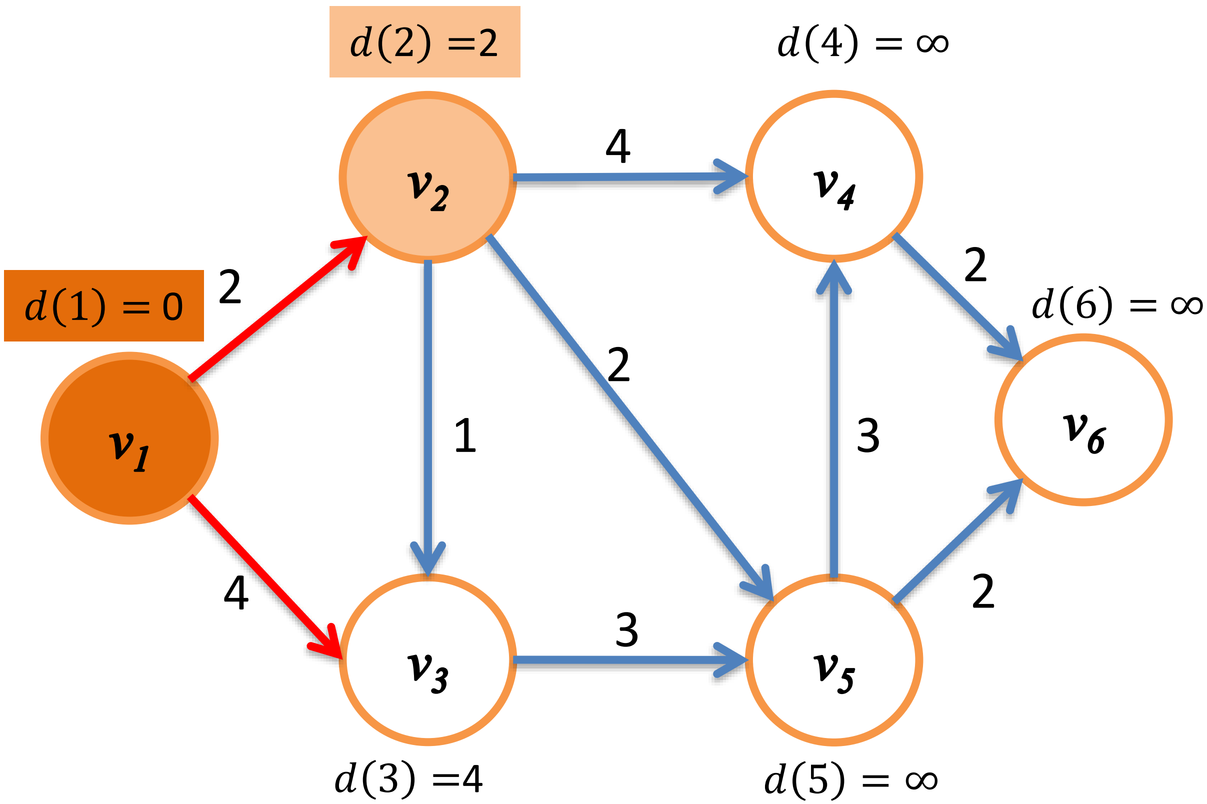
    end if
  end for
end while

$\boxed{\mathcal{S} = \{1,2\}}$ $\boxed{\bar{\mathcal{S}} = \{3,4,5,6\}}$



Algoritmo de Dijkstra

$\mathcal{S} = \{1,2\}$ $\quad\quad$ $\bar{\mathcal{S}} = \{3,4,5,6\}$

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
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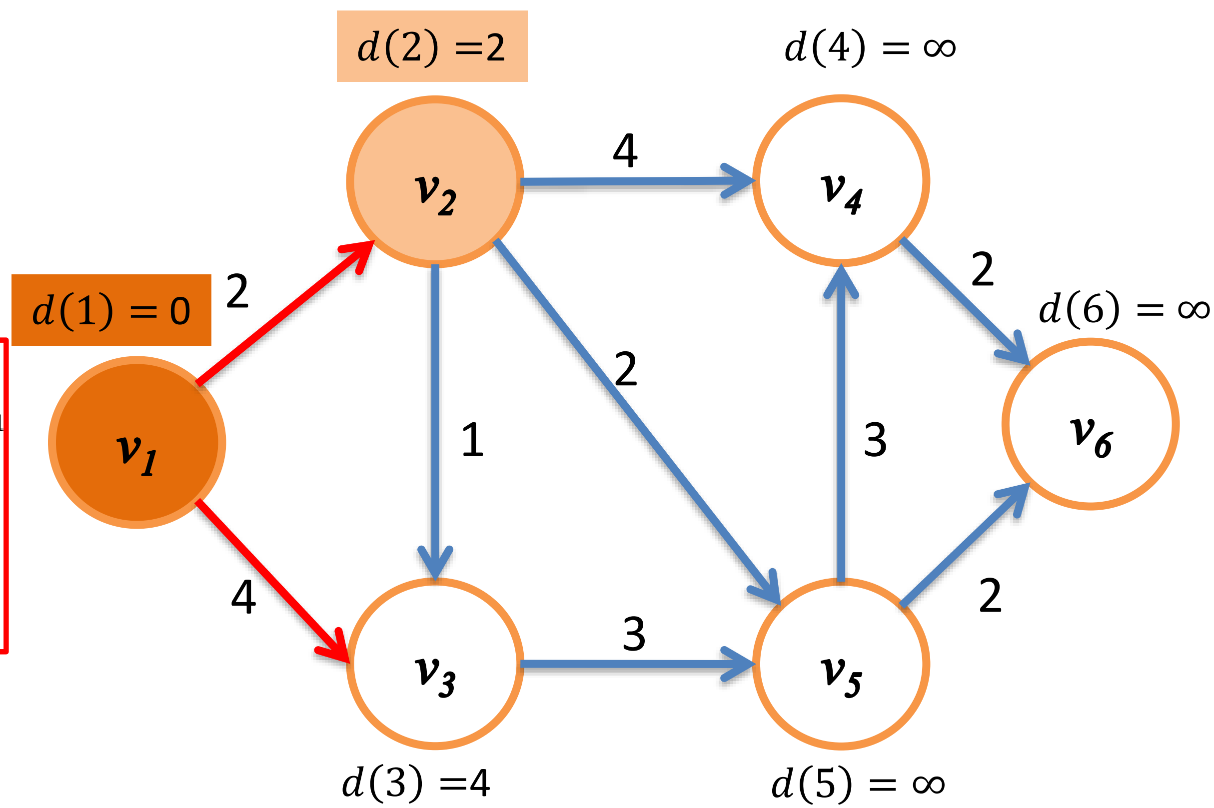

Algoritmo de Dijkstra

$\mathcal{S} = \{1{,}2\}$ $\qquad$ $\bar{\mathcal{S}} = \{3{,}4{,}5{,}6\}$

```
S ← ∅, S̄ ← N
d(i) ← ∞, i ∈ N, i ≠ s
d(s) ← 0, p(s) ← 0
while |S| < |N| do
    i ← arg min_{j∈S̄} {d(j)}
    d(i) ← min_{j∈S̄} {d(j)}
    S ← S ∪ {i}
    S̄ ← S̄ \ {i}
    for (i, j) ∈ A(i) do
        if d(j) > d(i) + c_ij then
            d(j) ← d(i) + c_ij
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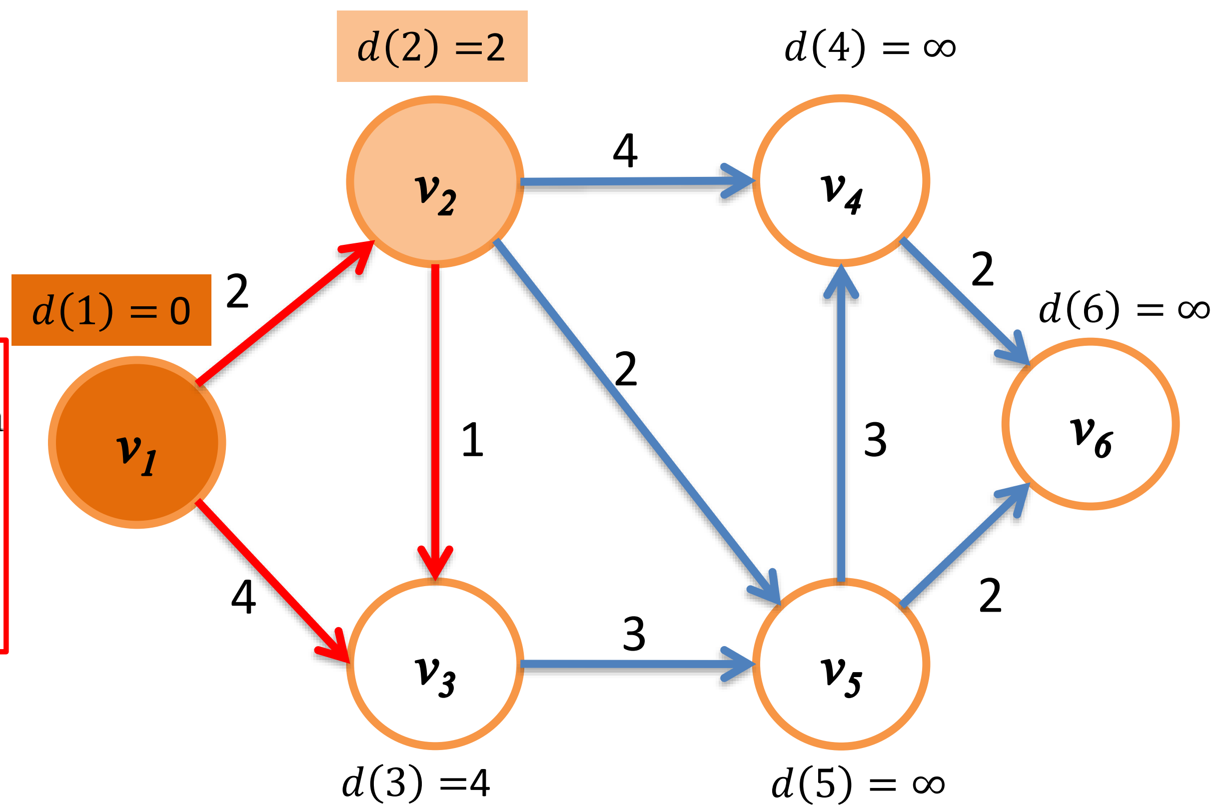
        end if
    end for
end while
```



Algoritmo de Dijkstra

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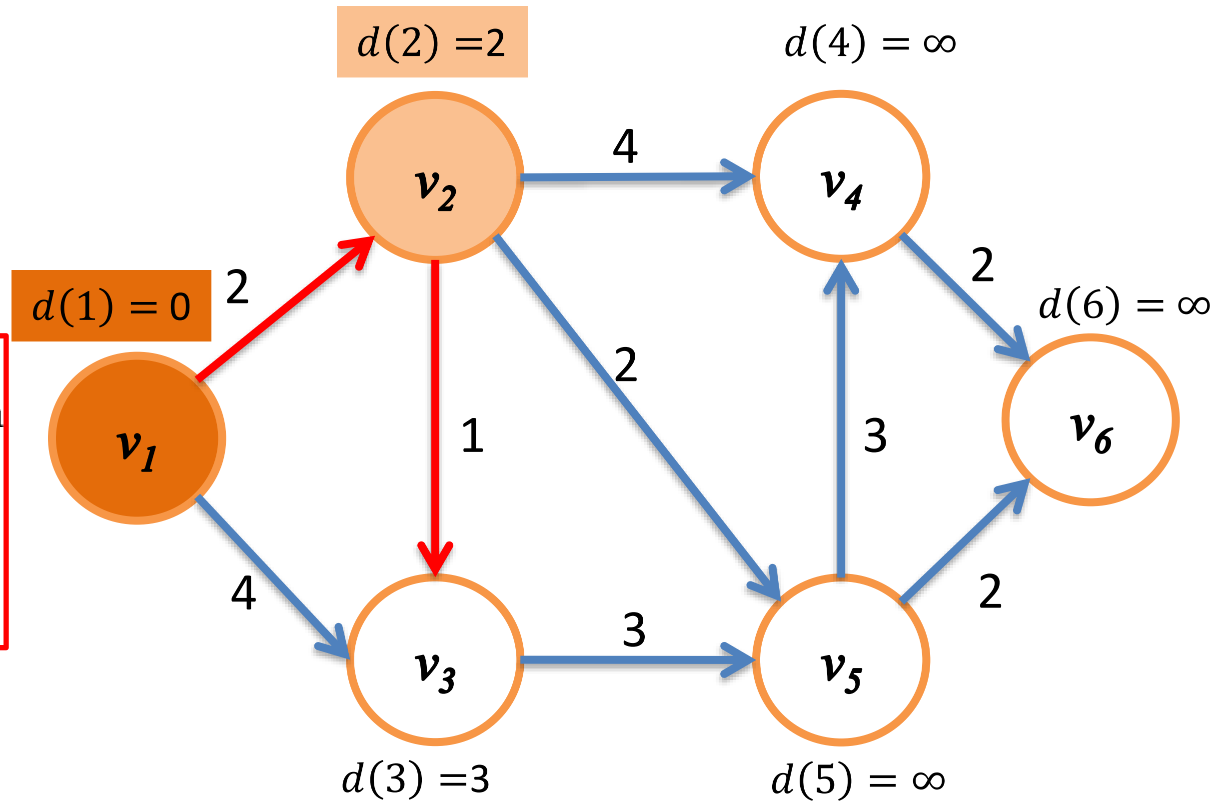

Algoritmo de Dijkstra

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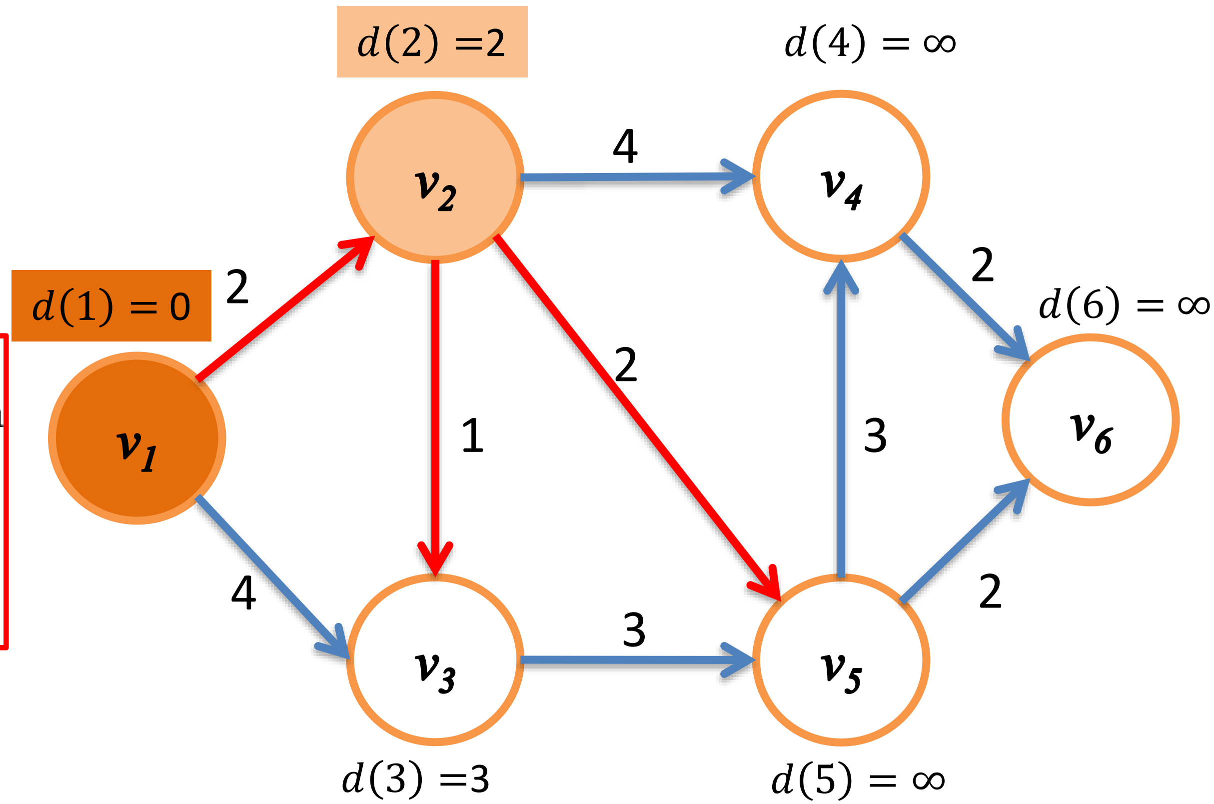

Algoritmo de Dijkstra

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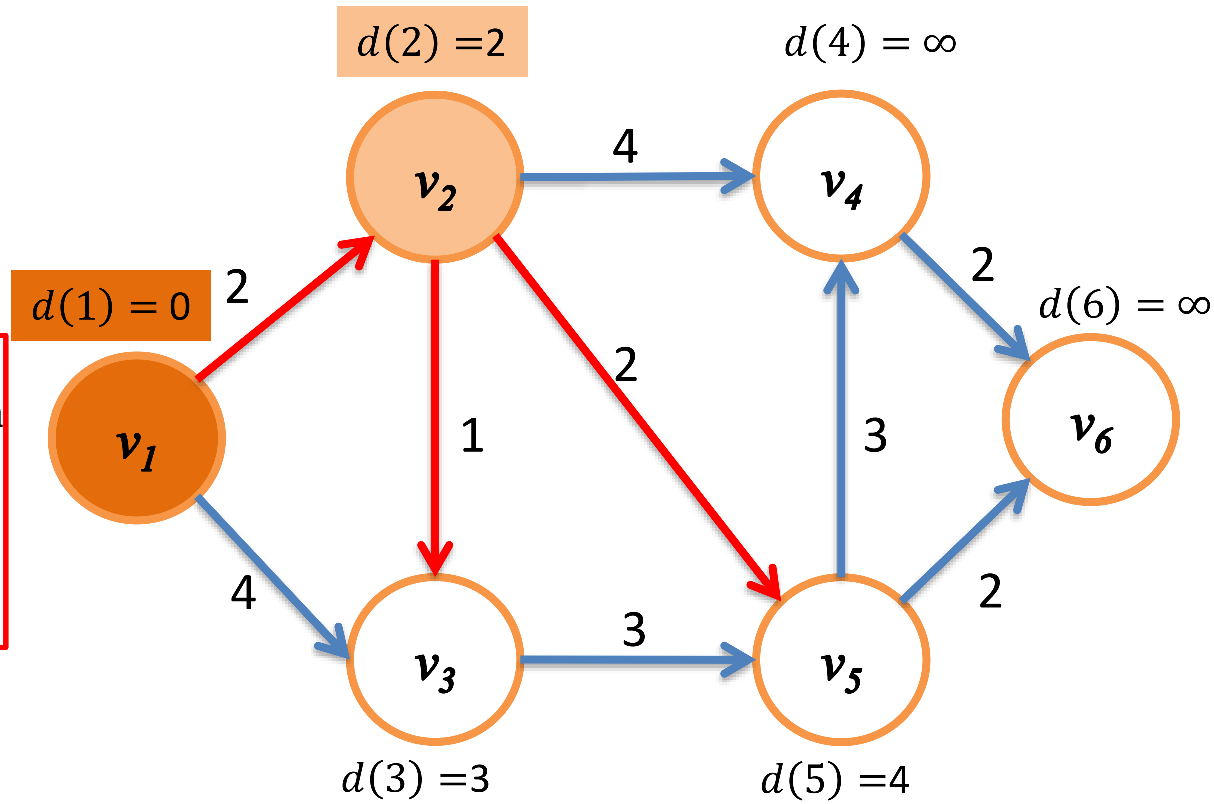

Algoritmo de Dijkstra

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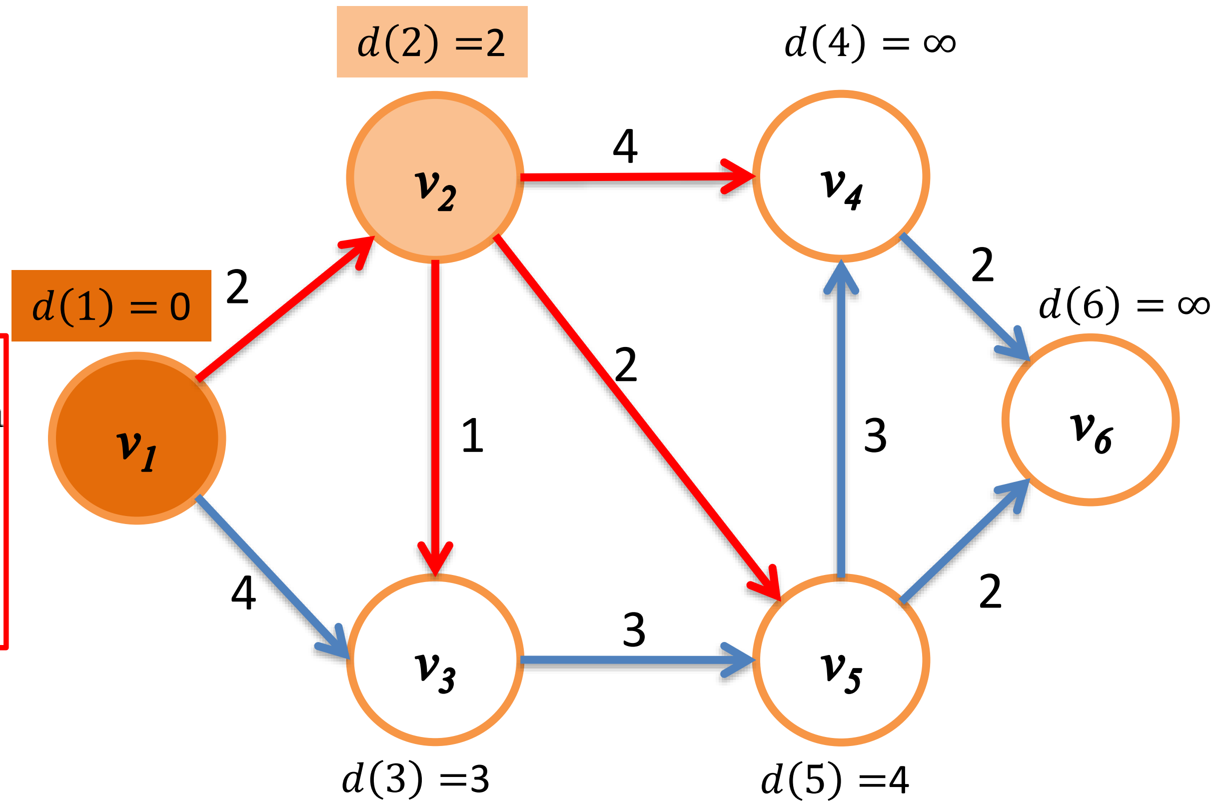

Algoritmo de Dijkstra

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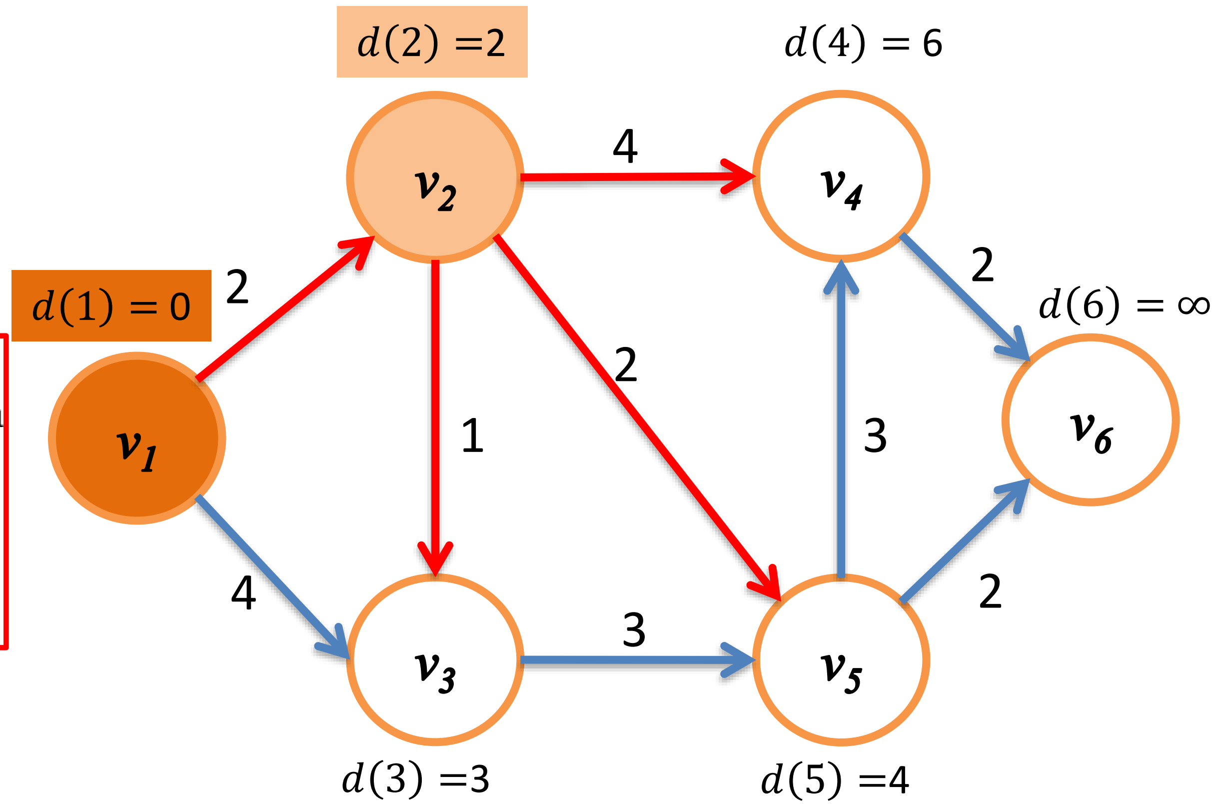
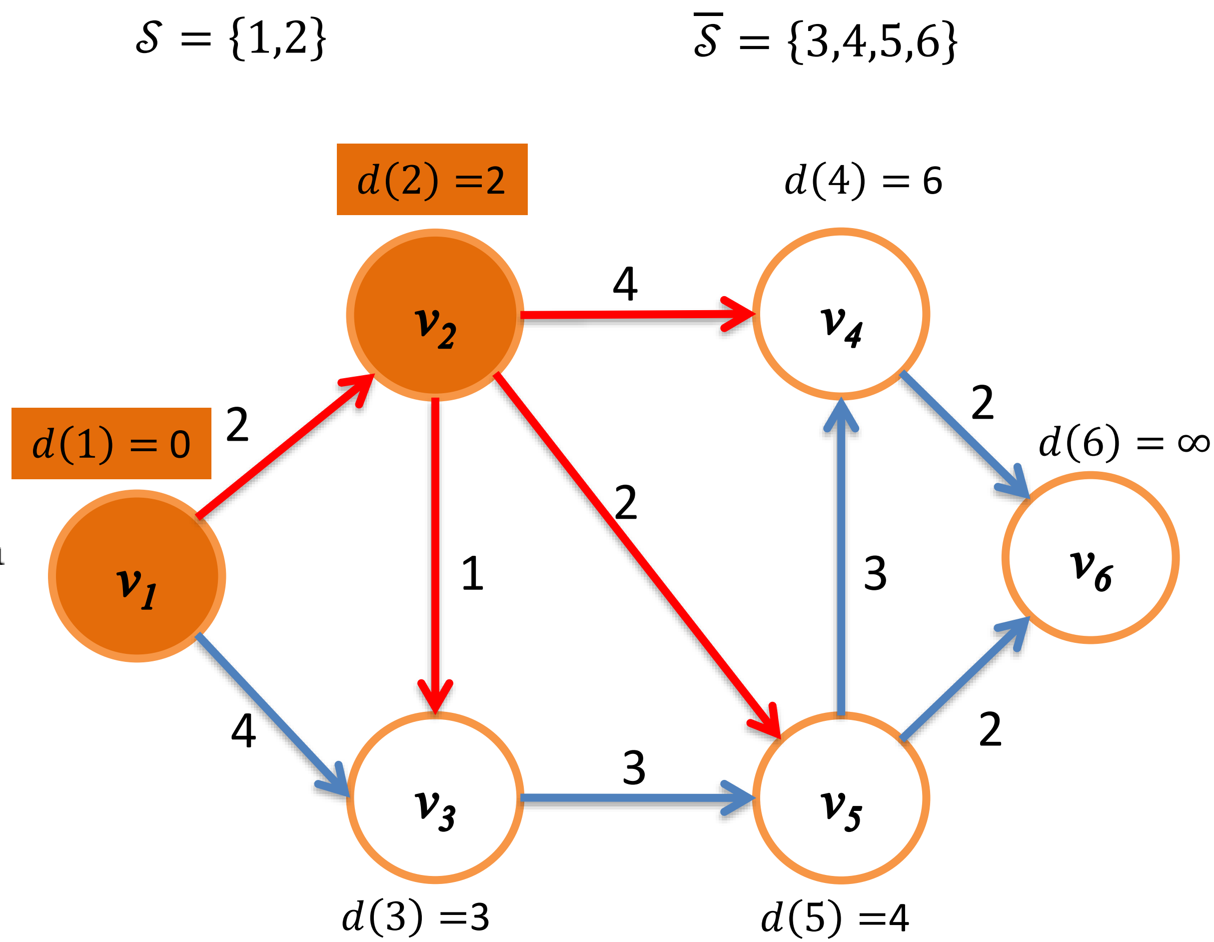
    d(i) ← min_{j∈S̄} {d(j)}
    S ← S ∪ {i}
    S̄ ← S̄ \ {i}
    for (i, j) ∈ A(i) do
        if d(j) > d(i) + c_ij then
            d(j) ← d(i) + c_ij
            p(j) ← i
        end if
    end for
end while
```



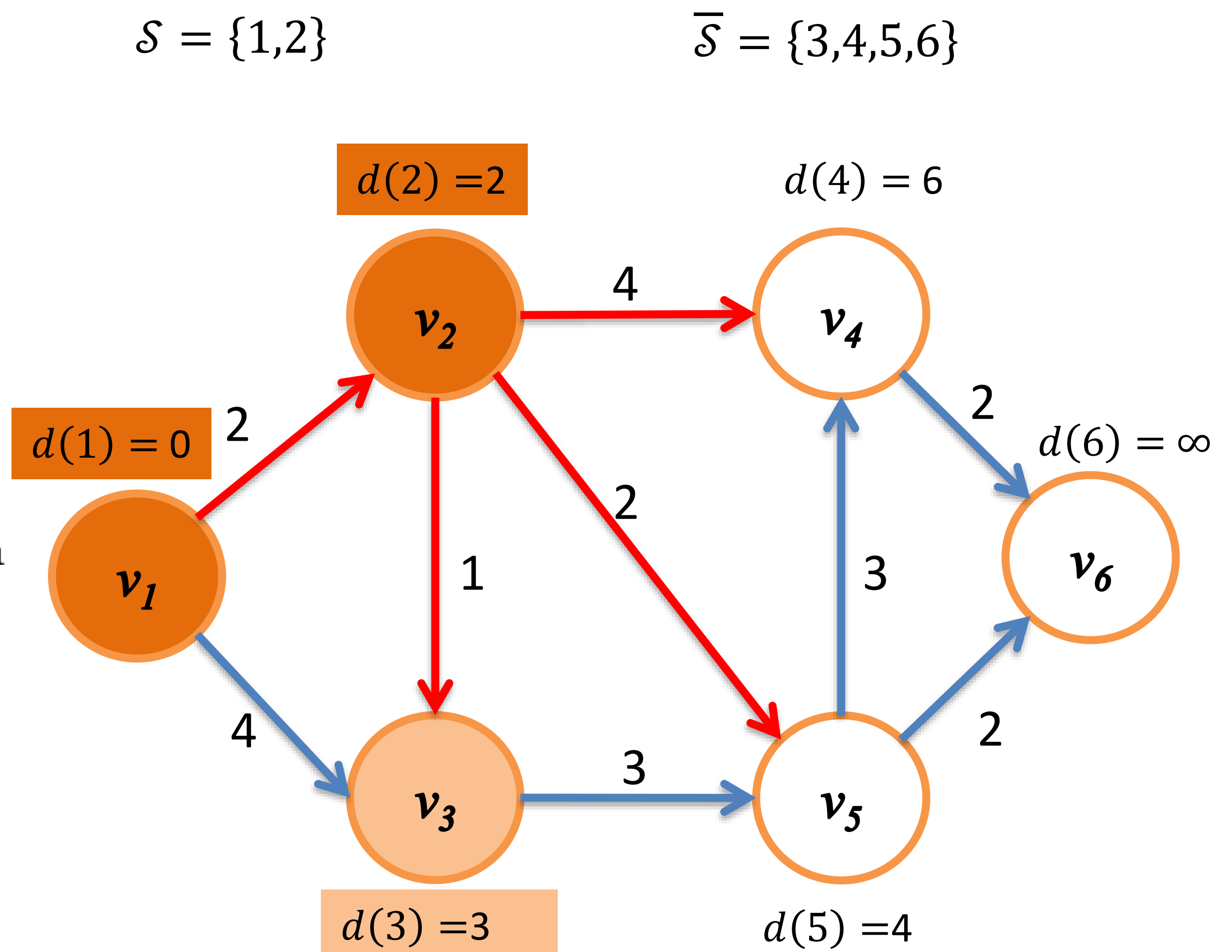
Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
$\boxed{\textbf{while } |\mathcal{S}| < |\mathcal{N}| \textbf{ do}}$
  $i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
  $d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
  $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
  $\bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}$
  for $(i,j) \in \mathcal{A}(i)$ **do**
    if $d(j) > d(i) + c_{ij}$ **then**
      $d(j) \leftarrow d(i) + c_{ij}$
      $p(j) \leftarrow i$
    end if
  end for
end while



Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ \ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
while $|\mathcal{S}| < |\mathcal{N}|$ **do**
  $i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
  $d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
  $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
  $\bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}$
  for $(i, j) \in \mathcal{A}(i)$ **do**
    if $d(j) > d(i) + c_{ij}$ **then**
      $d(j) \leftarrow d(i) + c_{ij}$
      $p(j) \leftarrow i$
    end if
  end for
end while

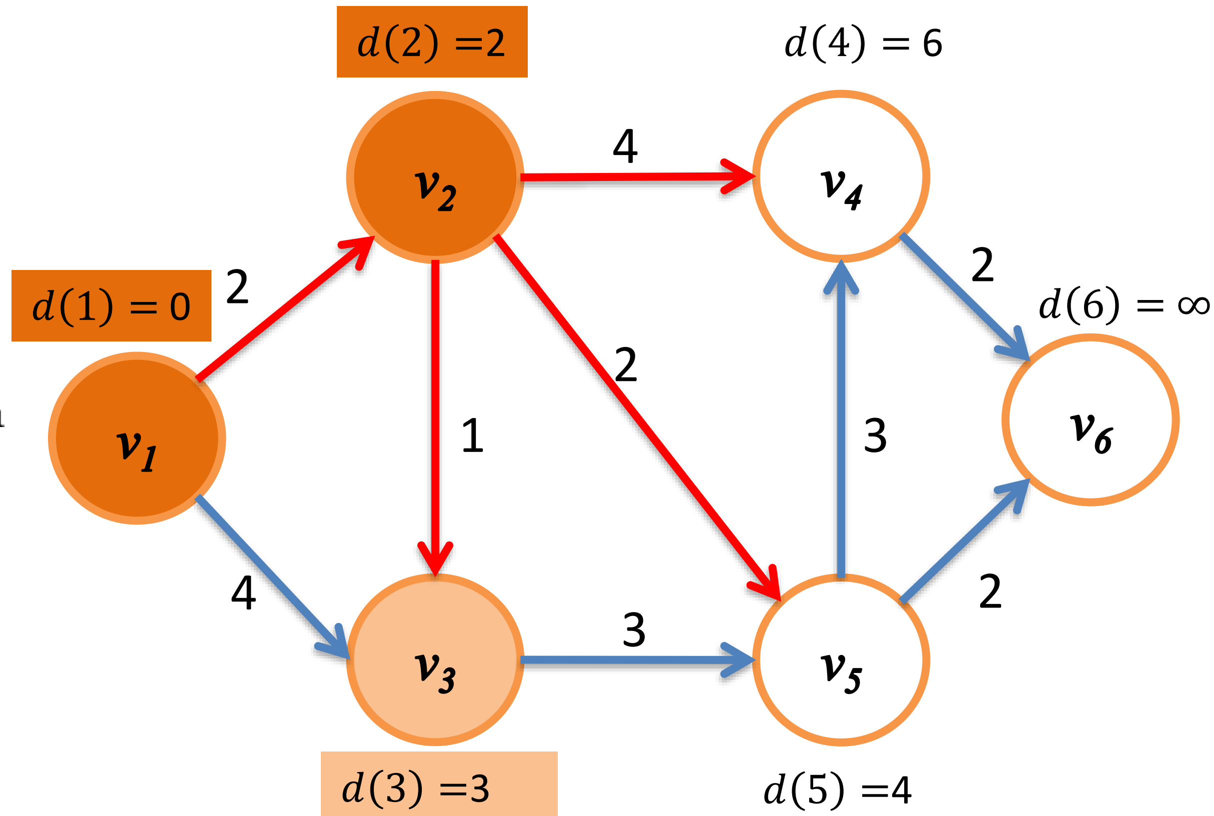


Algoritmo de Dijkstra

1: $\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
2: $d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
3: $d(s) \leftarrow 0,\ p(s) \leftarrow 0$
4: **while** $|\mathcal{S}| < |\mathcal{N}|$ **do**
5: $\quad i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
6: $\quad d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
7: $\quad \mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
8: $\quad \bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}$
9: $\quad$ **for** $(i,j) \in \mathcal{A}(i)$ **do**
10: $\quad\quad$ **if** $d(j) > d(i) + c_{ij}$ **then**
11: $\quad\quad\quad d(j) \leftarrow d(i) + c_{ij}$
12: $\quad\quad\quad p(j) \leftarrow i$
13: $\quad\quad$ **end if**
14: $\quad$ **end for**
15: **end while**

$\mathcal{S} = \{1,2,3\}$

$\bar{\mathcal{S}} = \{4,5,6\}$



Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
while $|\mathcal{S}| < |\mathcal{N}|$ **do**
  $i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
  $d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
  $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
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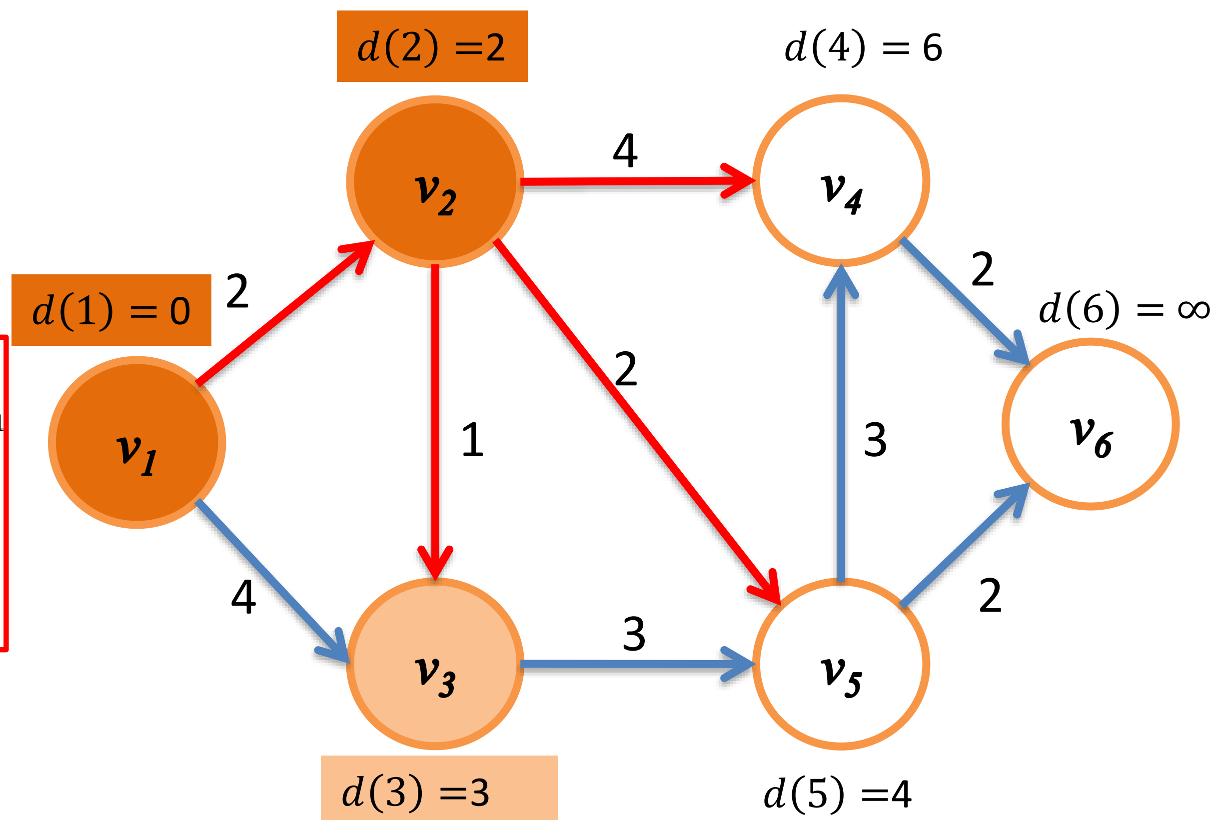
  for $(i, j) \in \mathcal{A}(i)$ **do**
    if $d(j) > d(i) + c_{ij}$ **then**
      $d(j) \leftarrow d(i) + c_{ij}$
      $p(j) \leftarrow i$
    end if
  end for
end while

$\mathcal{S} = \{1,2,3\}$ $\bar{\mathcal{S}} = \{4,5,6\}$



Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
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  $i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
  $d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
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    if $d(j) > d(i) + c_{ij}$ **then**
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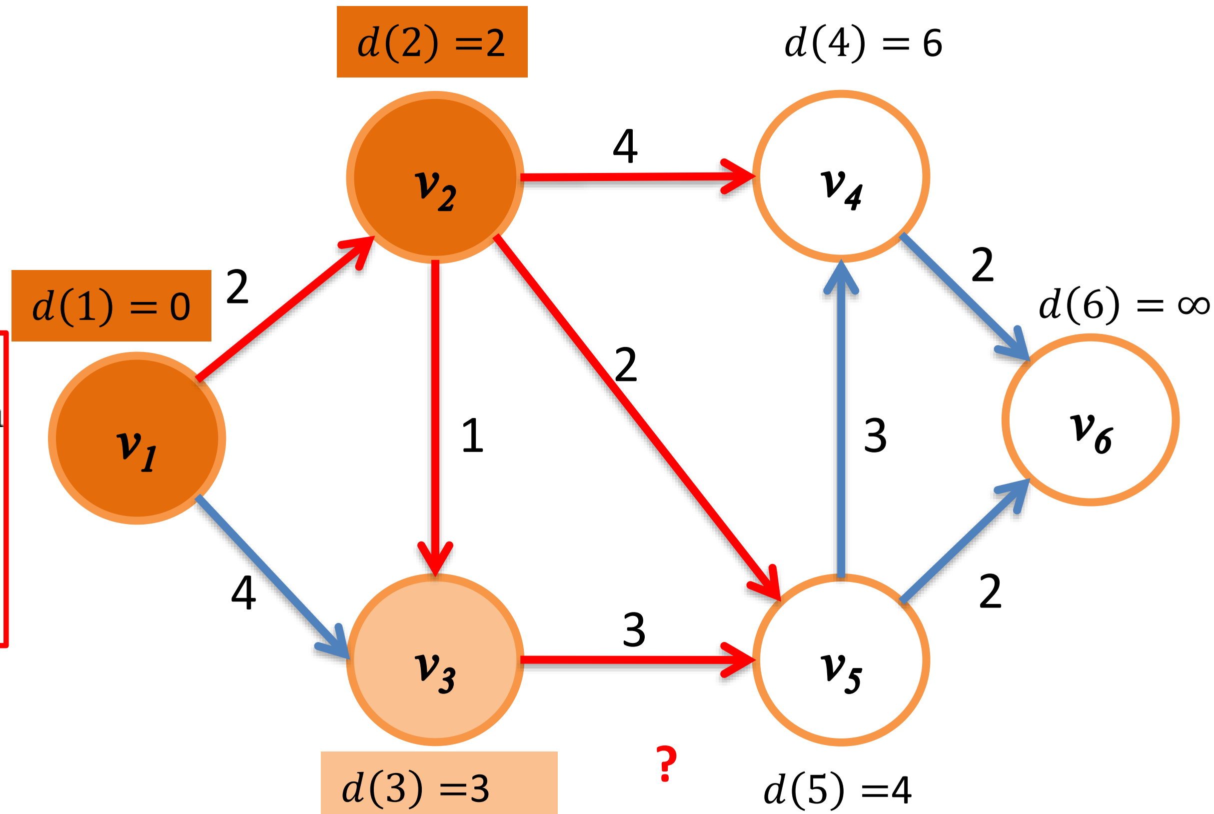
      $p(j) \leftarrow i$
    end if
  end for
end while

$\mathcal{S} = \{1,2,3\}$ $\bar{\mathcal{S}} = \{4,5,6\}$



Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
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  $d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
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  for $(i,j) \in \mathcal{A}(i)$ **do**
    if $d(j) > d(i) + c_{ij}$ **then**
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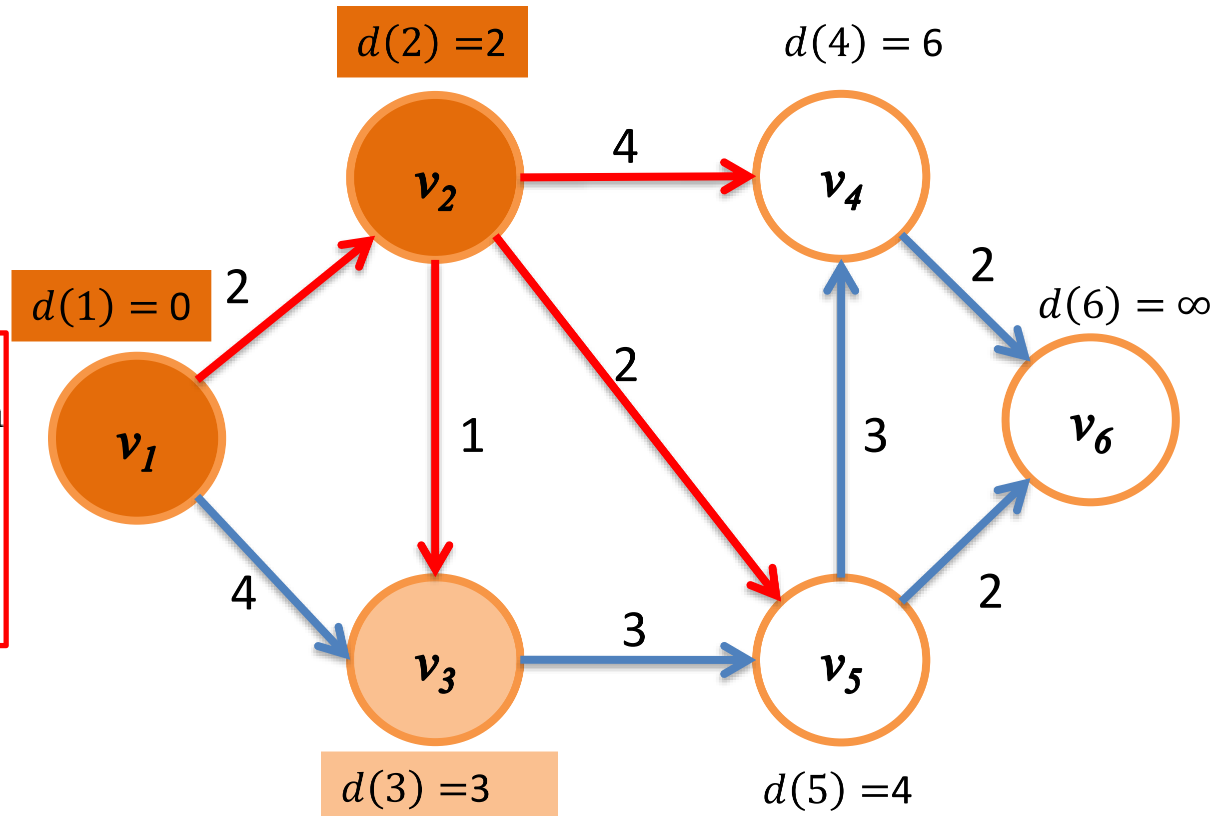
      $p(j) \leftarrow i$
    end if
  end for
end while

$\mathcal{S} = \{1,2,3\}$ $\bar{\mathcal{S}} = \{4,5,6\}$



Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
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$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
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  $i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
  $d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
  $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
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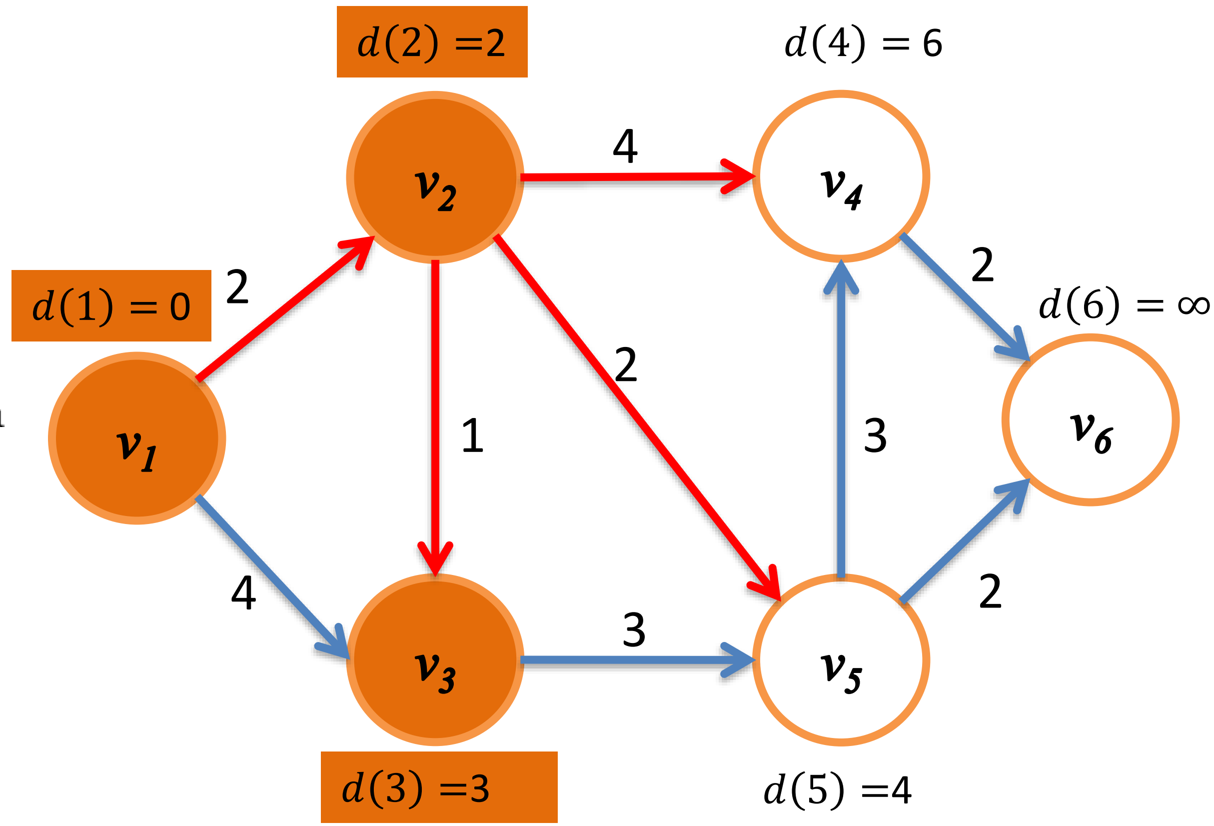
    if $d(j) > d(i) + c_{ij}$ **then**
      $d(j) \leftarrow d(i) + c_{ij}$
      $p(j) \leftarrow i$
    end if
  end for
end while

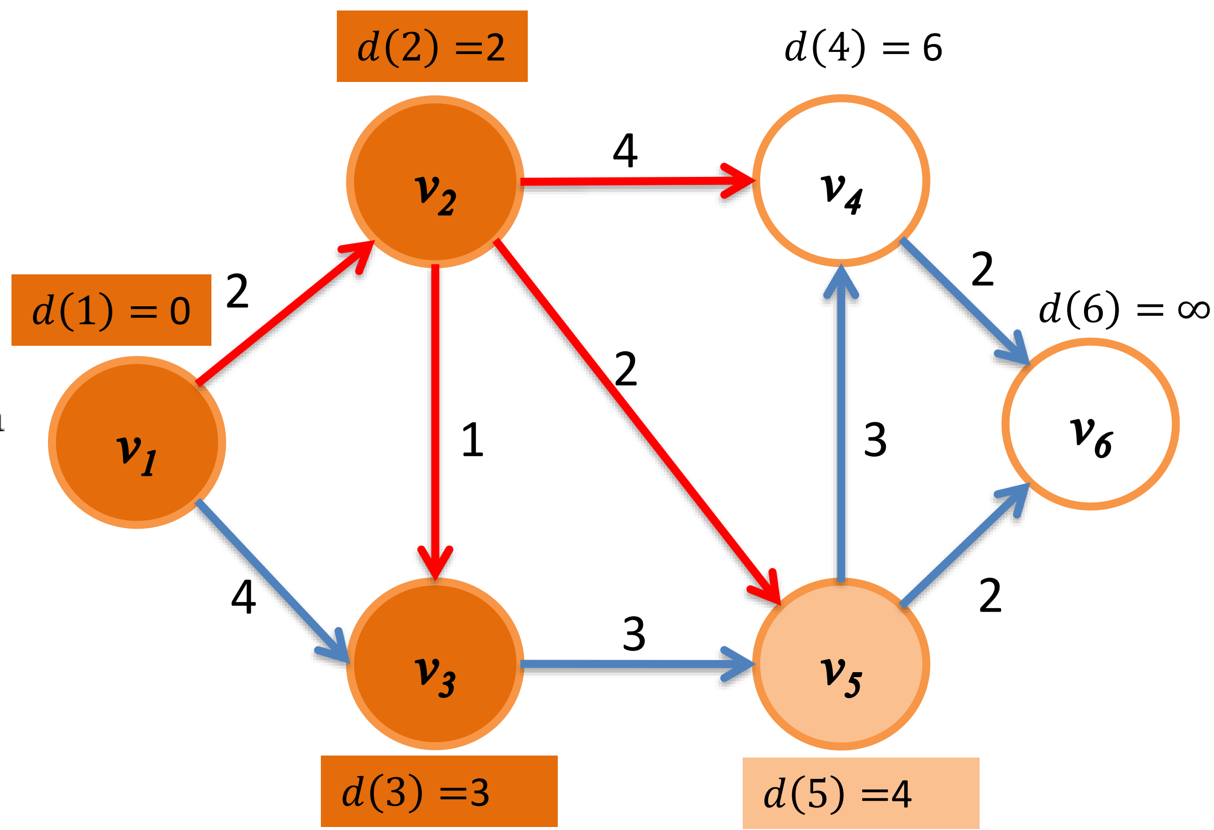
$\mathcal{S} = \{1,2,3\}$ $\bar{\mathcal{S}} = \{4,5,6\}$



Algoritmo de Dijkstra

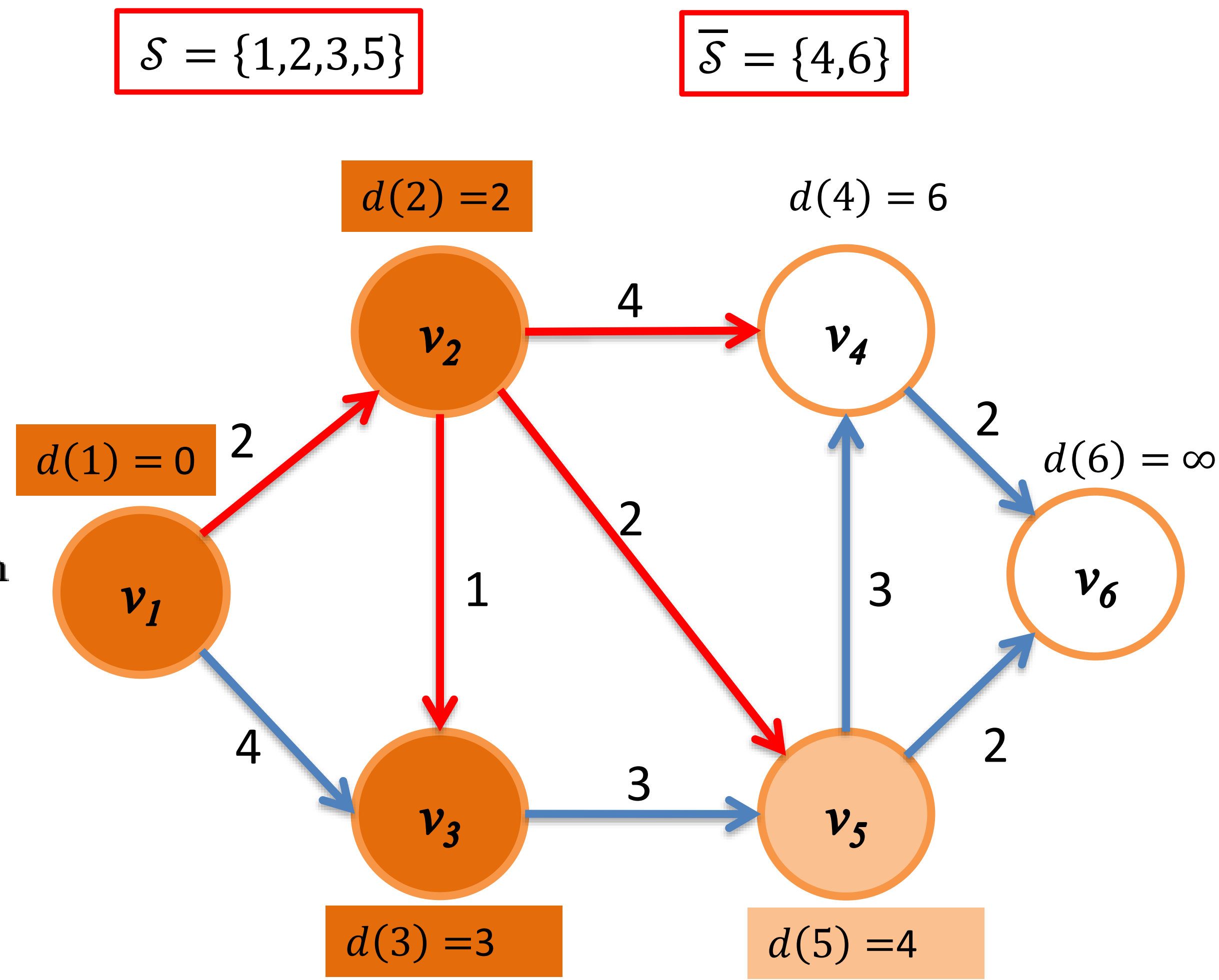
$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ \ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
while $|\mathcal{S}| < |\mathcal{N}|$ **do**
  $\boxed{i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}} \{d(j)\}}$
  $\boxed{d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}} \{d(j)\}}$
  $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
  $\bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}$
  for $(i,j) \in \mathcal{A}(i)$ **do**
    if $d(j) > d(i) + c_{ij}$ **then**
      $d(j) \leftarrow d(i) + c_{ij}$
      $p(j) \leftarrow i$
    end if
  end for
end while

$\mathcal{S} = \{1,2,3\}$ $\bar{\mathcal{S}} = \{4,5,6\}$



Algoritmo de Dijkstra

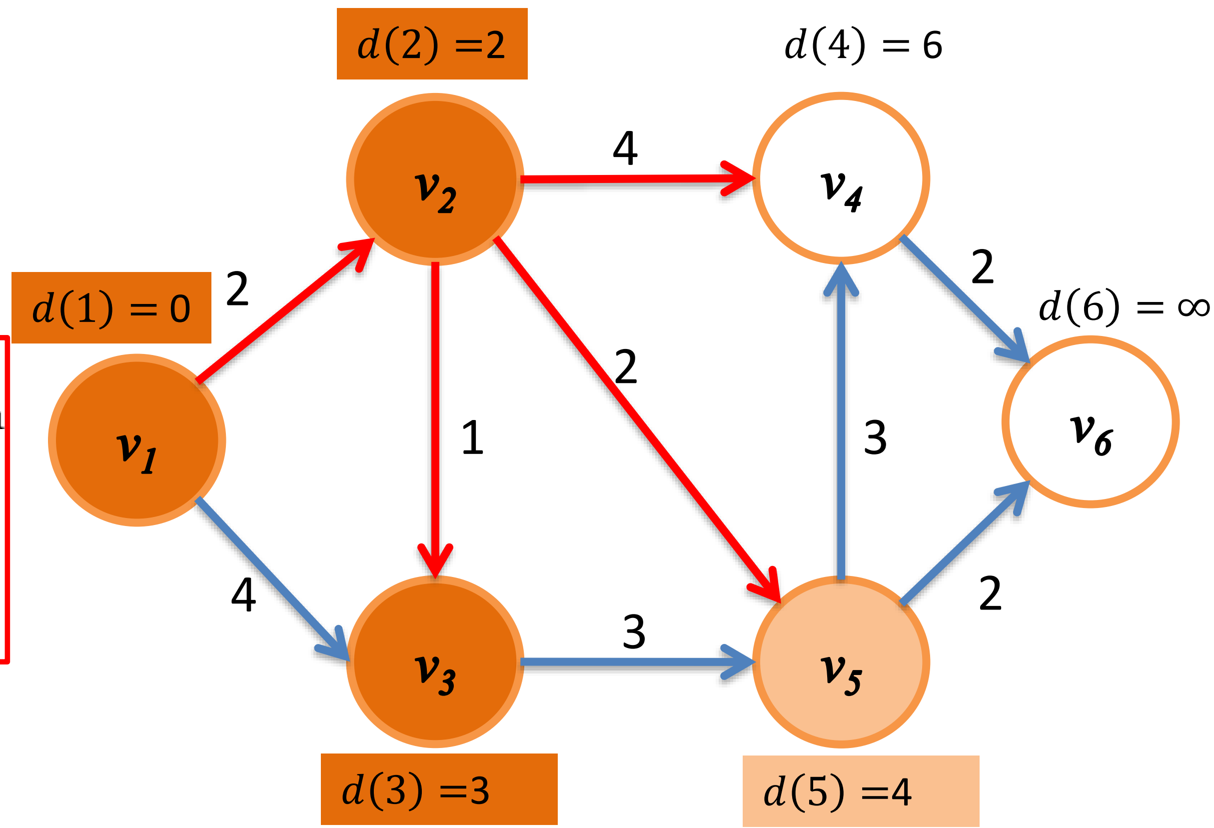
$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
while $|\mathcal{S}| < |\mathcal{N}|$ **do**
  $i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
  $d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
  $\boxed{\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}}$
  $\boxed{\bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}}$
  for $(i,j) \in \mathcal{A}(i)$ **do**
    if $d(j) > d(i) + c_{ij}$ **then**
      $d(j) \leftarrow d(i) + c_{ij}$
      $p(j) \leftarrow i$
    end if
  end for
end while



Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
while $|\mathcal{S}| < |\mathcal{N}|$ **do**
  $i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
  $d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
  $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
  $\bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}$
  for $(i, j) \in \mathcal{A}(i)$ **do**
    if $d(j) > d(i) + c_{ij}$ **then**
      $d(j) \leftarrow d(i) + c_{ij}$
      $p(j) \leftarrow i$
    end if
  end for
end while

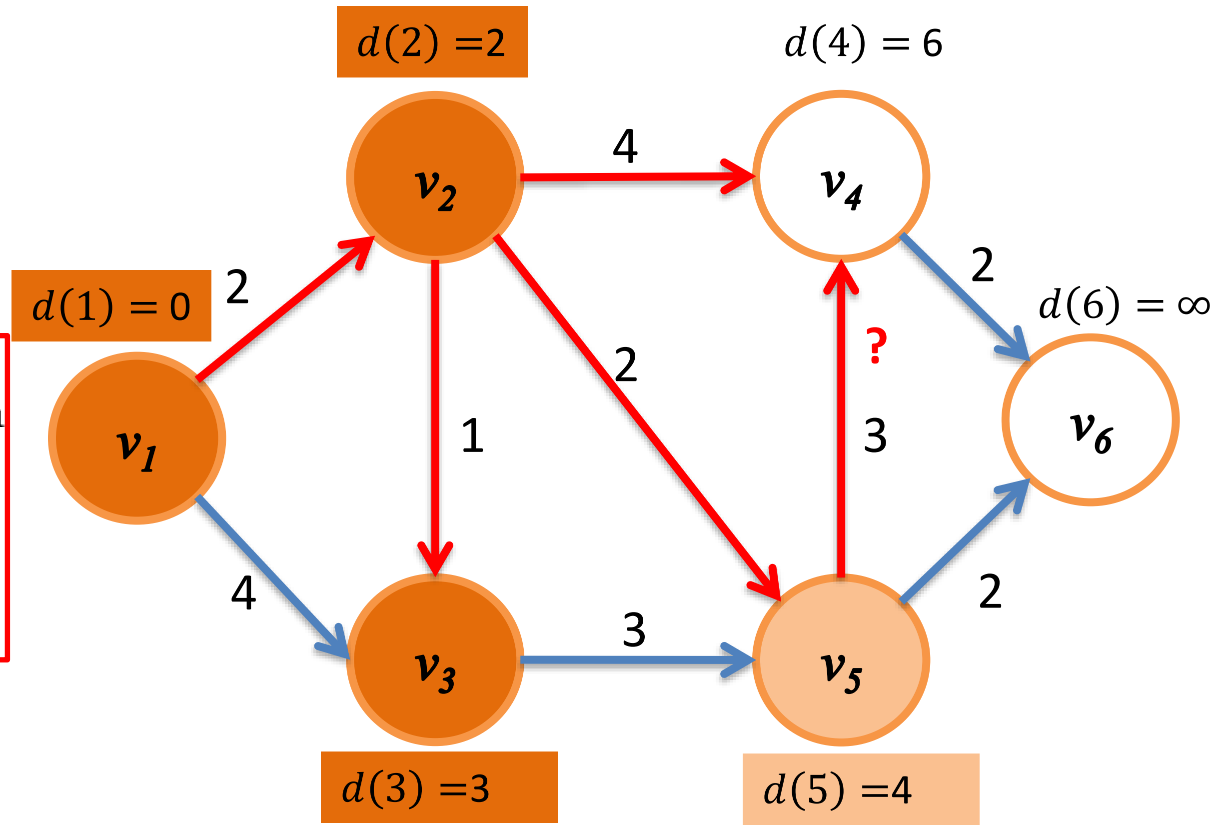
$\mathcal{S} = \{1,2,3,5\}$ $\bar{\mathcal{S}} = \{4,6\}$



Algoritmo de Dijkstra

$\mathcal{S} = \{1,2,3,5\}$ $\qquad$ $\bar{\mathcal{S}} = \{4,6\}$

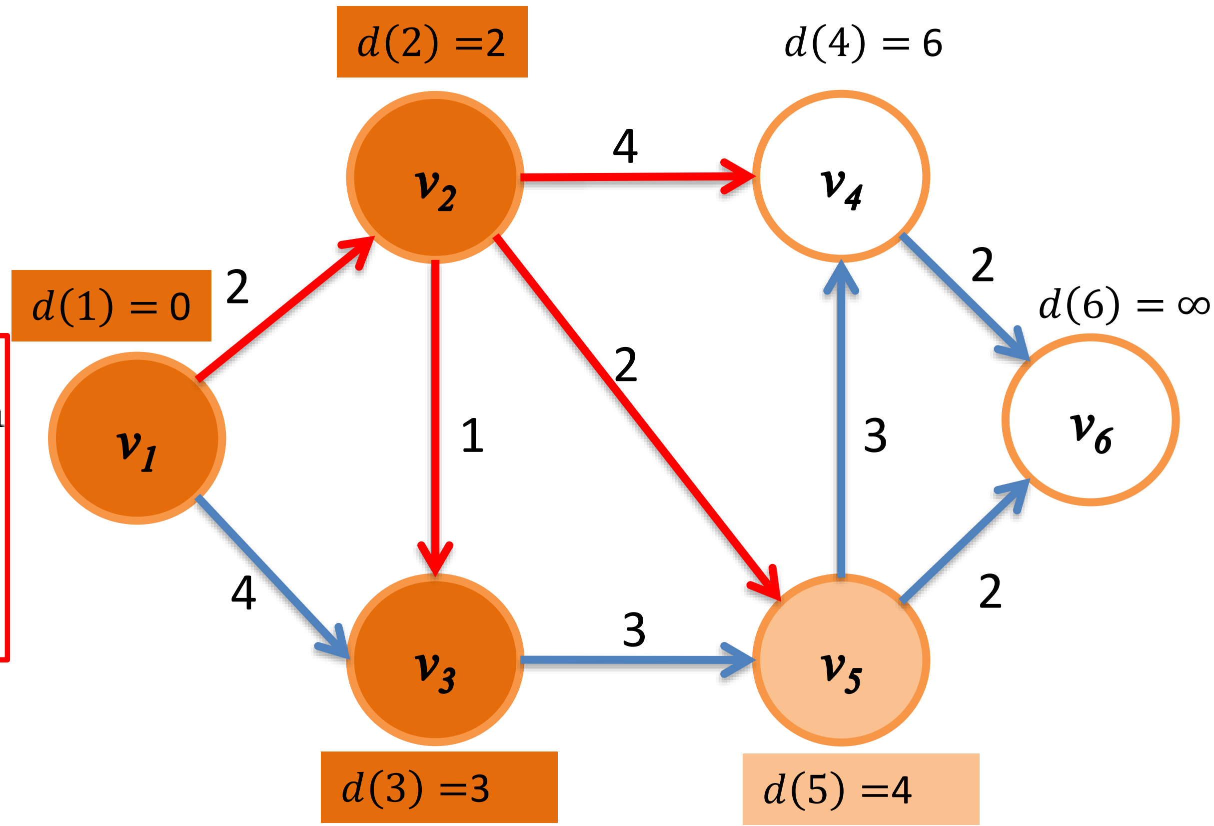
$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
while $|\mathcal{S}| < |\mathcal{N}|$ **do**
$\quad i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
$\quad d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
$\quad \mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
$\quad \bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}$
$\quad$ **for** $(i,j) \in \mathcal{A}(i)$ **do**
$\quad\quad$ **if** $d(j) > d(i) + c_{ij}$ **then**
$\quad\quad\quad d(j) \leftarrow d(i) + c_{ij}$
$\quad\quad\quad p(j) \leftarrow i$
$\quad\quad$ **end if**
$\quad$ **end for**
end while



Algoritmo de Dijkstra

$\mathcal{S} = \{1,2,3,5\}$ $\qquad$ $\bar{\mathcal{S}} = \{4,6\}$

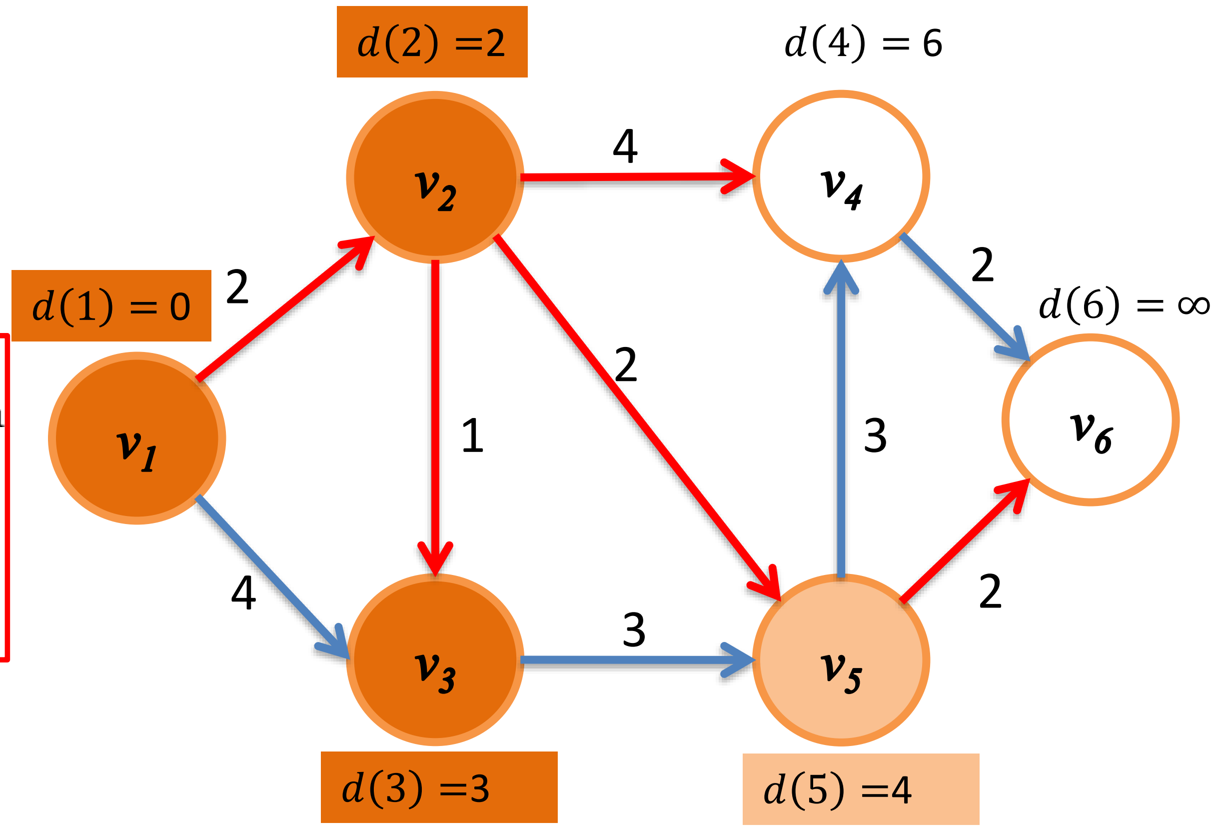
$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
while $|\mathcal{S}| < |\mathcal{N}|$ **do**
$\quad i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
$\quad d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}}\{d(j)\}$
$\quad \mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
$\quad \bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}$
$\quad$ **for** $(i,j) \in \mathcal{A}(i)$ **do**
$\quad\quad$ **if** $d(j) > d(i) + c_{ij}$ **then**
$\quad\quad\quad d(j) \leftarrow d(i) + c_{ij}$
$\quad\quad\quad p(j) \leftarrow i$
$\quad\quad$ **end if**
$\quad$ **end for**
end while



Algoritmo de Dijkstra

$$
\begin{array}{l}
\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N} \\
d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s \\
d(s) \leftarrow 0,\ p(s) \leftarrow 0 \\
\textbf{while } |\mathcal{S}| < |\mathcal{N}| \textbf{ do} \\
\quad i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}} \{d(j)\} \\
\quad d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}} \{d(j)\} \\
\quad \mathcal{S} \leftarrow \mathcal{S} \cup \{i\} \\
\quad \bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\} \\
\quad \textbf{for } (i,j) \in \mathcal{A}(i) \textbf{ do} \\
\quad\quad \textbf{if } d(j) > d(i) + c_{ij} \textbf{ then} \\
\quad\quad\quad d(j) \leftarrow d(i) + c_{ij} \\
\quad\quad\quad p(j) \leftarrow i \\
\quad\quad \textbf{end if} \\
\quad \textbf{end for} \\
\textbf{end while}
\end{array}
$$

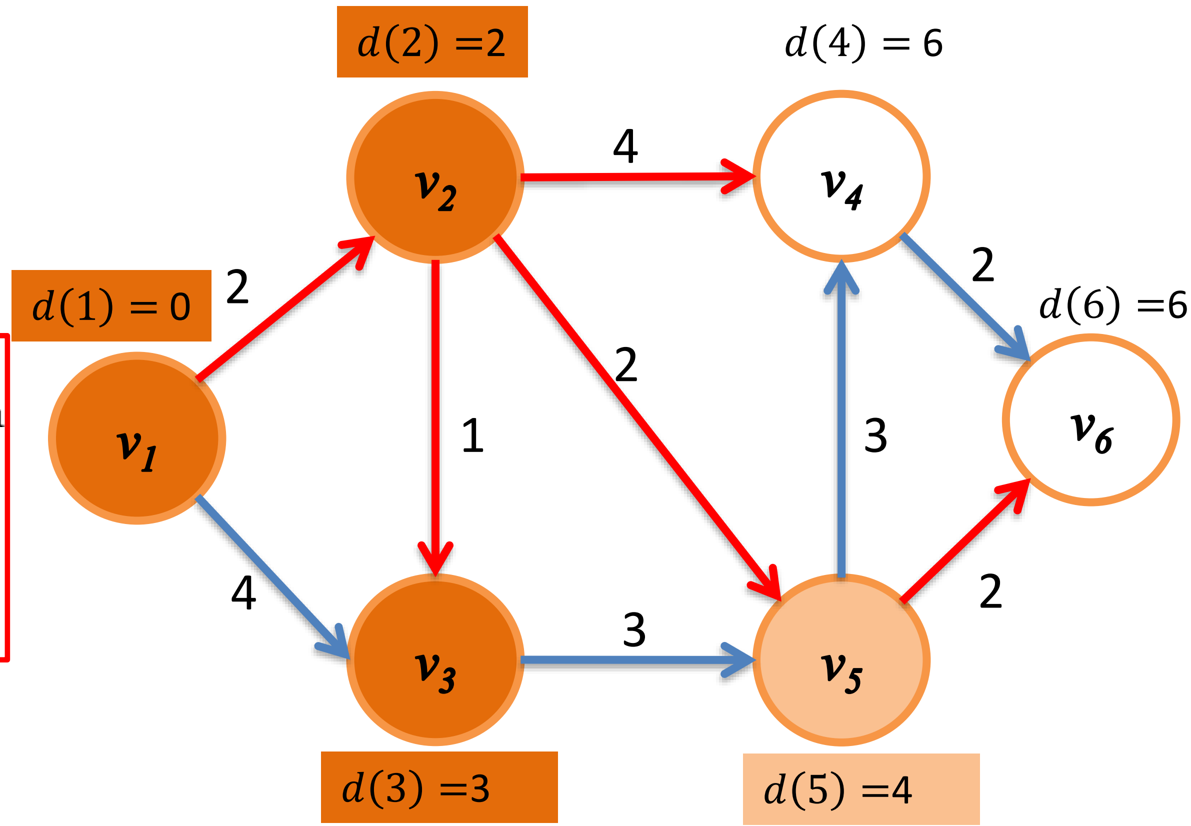
$\mathcal{S} = \{1,2,3,5\}$ $\quad\quad$ $\bar{\mathcal{S}} = \{4,6\}$



Algoritmo de Dijkstra

$\mathcal{S} = \{1,2,3,5\}$ $\qquad$ $\bar{\mathcal{S}} = \{4,6\}$

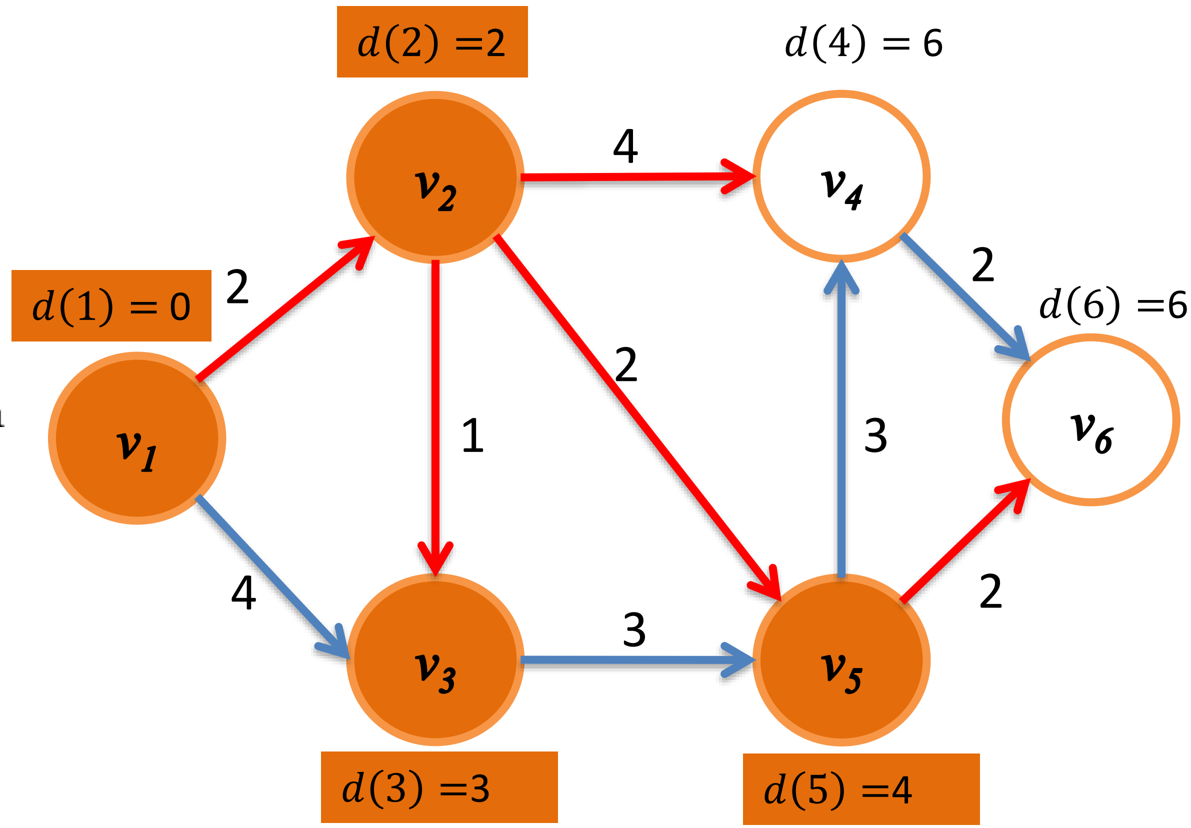
$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
while $|\mathcal{S}| < |\mathcal{N}|$ **do**
 $i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
 $d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
 $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
 $\bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}$
 for $(i,j) \in \mathcal{A}(i)$ **do**
  if $d(j) > d(i) + c_{ij}$ **then**
   $d(j) \leftarrow d(i) + c_{ij}$
   $p(j) \leftarrow i$
  end if
 end for
end while



Algoritmo de Dijkstra

```
S ← ∅, S̄ ← N
d(i) ← ∞, i ∈ N, i ≠ s
d(s) ← 0, p(s) ← 0
while |S| < |N| do
    i ← arg min_{j∈S̄} {d(j)}
    d(i) ← min_{j∈S̄} {d(j)}
    S ← S ∪ {i}
    S̄ ← S̄ \ {i}
    for (i, j) ∈ A(i) do
        if d(j) > d(i) + c_ij then
            d(j) ← d(i) + c_ij
            p(j) ← i
        end if
    end for
end while
```

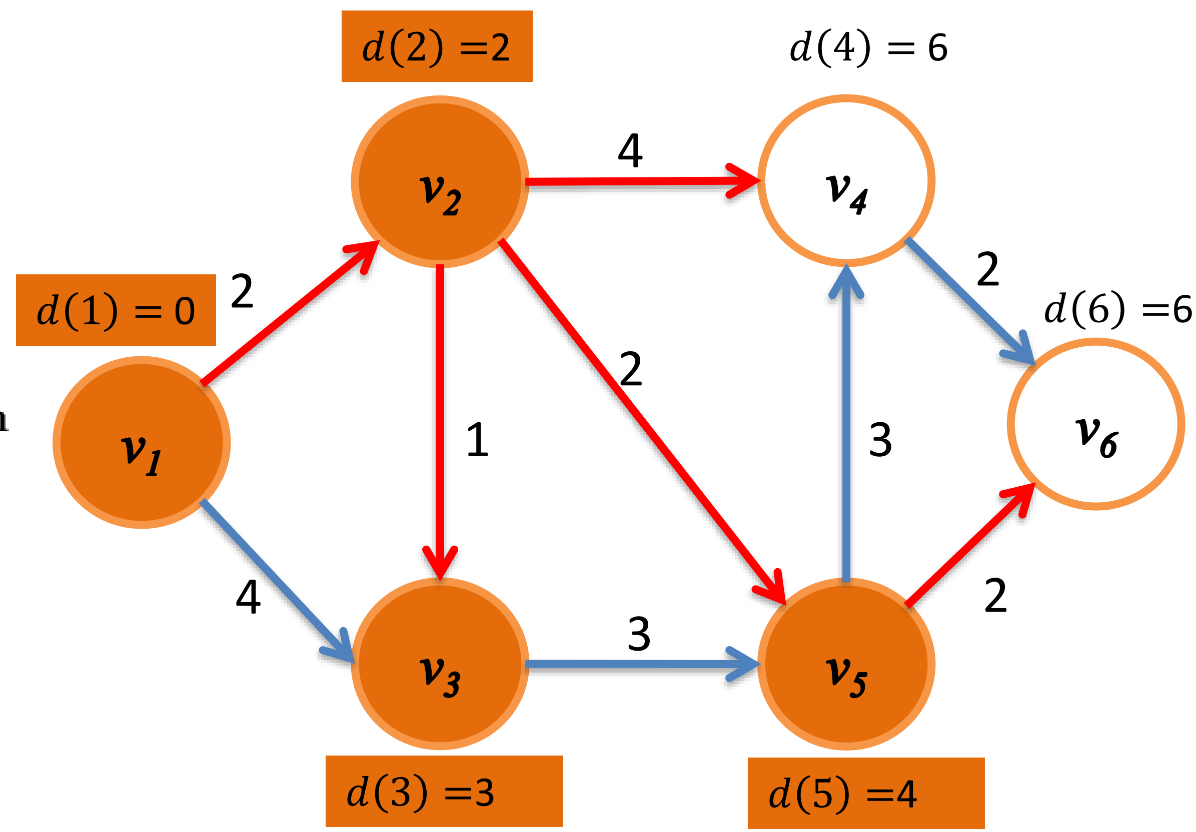
$\mathcal{S} = \{1,2,3,5\}$ $\bar{\mathcal{S}} = \{4,6\}$



Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ \ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ \ p(s) \leftarrow 0$
while $|\mathcal{S}| < |\mathcal{N}|$ **do**
$\quad \boxed{\begin{array}{l} i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}}\{d(j)\} \\ d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}}\{d(j)\} \end{array}}$ **?**
$\quad \mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
$\quad \bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}$
$\quad$ **for** $(i, j) \in \mathcal{A}(i)$ **do**
$\quad\quad$ **if** $d(j) > d(i) + c_{ij}$ **then**
$\quad\quad\quad d(j) \leftarrow d(i) + c_{ij}$
$\quad\quad\quad p(j) \leftarrow i$
$\quad\quad$ **end if**
$\quad$ **end for**
end while

$\mathcal{S} = \{1,2,3,5\}$ $\quad\quad$ $\bar{\mathcal{S}} = \{4,6\}$



Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
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  $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
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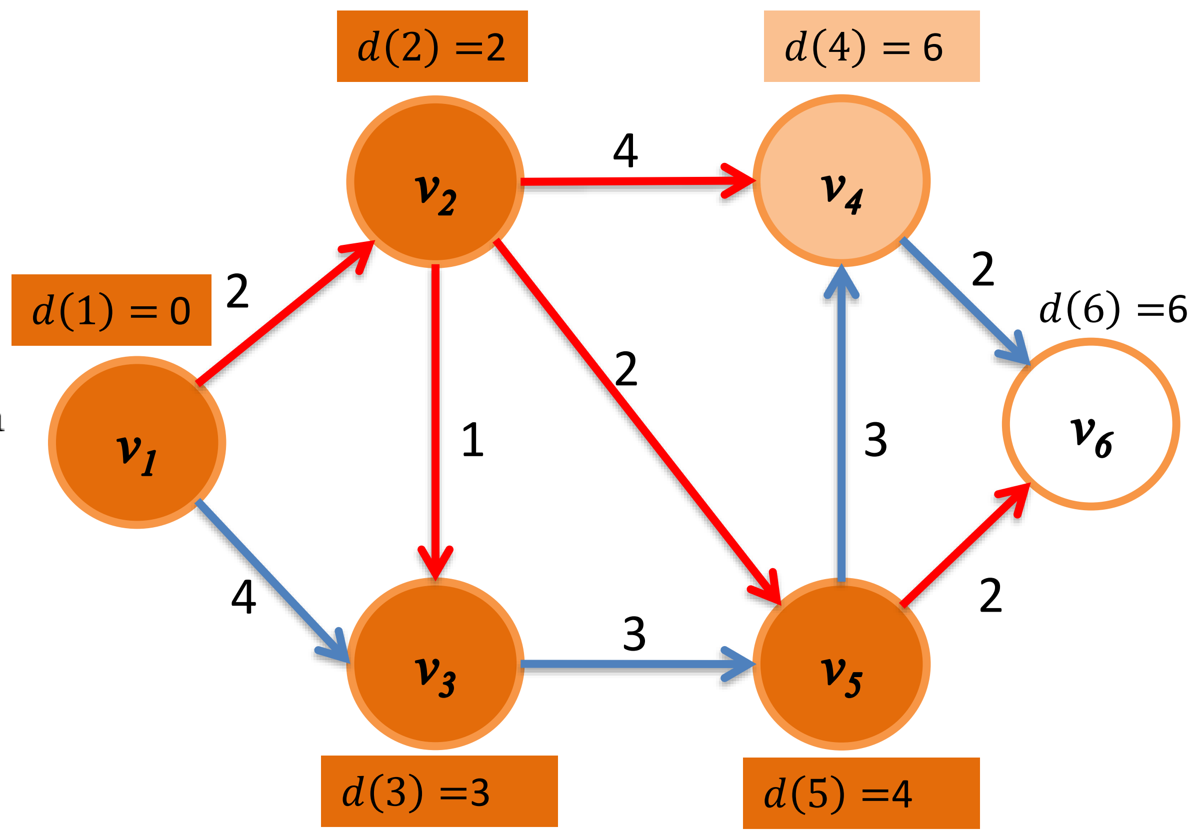
  for $(i, j) \in \mathcal{A}(i)$ **do**
    if $d(j) > d(i) + c_{ij}$ **then**
      $d(j) \leftarrow d(i) + c_{ij}$
      $p(j) \leftarrow i$
    end if
  end for
end while

?

$\mathcal{S} = \{1,2,3,5\}$ $\bar{\mathcal{S}} = \{4,6\}$



Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
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      $d(j) \leftarrow d(i) + c_{ij}$
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    end if
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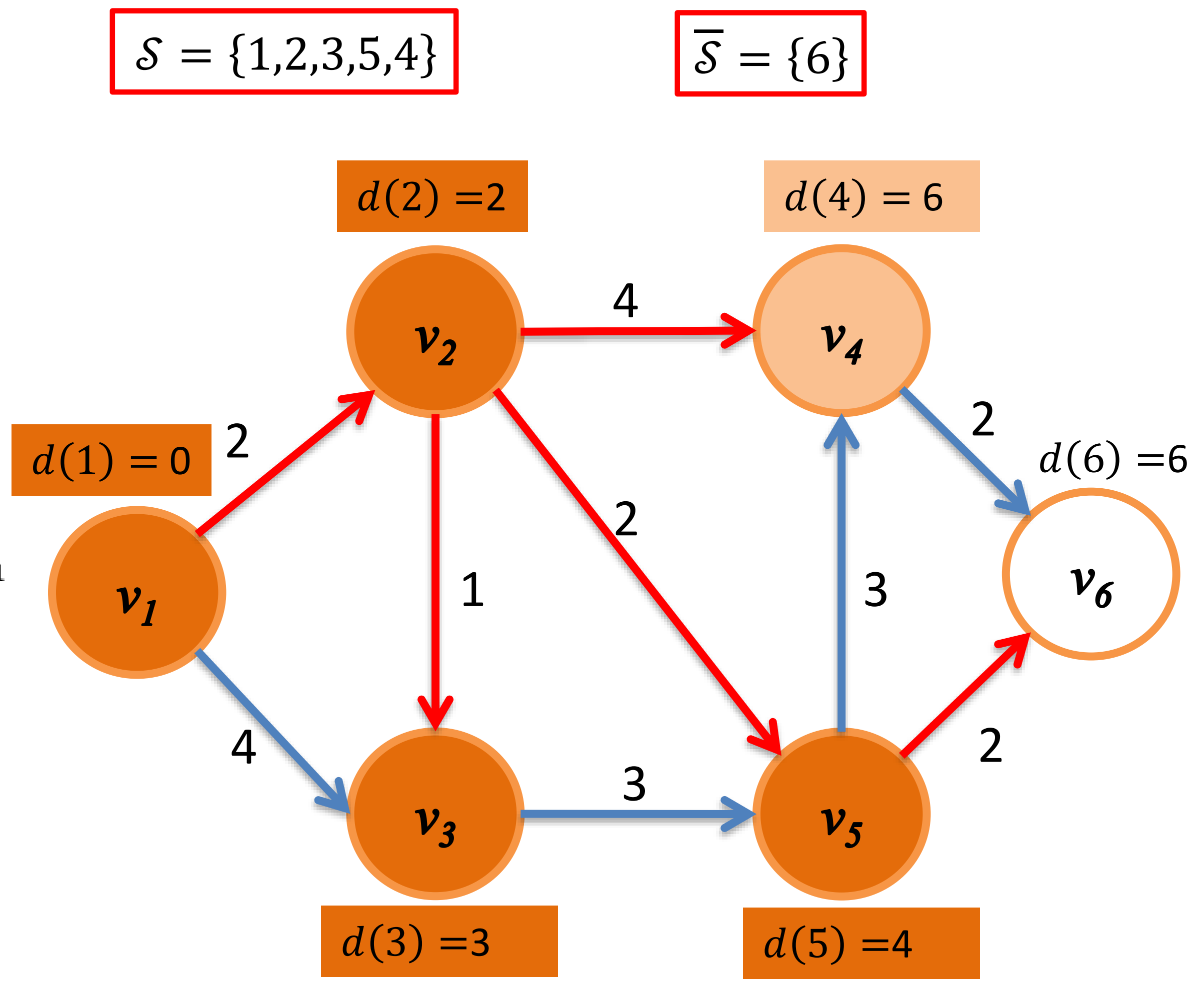
end while



Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
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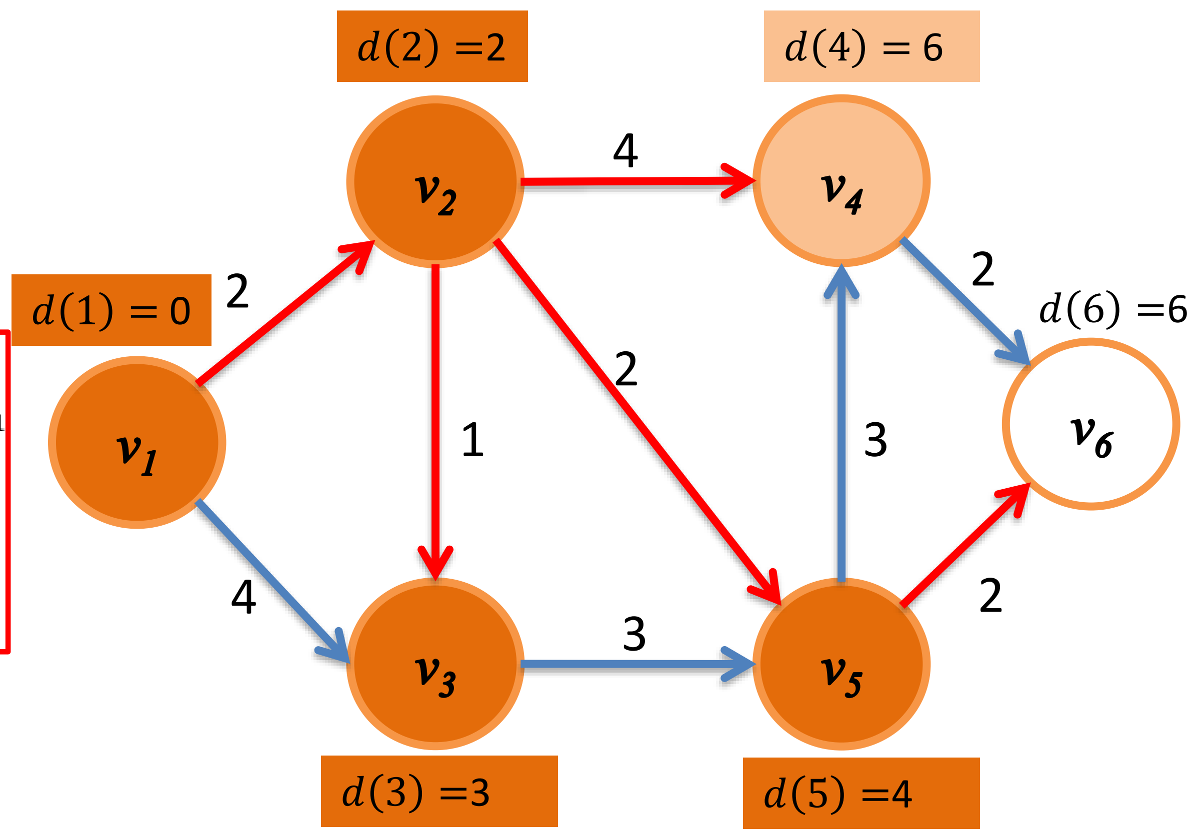
    for $(i,j) \in \mathcal{A}(i)$ **do**
        if $d(j) > d(i) + c_{ij}$ **then**
            $d(j) \leftarrow d(i) + c_{ij}$
            $p(j) \leftarrow i$
        end if
    end for
end while

$\mathcal{S} = \{1,2,3,5,4\}$ $\bar{\mathcal{S}} = \{6\}$



Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
$d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
$d(s) \leftarrow 0,\ p(s) \leftarrow 0$
while $|\mathcal{S}| < |\mathcal{N}|$ **do**
  $i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
  $d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
  $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
  $\bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}$
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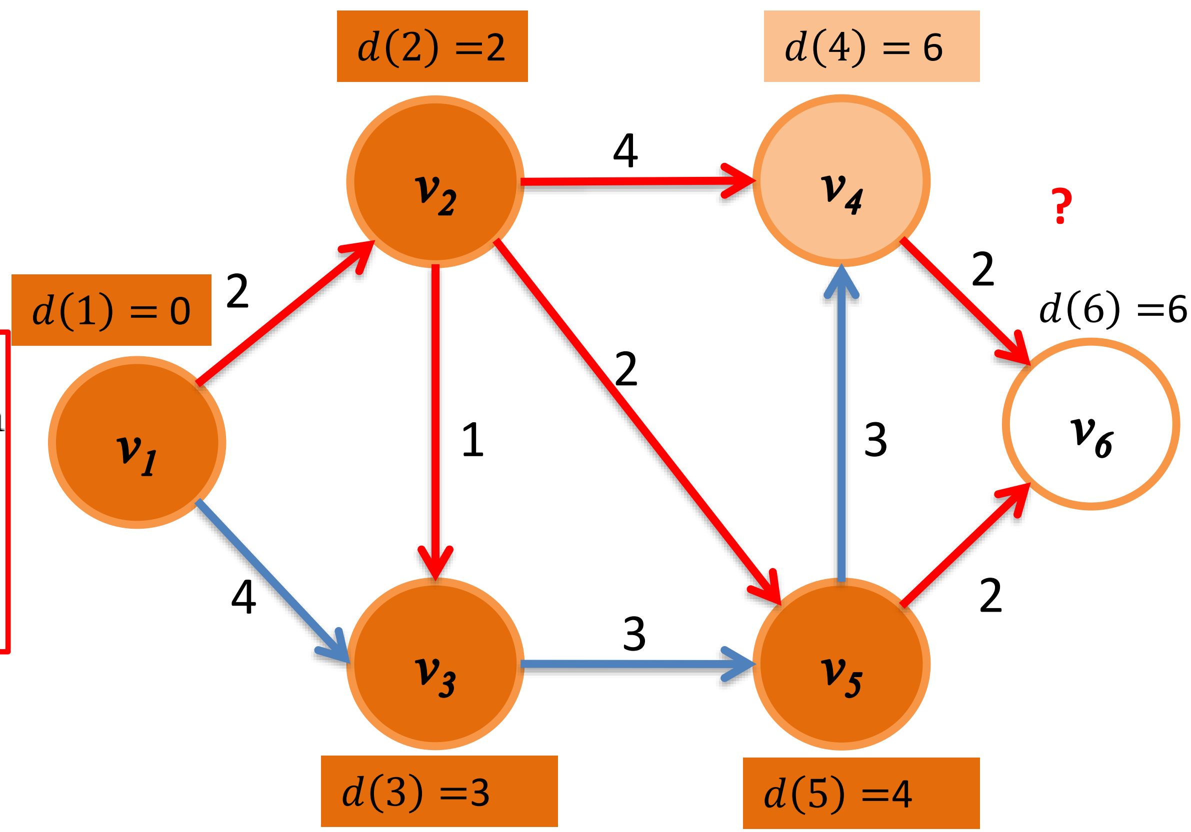

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$\quad\quad$ **end if**
$\quad$ **end for**
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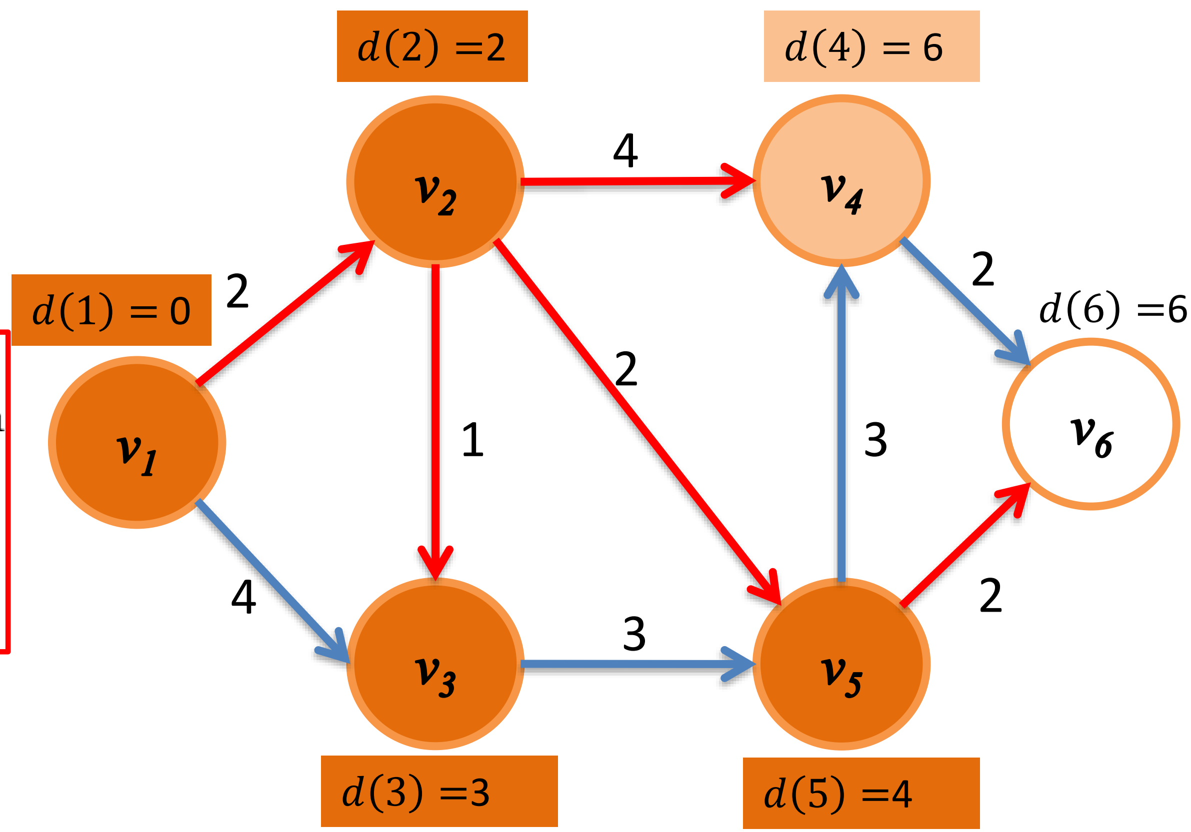
$\mathcal{S} = \{1,2,3,5,4\}$ $\quad\quad$ $\bar{\mathcal{S}} = \{6\}$



Algoritmo de Dijkstra

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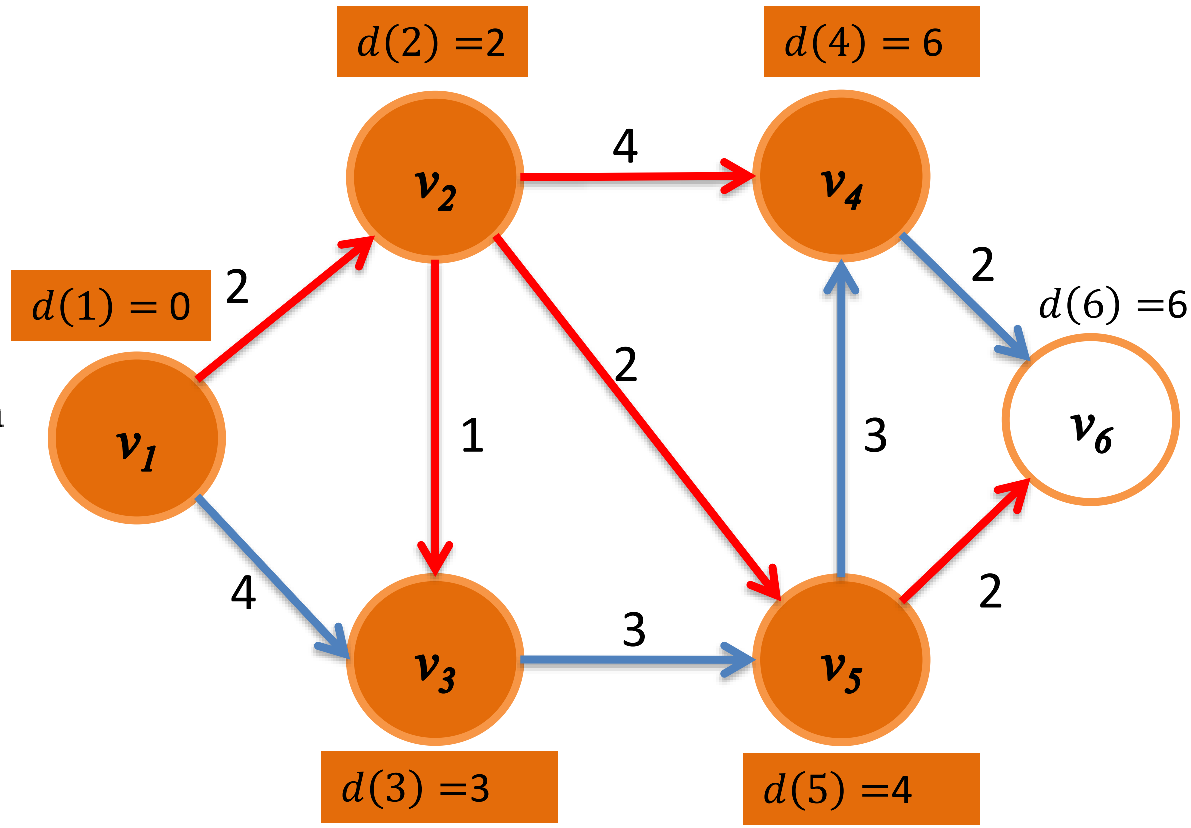
$\mathcal{S} = \{1,2,3,5,4\}$ $\bar{\mathcal{S}} = \{6\}$



Algoritmo de Dijkstra

$\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
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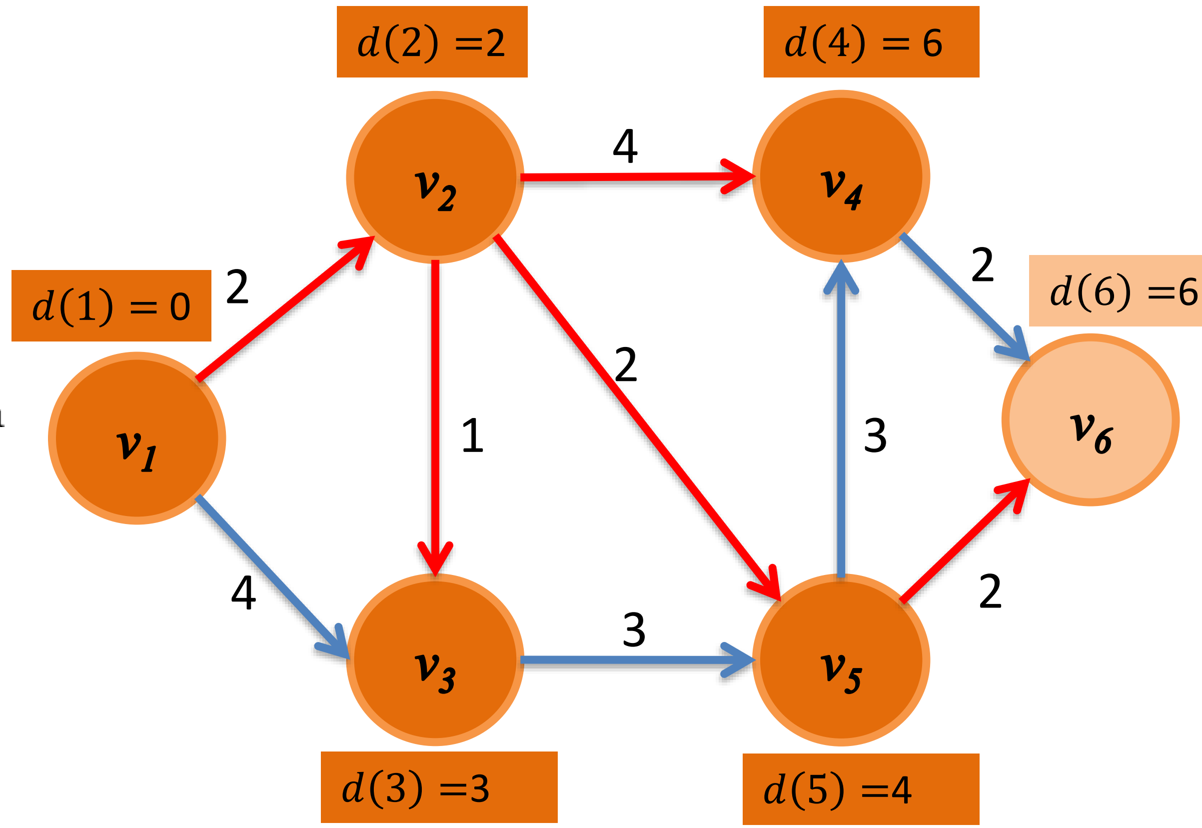
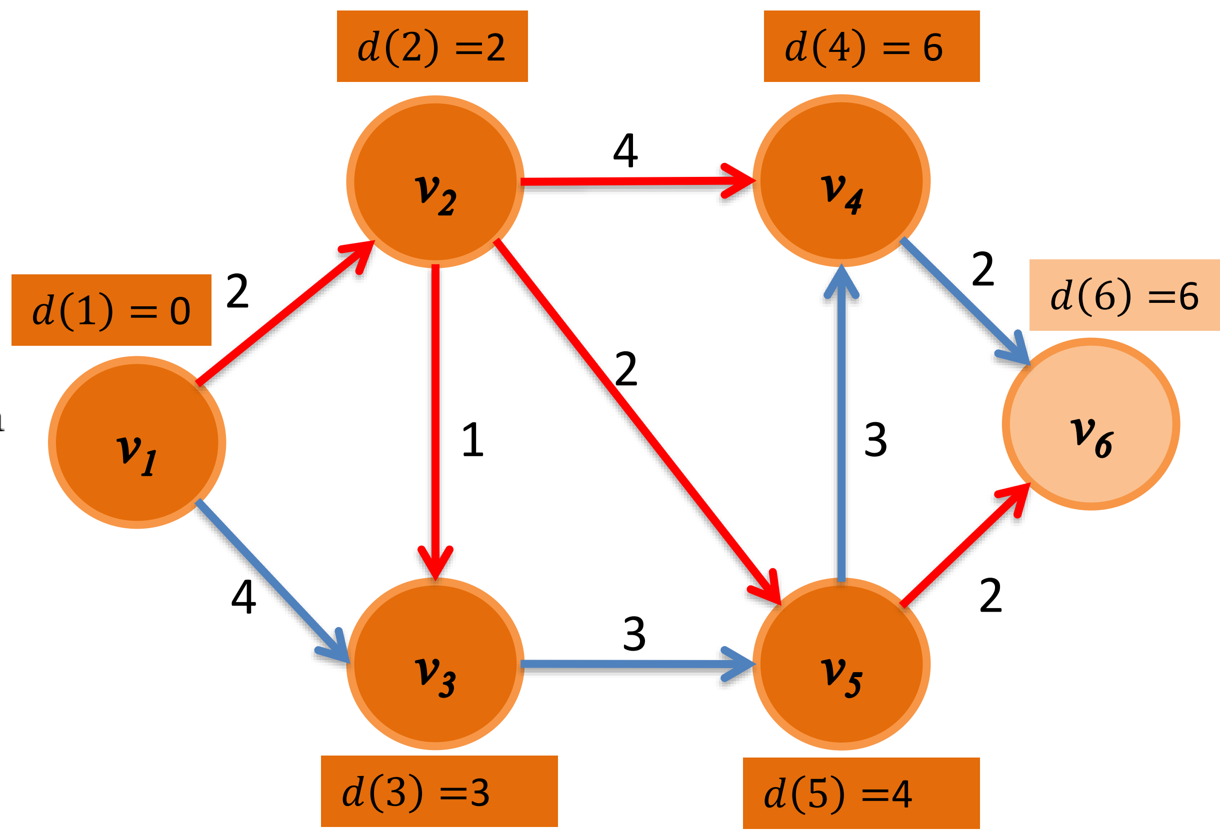


Algoritmo de Dijkstra

1: $\mathcal{S} \leftarrow \emptyset,\ \bar{\mathcal{S}} \leftarrow \mathcal{N}$
2: $d(i) \leftarrow \infty,\ i \in \mathcal{N}, i \neq s$
3: $d(s) \leftarrow 0,\ p(s) \leftarrow 0$
4: **while** $|\mathcal{S}| < |\mathcal{N}|$ **do**
5: $\quad i \leftarrow \arg\min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
6: $\quad d(i) \leftarrow \min_{j \in \bar{\mathcal{S}}} \{d(j)\}$
7: $\quad \mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
8: $\quad \bar{\mathcal{S}} \leftarrow \bar{\mathcal{S}} \setminus \{i\}$
9: $\quad$ **for** $(i, j) \in \mathcal{A}(i)$ **do**
10: $\quad\quad$ **if** $d(j) > d(i) + c_{ij}$ **then**
11: $\quad\quad\quad d(j) \leftarrow d(i) + c_{ij}$
12: $\quad\quad\quad p(j) \leftarrow i$
13: $\quad\quad$ **end if**
14: $\quad$ **end for**
15: **end while**

$\mathcal{S} = \{1,2,3,5,4,6\}$ $\quad \bar{\mathcal{S}} = \{\ \}$



Algoritmo de Dijkstra

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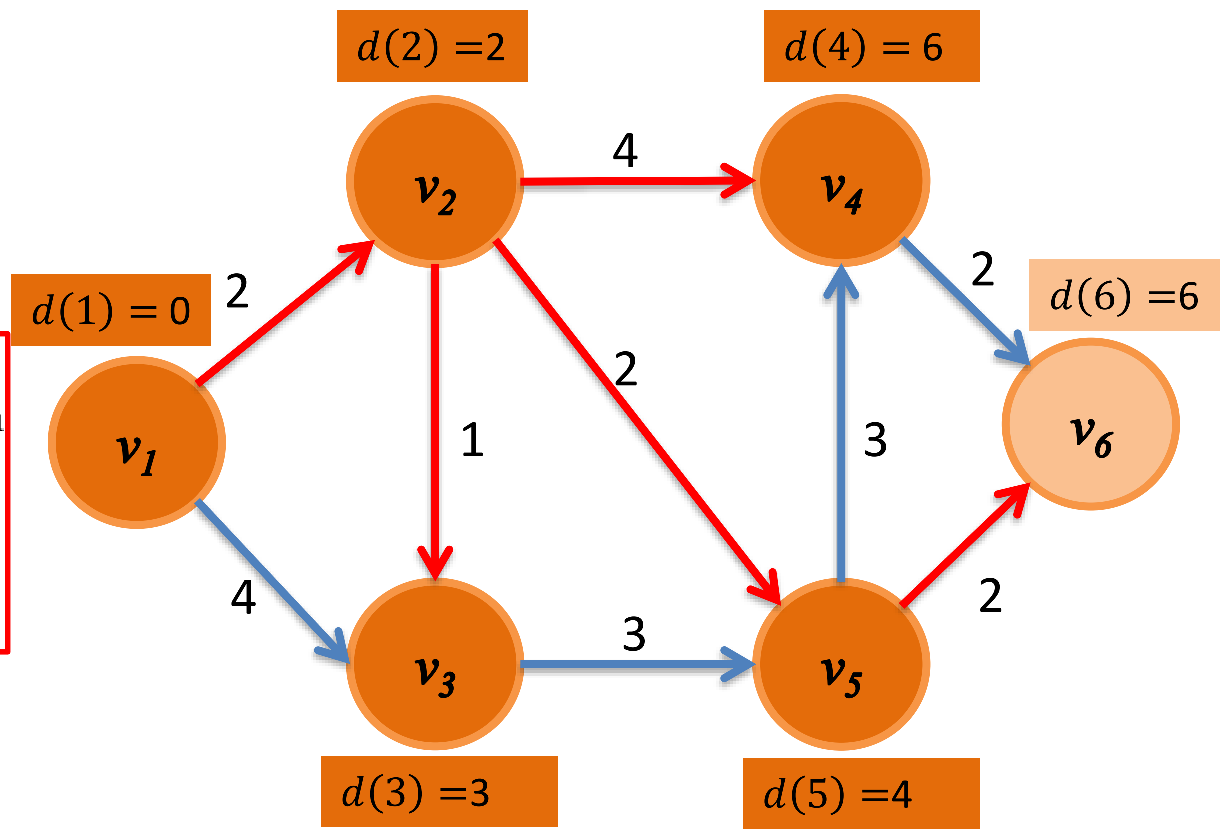
$\mathcal{S} = \{1,2,3,5,4,6\}$ $\bar{\mathcal{S}} = \{\ \}$



Algoritmo de Dijkstra

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$\qquad \textbf{end if}$
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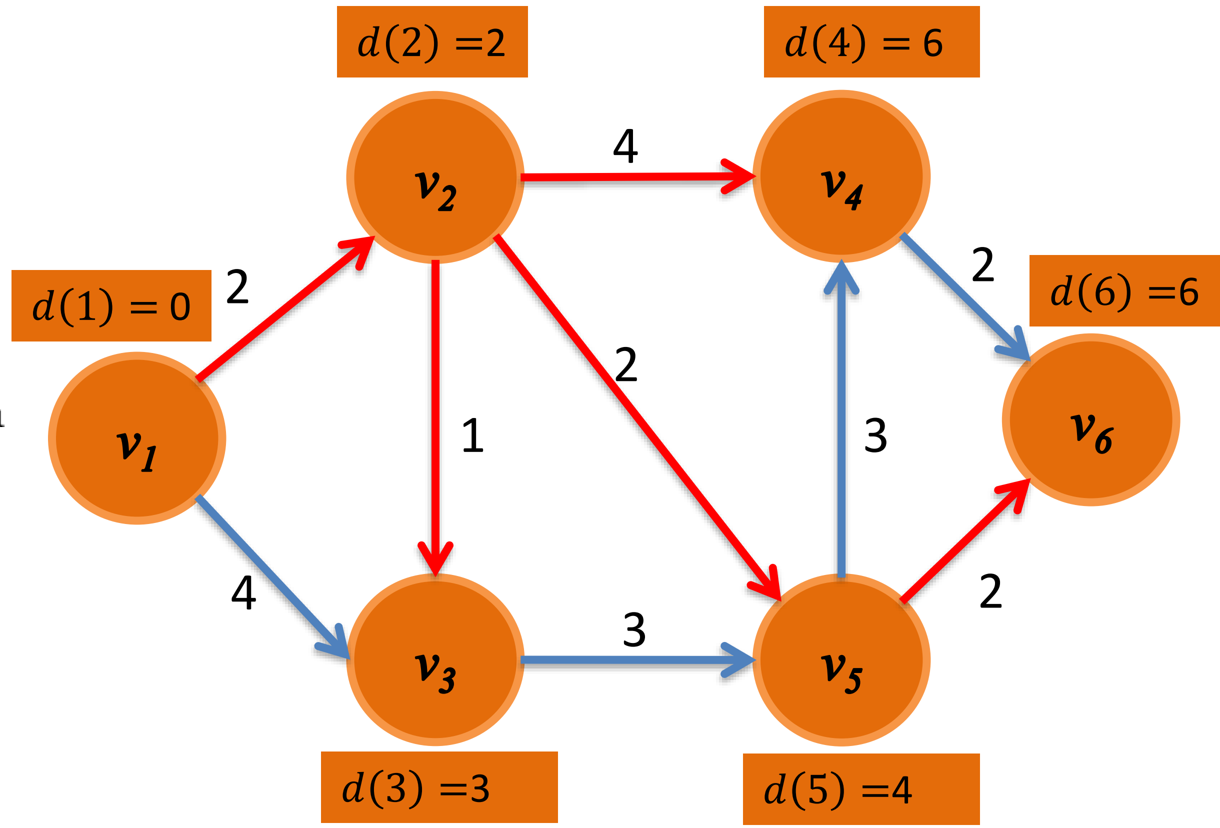
$\textbf{end while}$



Algoritmo de Dijkstra

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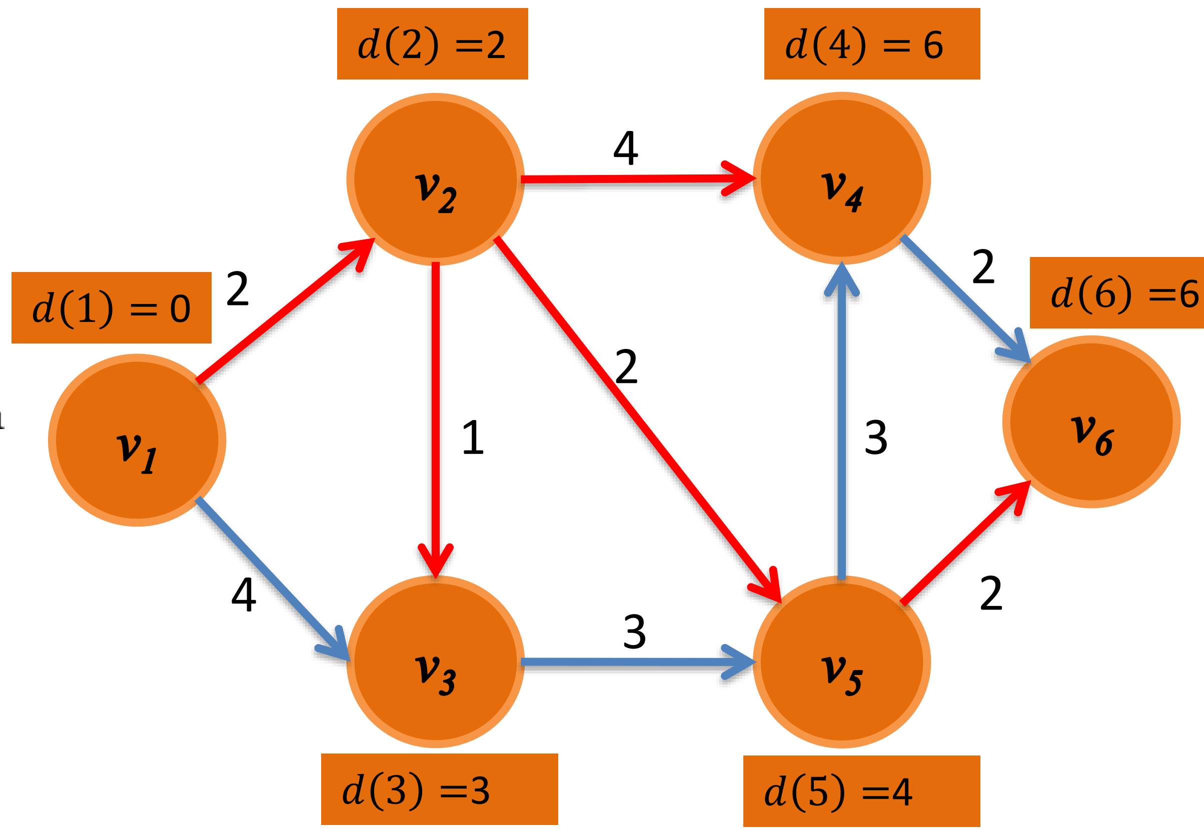
$\quad\quad$ **end if**
$\quad$ **end for**
end while

$\mathcal{S} = \{1,2,3,5,4,6\}$ $\qquad$ $\bar{\mathcal{S}} = \{\ \ \}$



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Algoritmo de Dijkstra: complejidad

- Cuántas operaciones elementales?
  - Escogencia de nodos (con etiqueta temporal)

$$i \leftarrow \arg\min_{j \in \bar{S}} \{d(j)\}$$

  - Actualización de etiquetas

$$
\begin{array}{l}
\textbf{for } (i,j) \in \mathcal{A}(i) \textbf{ do} \\
\quad \textbf{if } d(j) > d(i) + c_{ij} \textbf{ then} \\
\quad\quad d(j) \leftarrow d(i) + c_{ij} \\
\quad\quad p(i) \leftarrow i \\
\quad \textbf{end if} \\
\textbf{end for}
\end{array}
$$

Algoritmo de Dijkstra: complejidad

- Nodos examinados: $n=|\mathcal{N}|$
- Debe examinar cada uno de los nodos con etiqueta temporal

$$n+(n-1)+(n-2)+\ldots+1=\frac{n(n+1)}{2}=O\left(n^2\right)$$

- Actualización de etiquetas temporales

$$\sum_{i\in N}\left|A(i)\right|=m=O(m)$$

- Complejidad

$$O\left(n^2\right)+O\left(m\right)=O\left(n^2\right)$$

Agenda

- Part I: fundamentals
  - Dijkstra
  - **A***
  - Labeling algorithms
  - Branch and bound
- Part II: intuition
- Part III: extensions
- Part IV: applications
- Part V: perspectives

A*: A-star

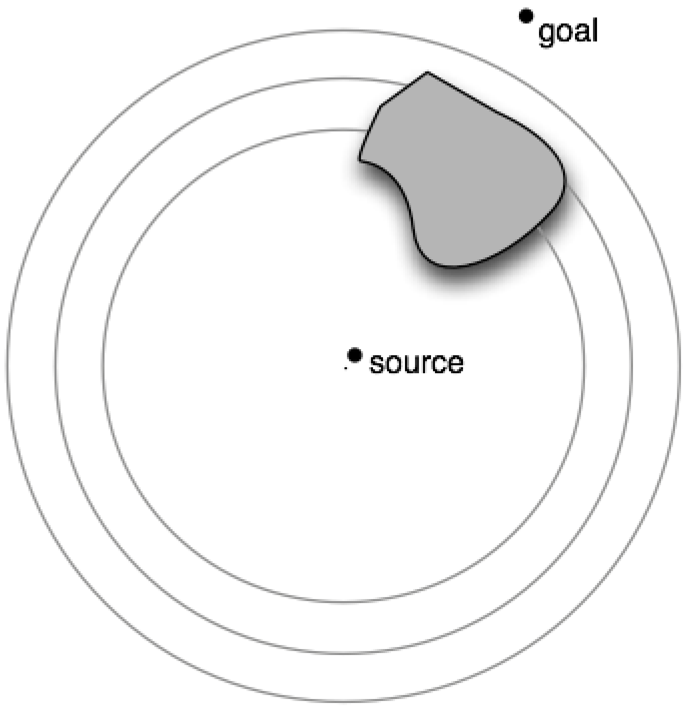
Intuition

- Finds a path to a particular vertex in a weighted graph better than Dijkstra's.
- Dijkstra's algorithm greedily visits nodes by increasing distance from initial vertex.
- Dijkstra's spends as much time exploring nodes that are *along* the optimal path as it does on exploring nodes that are in the *opposite direction* from the desired goal.
- The A* search algorithm directs the search towards the desired goal rather than simply based on distance from the initial vertex.

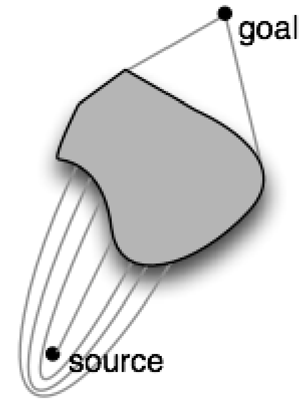
Reference:
Cornell U.: Data structures and functional programming
http://www.cs.cornell.edu/courses/cs312/2007sp

A*: A-star

Dijkstra vs. A*



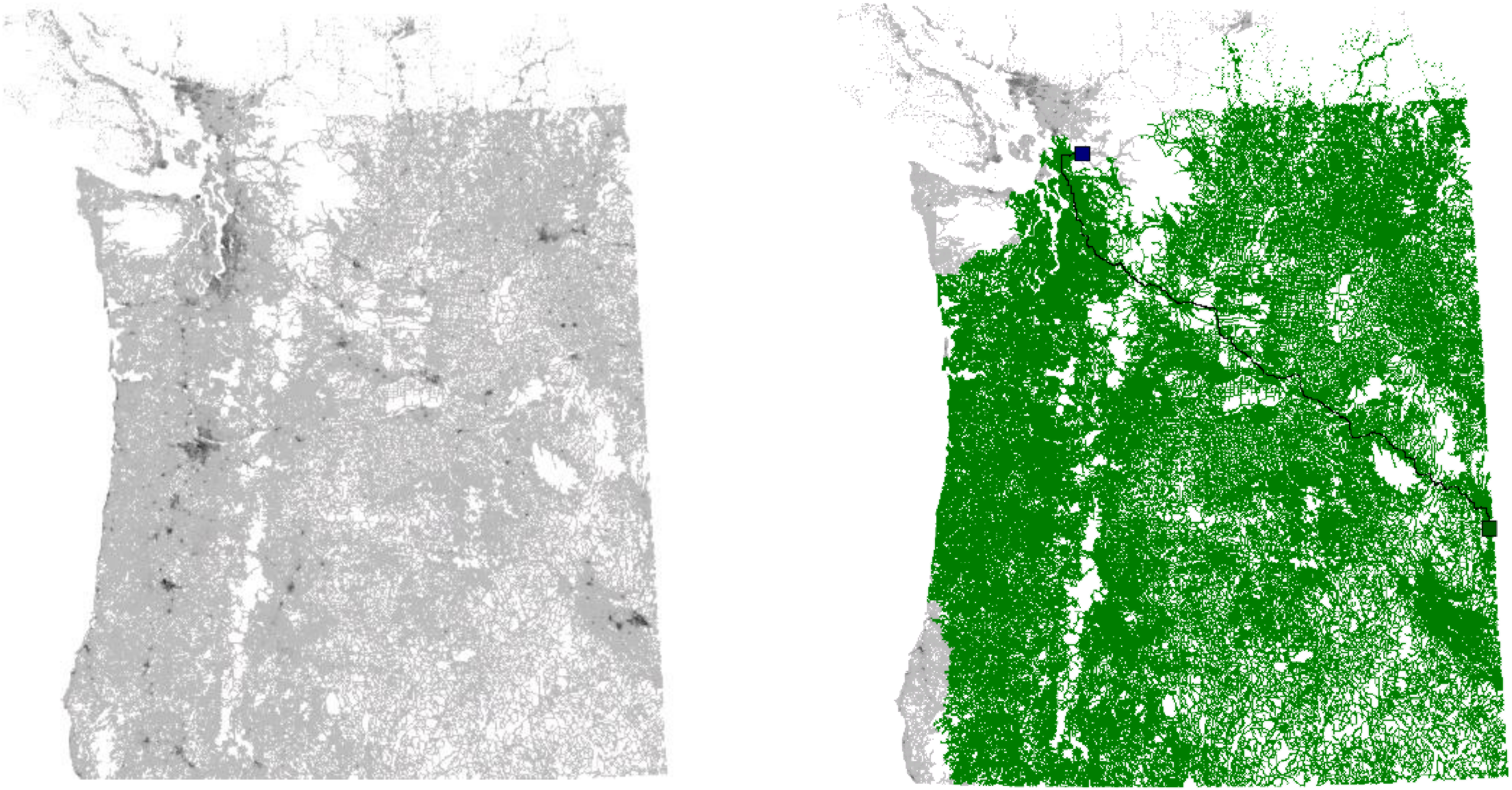
Dijkstra's algorithm



A* algorithm

A*: A-star

Dijkstra's behavior in large-scale road networks



1.6M vertices, 3.8M arcs, travel time metric.

Reference:

A. Goldberg (2009): Reach for A*

A*: A-star

Heuristic function

- Key idea is to define a **heuristic function** $h(v)$ that estimates how far a given vertex v is from the goal vertex.

- Run Dijkstra's shortest path algorithm with the notion of distance defined as the *sum* of the real distance and the heuristic function: $dist(v) + h(v)$

- Chooses vertex that minimizes the sum $dist(v)+ h(v)$.

- If heuristic function is perfectly accurate (correctly estimates the remaining distance from node v to the desired goal), all nodes along the optimum path have exactly the same value for $dist(v)+ h(v)$.
  - They will be dequeued sequentially off the priority queue without any other nodes being considered. So optimal path will be found as efficiently as is possible.
  - Perfectly accurate heuristic functions are not feasible in general. But even an approximately correct heuristic function can improve search performance dramatically.

A*: A-star

Heuristic function properties

- A heuristic function is **admissible** if it never overestimates the distance to the goal vertex.
  - When the heuristic function is applied to the goal vertex itself, it must return zero.

- A heuristic function is **monotonic** if the combined distance+heuristic from the initial vertex never decreases along any path.
  - This is actually a stronger property than admissibility
  - If a function is monotonic, it must also be admissible.

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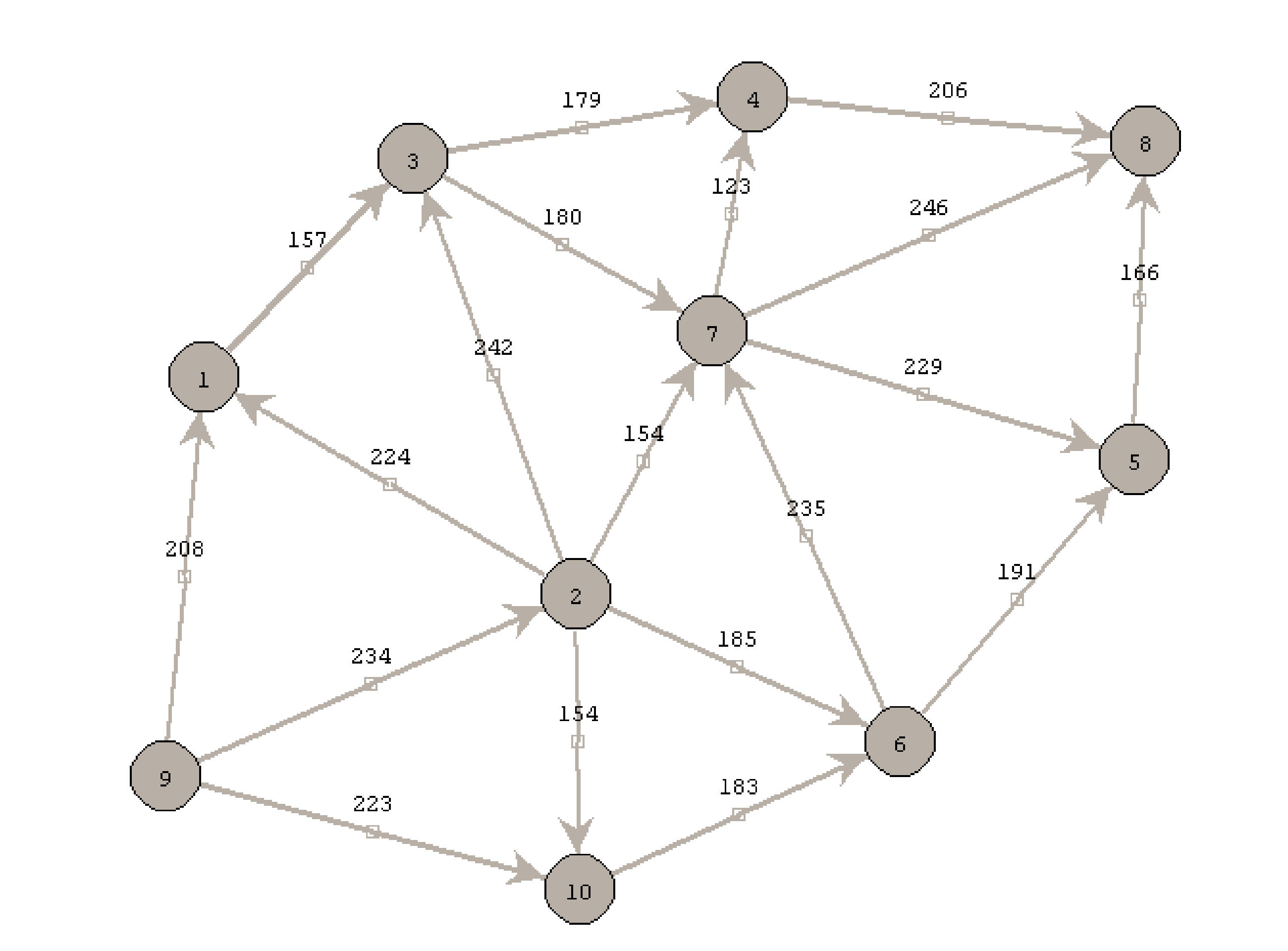
Algoritmo A*

Ejercicio en clase



Universidad de los Andes
Colombia

Algoritmo A*

Ejercicio en clase

Nodos

```
GdnIndex,GdnX,GdnY
int,int,int
1,182,271
2,376,384
3,291,157
4,468,125
5,667,314
6,545,461
7,447,247
8,673,148
9,162,479
10,378,538
```

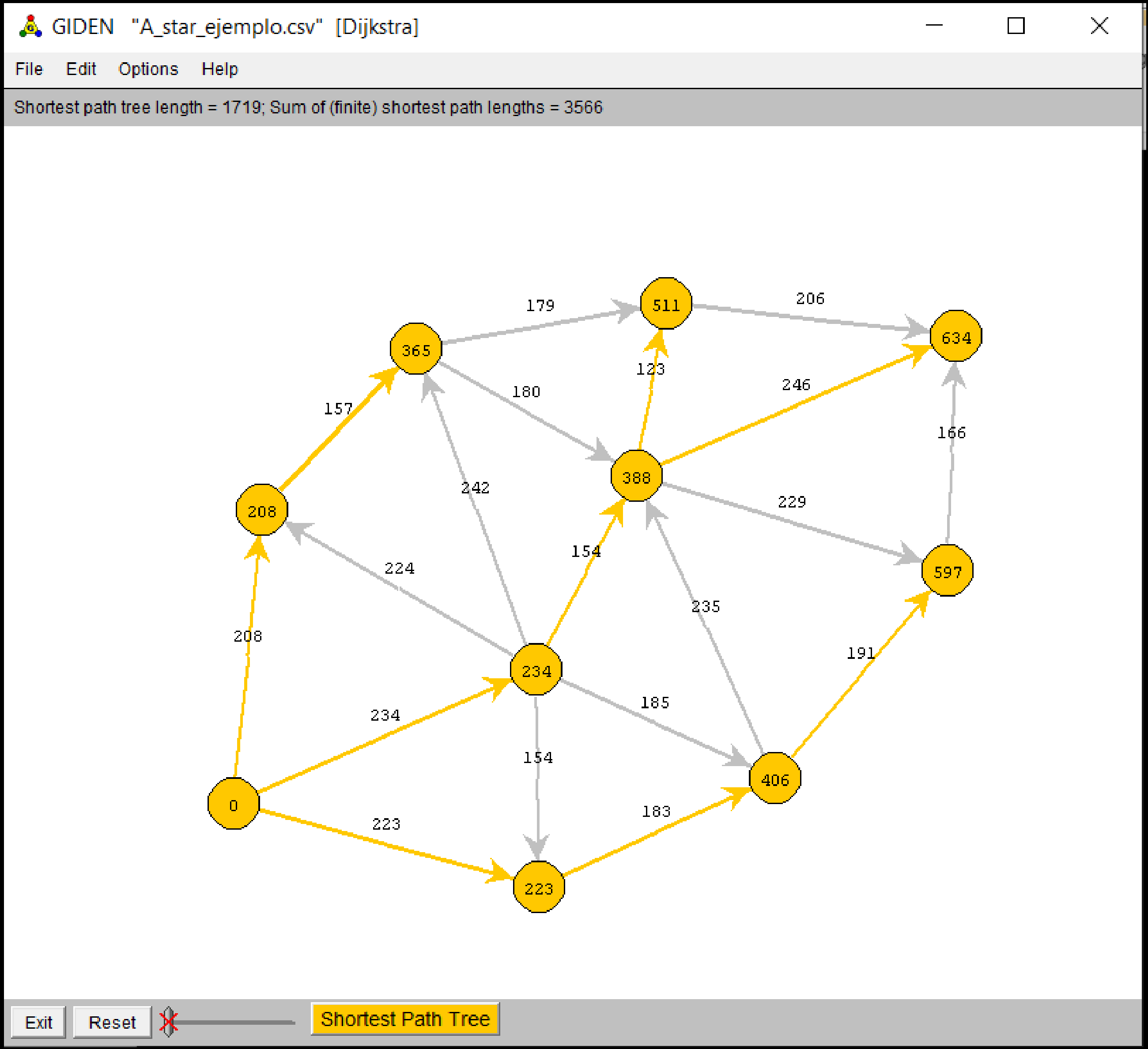
Arcos

```
GdnIndex,GdnSource,GdnTarget
int,int,int
1,9,1
2,9,2
3,9,10
4,2,10
5,2,6
6,10,6
7,2,1
8,2,3
9,2,7
10,1,3
11,3,7
12,7,4
13,3,4
14,7,8
15,4,8
16,7,5
17,6,7
18,6,5
19,5,8
```

Universidad de los Andes
Colombia

Algoritmo A*

Ejercicio en clase



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A*: A-star

What to expect from A*?

1.6M vertices, 3.8M arcs, travel time metric.

Reference:

A. Goldberg (2009): Reach for A*

Agenda

- Part I: fundamentals
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  - Labeling algorithms
  - **Branch and bound**
- Part II: intuition
- Part III: extensions
- Part IV: applications
- Part V: perspectives

Ramificación y acotamiento
Principios

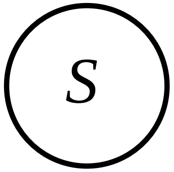
Proposition 7.1:
Let $S = S_1 \cup \ldots \cup S_K$ be a decomposition of S into smaller sets, and let $z^k = \max\{\mathbf{cx} : \mathbf{x} \in S_k\}$ for $k = 1, \ldots, K$. Then $z = \max_k z^k$.

- Descomposición del espacio de solución en conjuntos más pequeños
- Escogencia del mejor valor de la función

Fuente:
Wolsey. (1998). Integer Programming. Wiley.

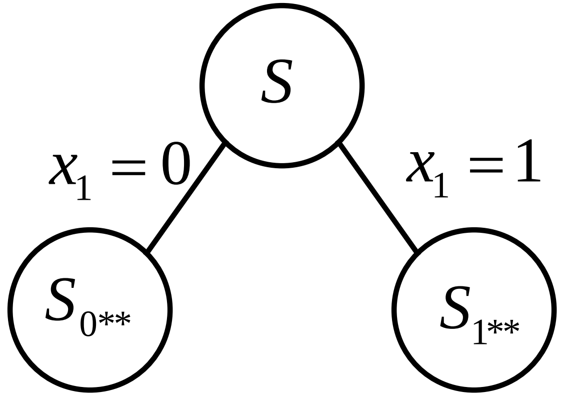
Ramificación y acotamiento
Ejemplo

$$S \subseteq \{0,1\}^3$$



Ramificación y acotamiento
Ejemplo

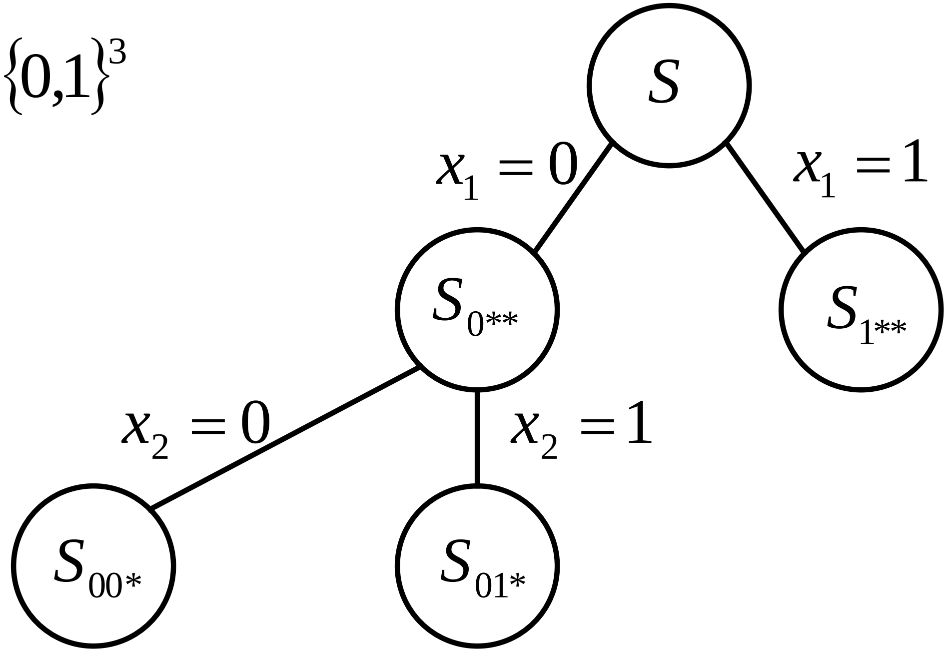
$S \subseteq \{0,1\}^3$



Ramificación y acotamiento

Ejemplo

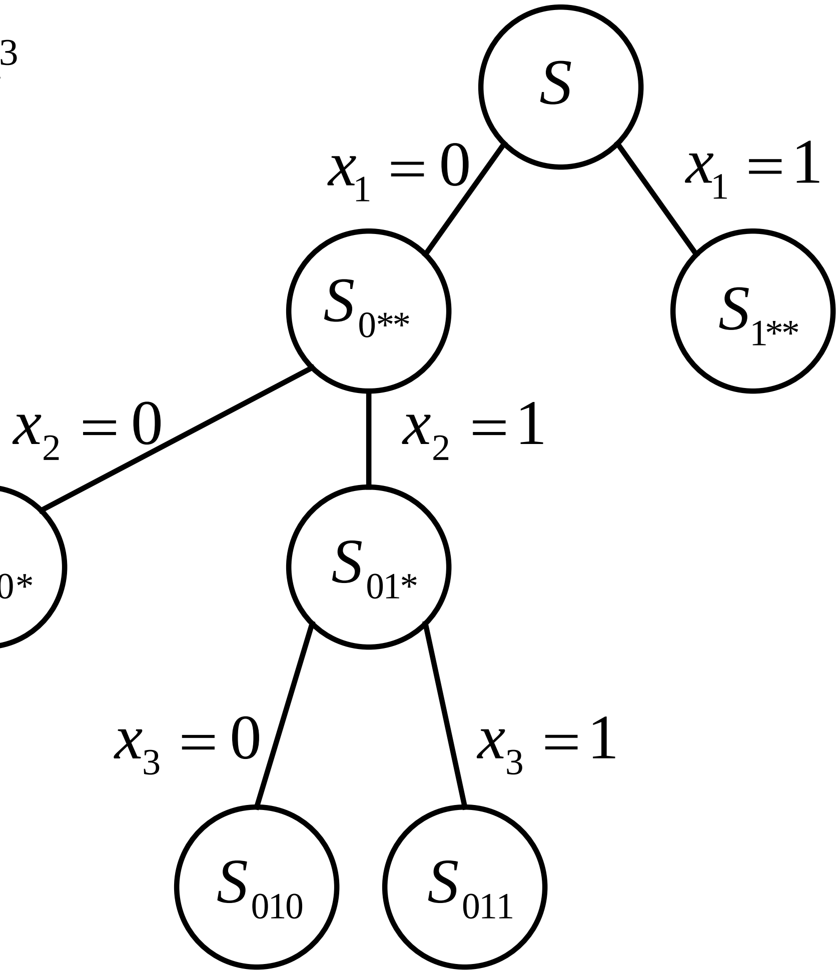
$S \subseteq \{0,1\}^3$



Ramificación y acotamiento

Ejemplo

$S \subseteq \{0,1\}^3$



Ramificación y acotamiento
Ejemplo

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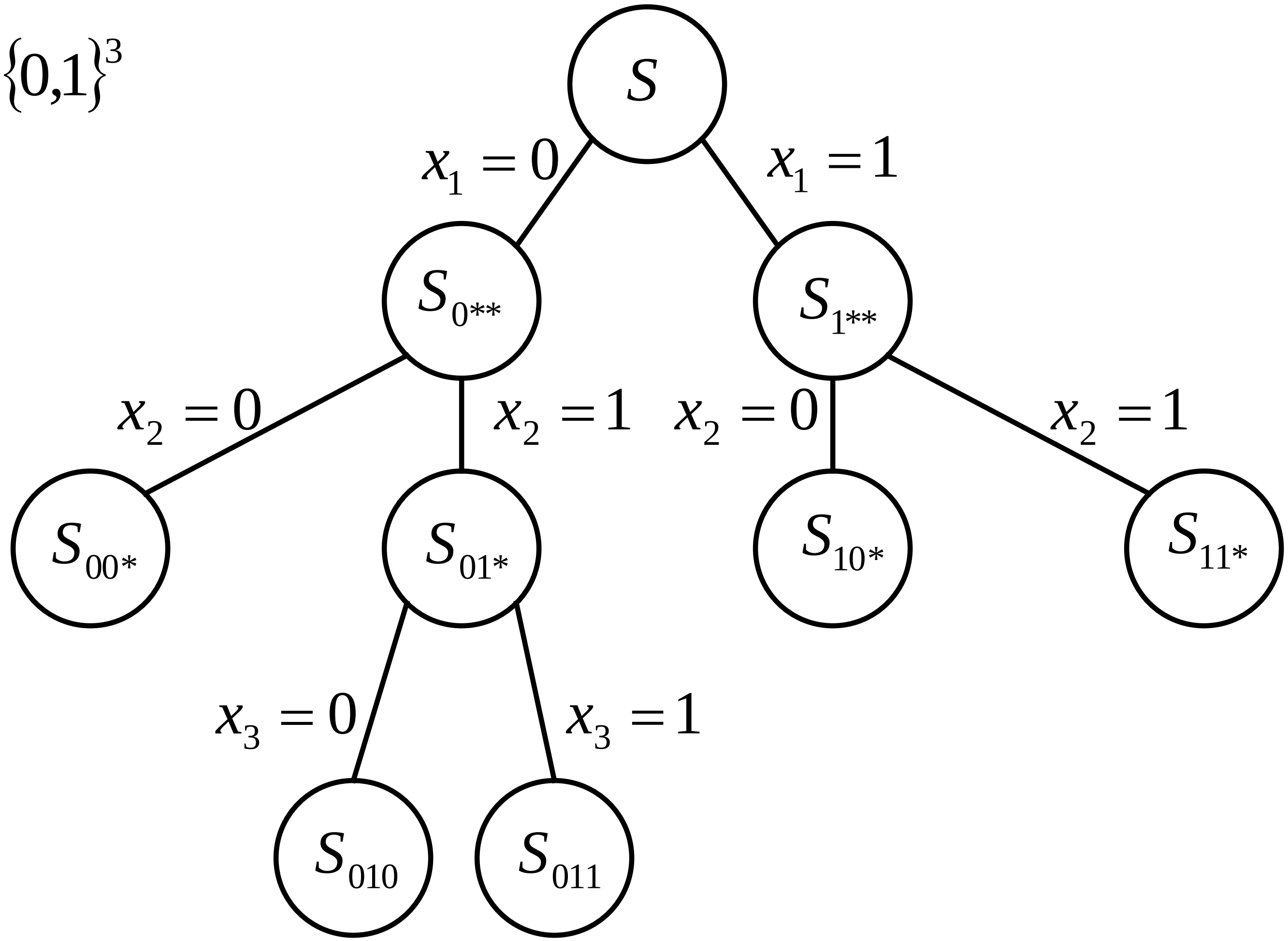
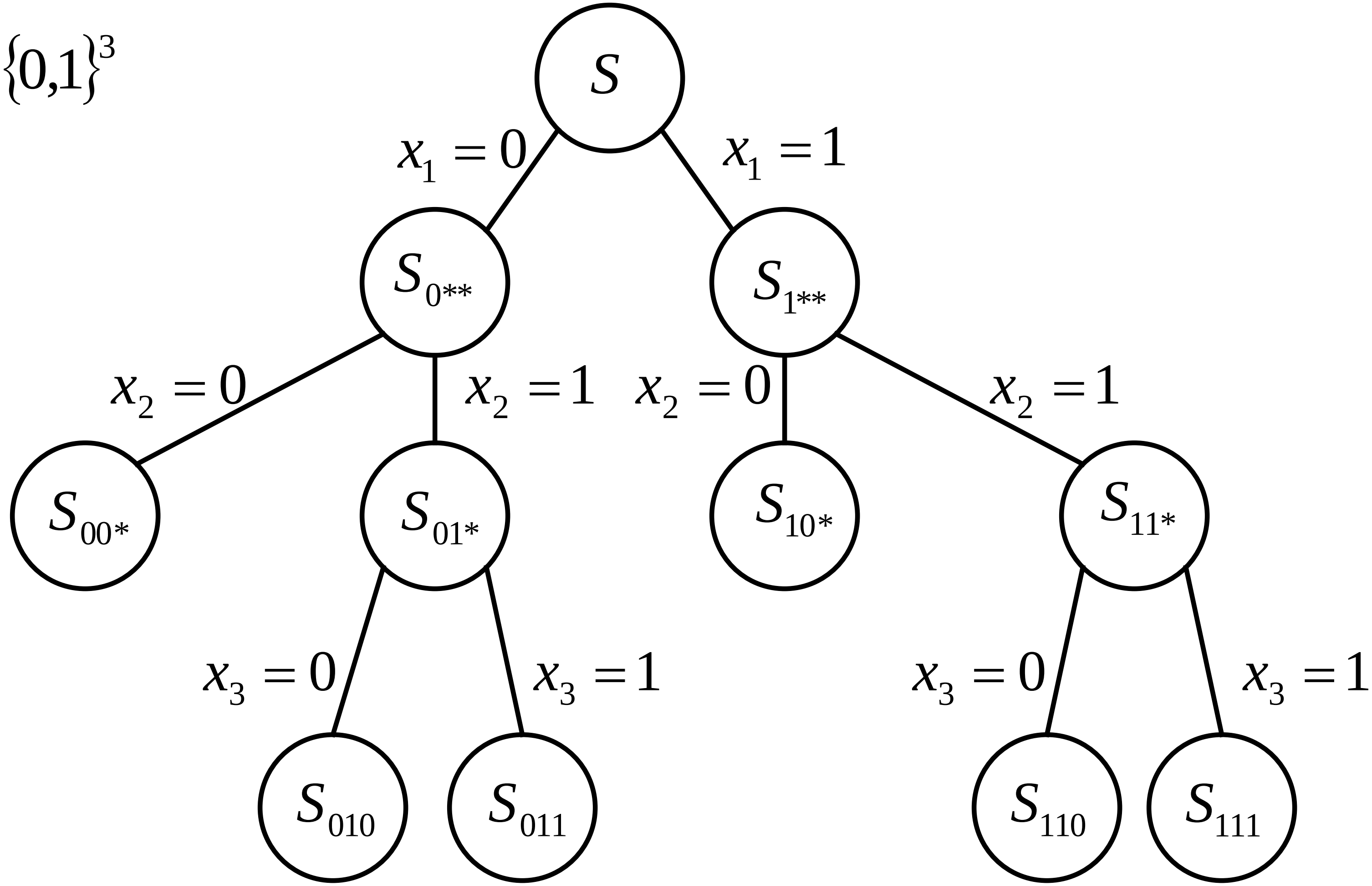


Ramificación y acotamiento
Ejemplo

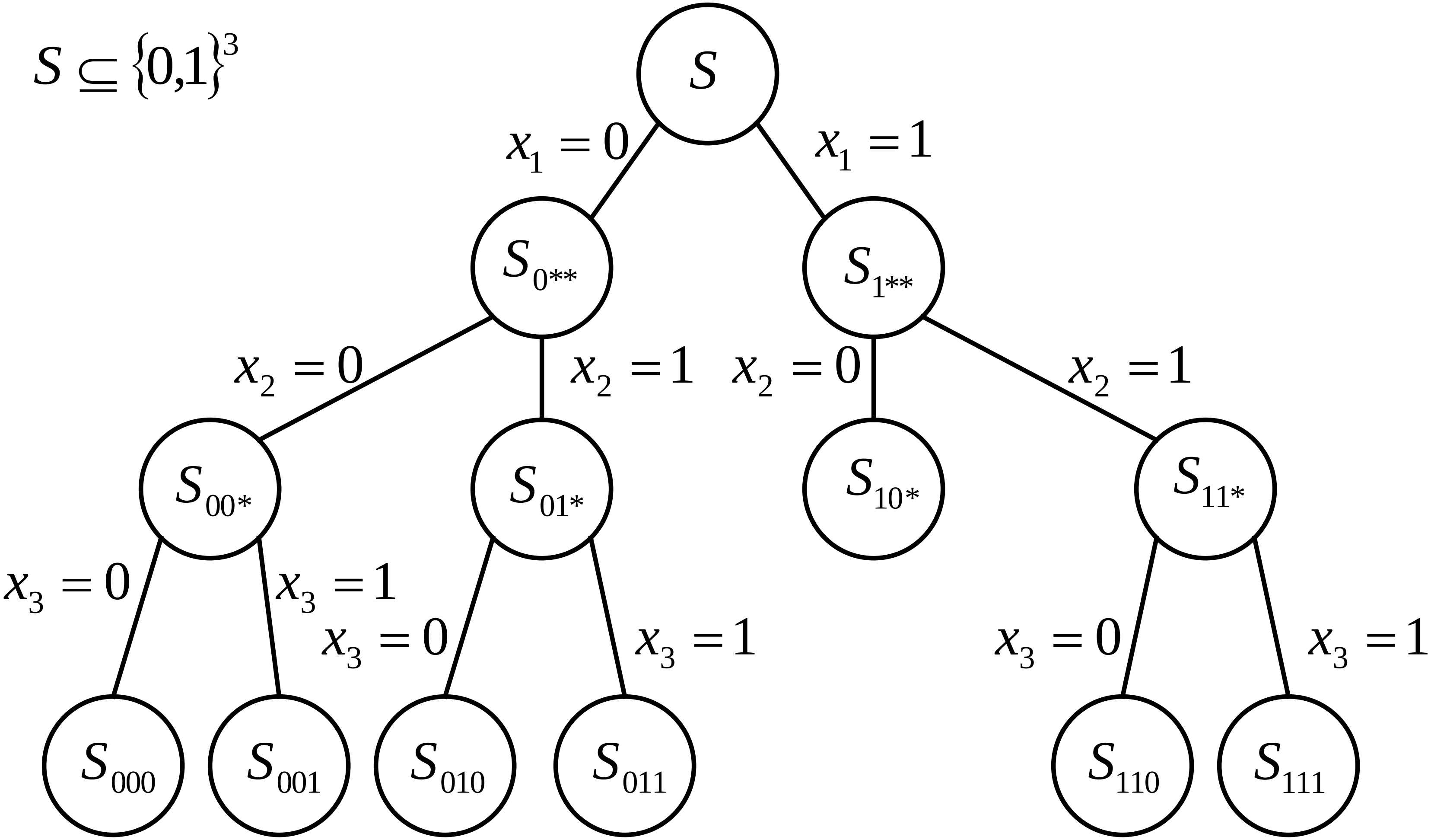
$S \subseteq \{0,1\}^3$



Ramificación y acotamiento

Ejemplo

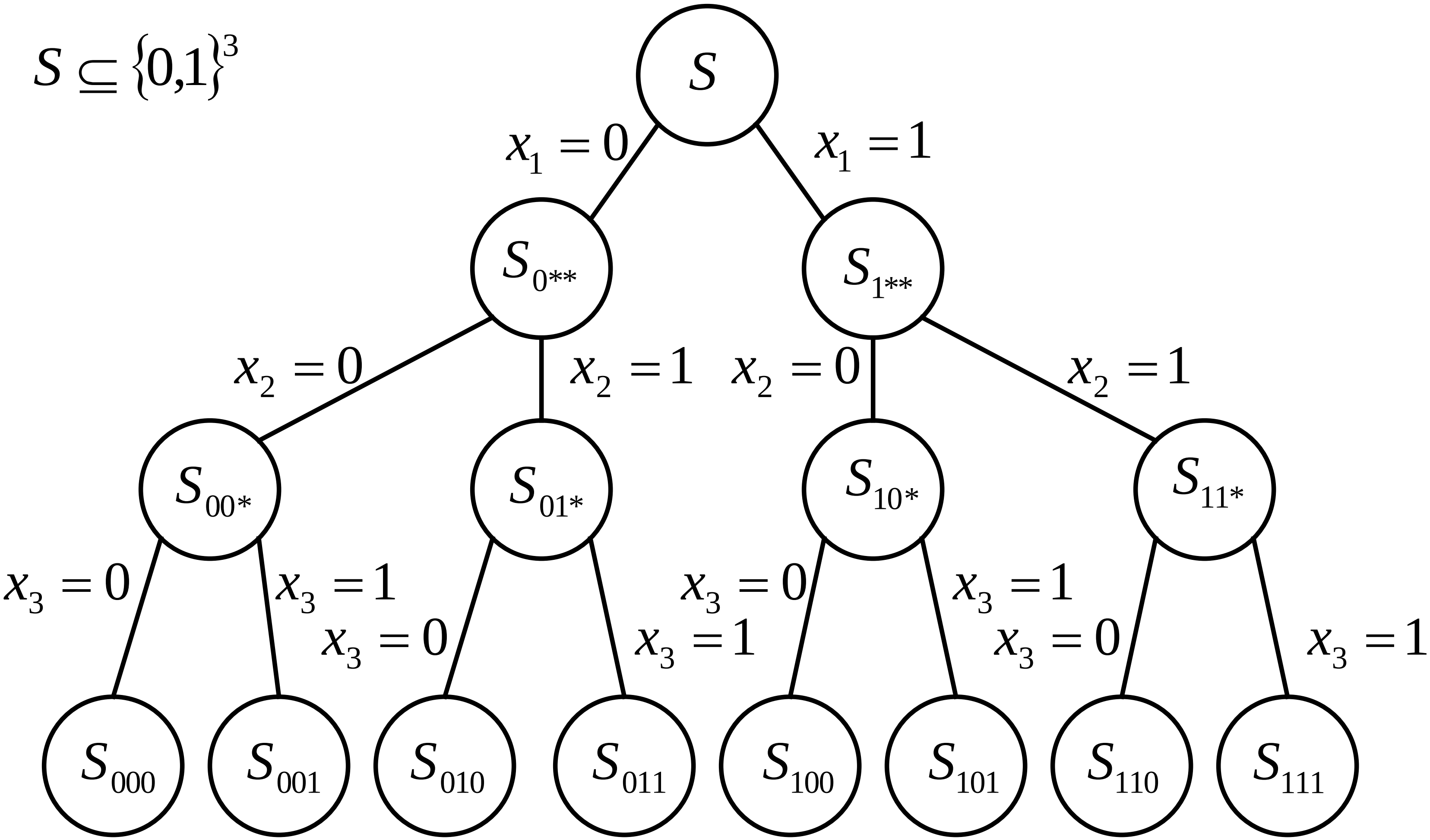
$$S \subseteq \{0,1\}^3$$



Ramificación y acotamiento

Ejemplo

$$S \subseteq \{0,1\}^3$$



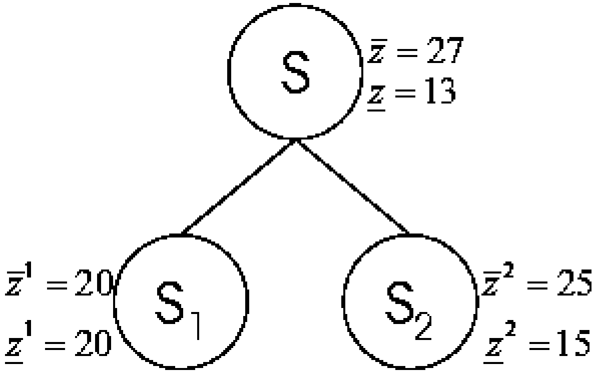
Ramificación y acotamiento
Cotas

Proposition 7.2:
Let $S = S_1 \cup \ldots \cup S_K$ be a decomposition of S into smaller sets, and let $z^k = \max\{\mathbf{cx} : \mathbf{x} \in S_k\}$ for $k = 1, \ldots, K$, $\overline{z}^k$ be an upper bound on z^k and $\underline{z}^k$ be a lower bound on z^k. Then $\overline{z} = \max_k \overline{z}^k$ is an upper bound on z and $\underline{z} = \max_k \underline{z}^k$ is a lower bound on z.

- Función objetivo
  - Cota superior
  - Cota inferior
- ¿Para qué las cotas?
  - Podar ramas del árbol
  - Búsqueda eficiente

Ramificación y acotamiento

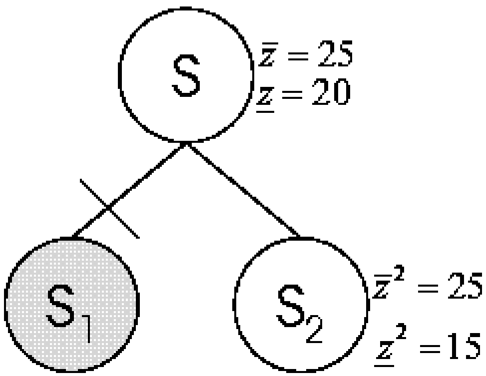
Podas



$$\overline{z} = \max_k \overline{z}^k = \max\{20, 25\} = 25$$
$$\underline{z} = \max_k \underline{z}^k = \max\{20, 15\} = 20$$

Ramificación y acotamiento

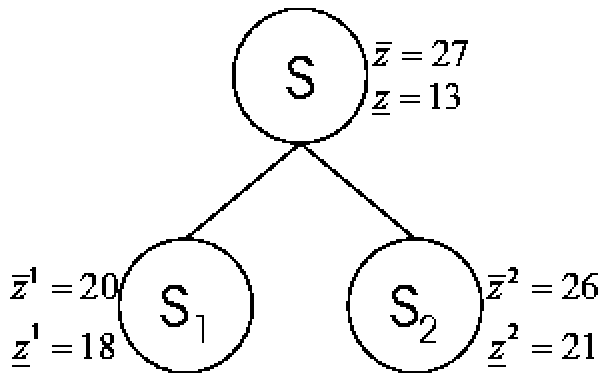
Podas



- **Nodo S_1 (y sus descendentes) podado por optimalidad**

Ramificación y acotamiento

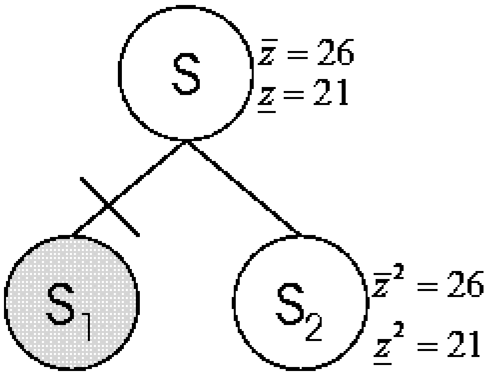
Podas



$$\overline{z} = \max_k \overline{z}^k = \max\{20, 26\} = 26$$
$$\underline{z} = \max_k \underline{z}^k = \max\{18, 21\} = 21$$

Ramificación y acotamiento

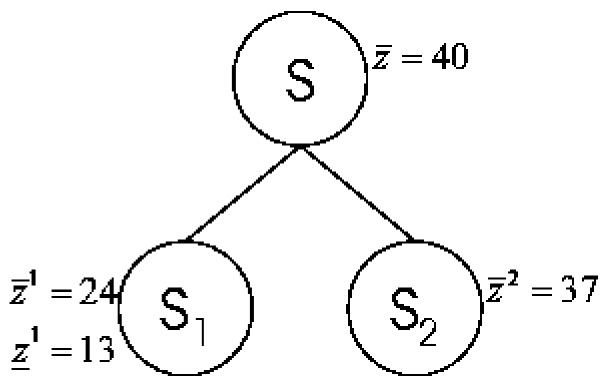
Podas



- **Nodo S_1 (y sus descendentes) podado por cota**

Ramificación y acotamiento

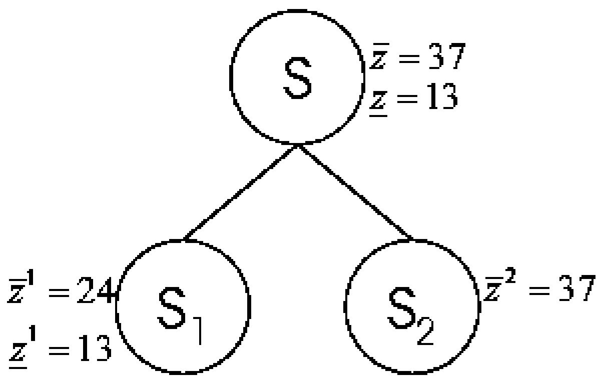
Podas



$$\overline{z} = \max_k \overline{z}^k = \max\{24, 37\} = 37$$
$$\underline{z} = \max_k \underline{z}^k = \max\{13, -\infty\} = 13$$

Ramificación y acotamiento

Podas



- **No es posible podar el árbol**

Ramificación y acotamiento

¿Cómo podar el árbol?

- Por optimalidad
  - Se resuelve:

$$z^t = \{\max \mathbf{cx} : \mathbf{x} \in S_t\}$$

- Por cota

$$\overline{z}^t \leq \underline{z}$$

- Por infactibilidad

$$S_t = \emptyset$$

Agenda

- Part I: fundamentals
  - Dijkstra
  - A*
  - **Labeling algorithms**
  - Branch and bound
- Part II: intuition
- Part III: extensions
- Part IV: applications
- Part V: perspectives

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$$\min \sum_{a \in A} c_a x_a$$

$$\text{s.t.} \sum_{a \in \delta^+(i)} x_a - \sum_{a \in \delta^-(i)} x_a = \begin{cases} 1, & \text{if } i = s \\ -1, & \text{if } i = t \\ 0, & \text{if } i \in V \setminus \{s, t\} \end{cases} \qquad \forall i \in V$$

$$\sum_{a \in A} w_a x_a \leqslant W$$

$$x_a \in \{0, 1\}, \qquad \forall a \in A$$

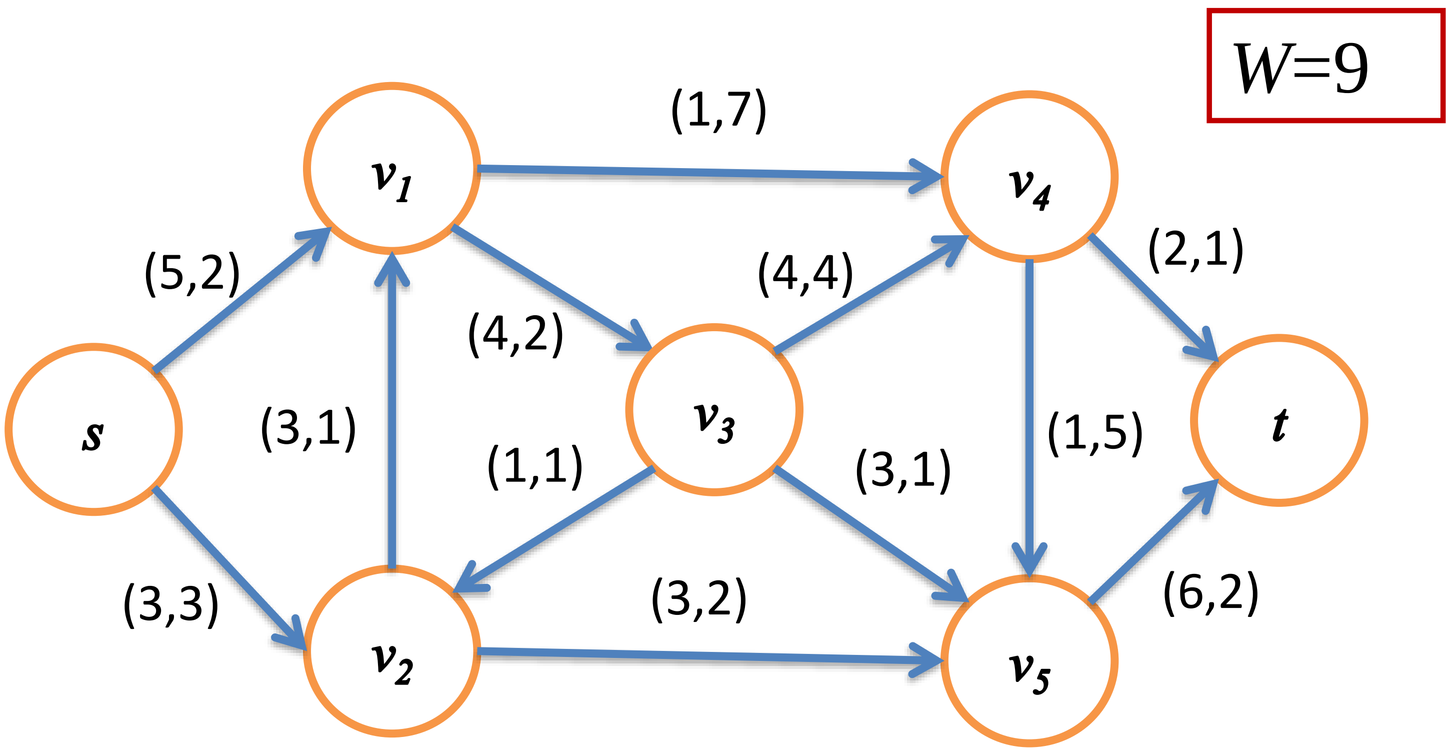
- The WCSPP is an NP-hard problem, even if we consider the case when the graph is acyclic and all weights and costs are positive (Garey and Johnson, 1979).

References:
Dumistrescu and Boland (2001)
Desrochers and Soumis (1988)

Algoritmo de Labeling

Ejercicio en clase

PARE



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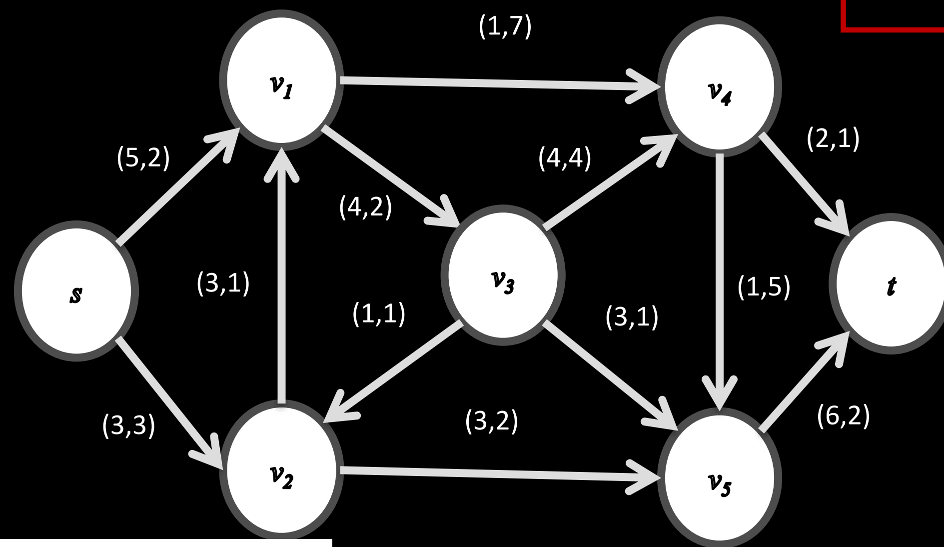
WCSPP - LABELING

EJERCICIO EN CLASE

W=9



Algorithm 1. *The label setting algorithm*

Step 0: *Initialization*
Set $L_s = \{(0, 0)\}$ and $L_i = \varnothing$ for all $i \in V \backslash \{s\}$.
Initialize I_i accordingly for each $i \in V$.
Set $T_i = \varnothing$ for each $i \in V$.

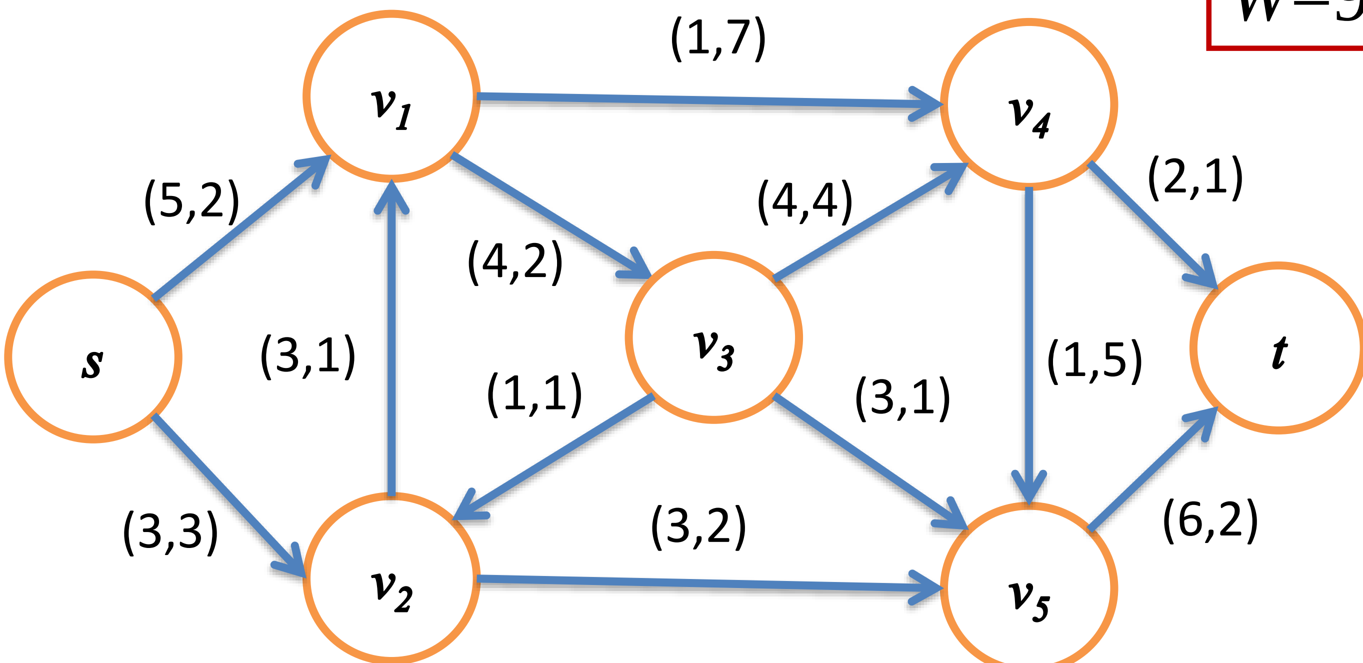
Step 1: *Selection of the label to be treated*
if $\bigcup_{i \in V}(I_i \backslash T_i) = \varnothing$ **then** STOP; all efficient labels have been generated
else choose $i \in V$ and $k \in I_i \backslash T_i$ so that W_i^k is minimal.

Step 2: *Treatment of label* (W_i^k, C_i^k)
for all $(i, j) \in \delta^+(i)$ with $W_i^k + w_{ij} \leqslant W$ **do**
  if $(W_i^k + w_{ij}, C_i^k + c_{ij})$ is not dominated
    by (W_j^l, C_j^l) for any $l \in I_j$
  then
    Set $L_j = L_j \cup \{(W_i^k + w_{ij}, C_i^k + c_{ij})\}$.
    Update I_j accordingly.
Set $T_i := T_i \cup \{k\}$.
go to Step 1.

Labeling algorithms

WCSPP - example

$W=9$



i	$\delta^+(i)$
s	$\{1,2\}$
1	$\{3,4\}$
2	$\{1,5\}$
3	$\{2,4,5\}$
4	$\{5,t\}$
5	$\{t\}$
t	$\emptyset$

Labeling algorithms

WCSPP - example

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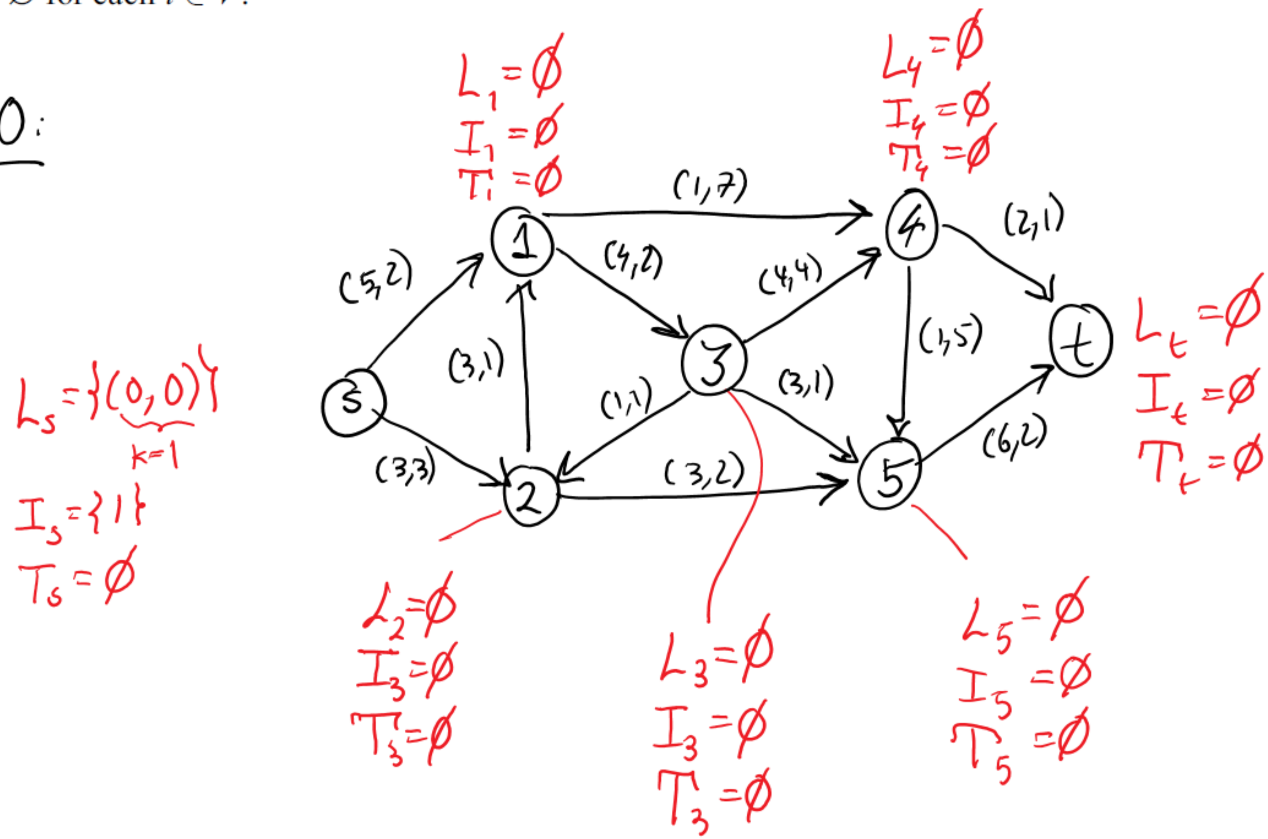
Algorithm 1. *The label setting algorithm*

Step 0: *Initialization*

Set $L_s = \{(0, 0)\}$ and $L_i = \varnothing$ for all $i \in V \setminus \{s\}$.
Initialize I_i accordingly for each $i \in V$.
Set $T_i = \varnothing$ for each $i \in V$.

Step 0:

$\boxed{T = 9}$



Labeling algorithms

WCSPP - example

Step 1: *Selection of the label to be treated*

if $\bigcup_{i\in V}(I_i \backslash T_i) = \emptyset$ **then** STOP; all efficient labels have been generated

else choose $i \in V$ and $k \in I_i \backslash T_i$ so that W_i^k is minimal.

Step 1:

$$(s,1) \leftarrow \arg \min_{i\in V,\, k\in I_i \backslash T_i} \underbrace{\{0\}}_{i=s,\, k=1}$$

Labeling algorithms

WCSPP - example

Step 2: *Treatment of label* (W_i^k, C_i^k)

for all $(i, j) \in \delta^+(i)$ with $W_i^k + w_{ij} \leqslant W$ **do**

if $(W_i^k + w_{ij}, C_i^k + c_{ij})$ is not dominated by (W_j^l, C_j^l) for any $l \in I_j$

then

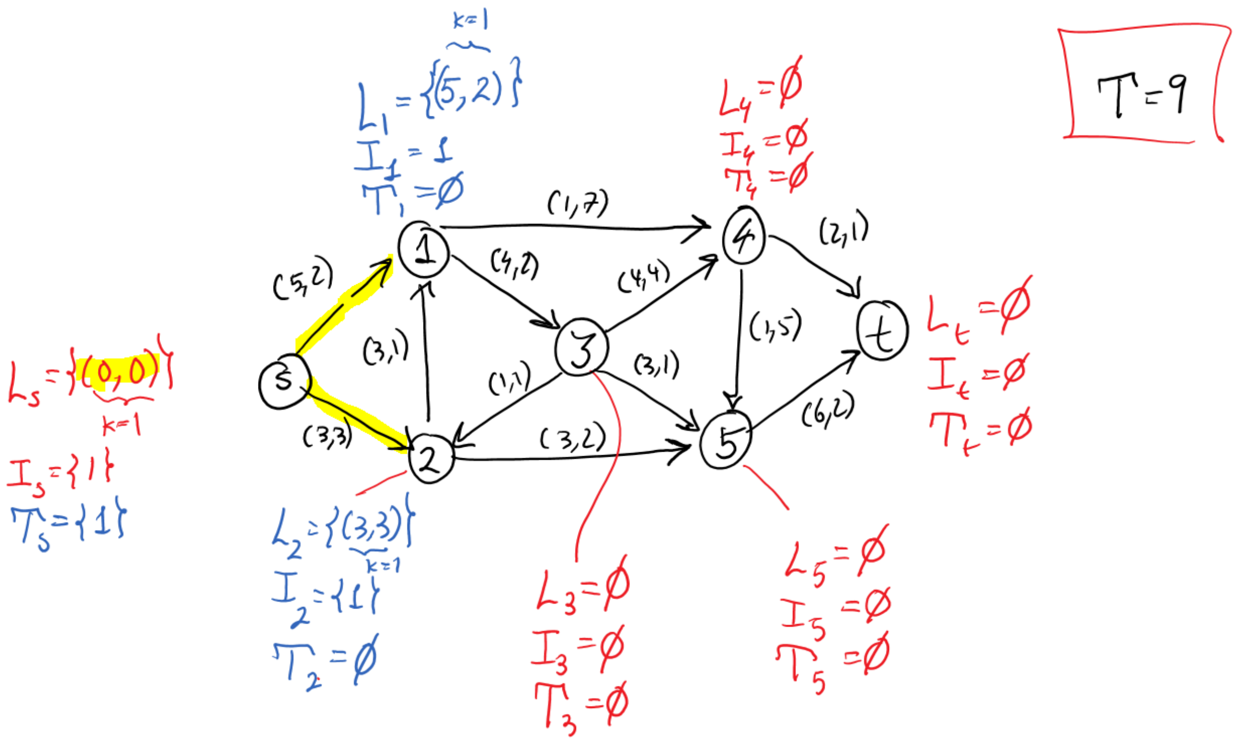
Set $L_j = L_j \cup \{(W_i^k + w_{ij}, C_i^k + c_{ij})\}$.

Update I_j accordingly.

Set $T_i := T_i \cup \{k\}$.

go to Step 1.

Step 2:



Labeling algorithms

WCSPP - example

Step 1: *Selection of the label to be treated*

if $\bigcup_{i \in V}(I_i \backslash T_i) = \emptyset$ **then** STOP; all efficient labels have been generated

else choose $i \in V$ and $k \in I_i \backslash T_i$ so that W_i^k is minimal.

Step 1:

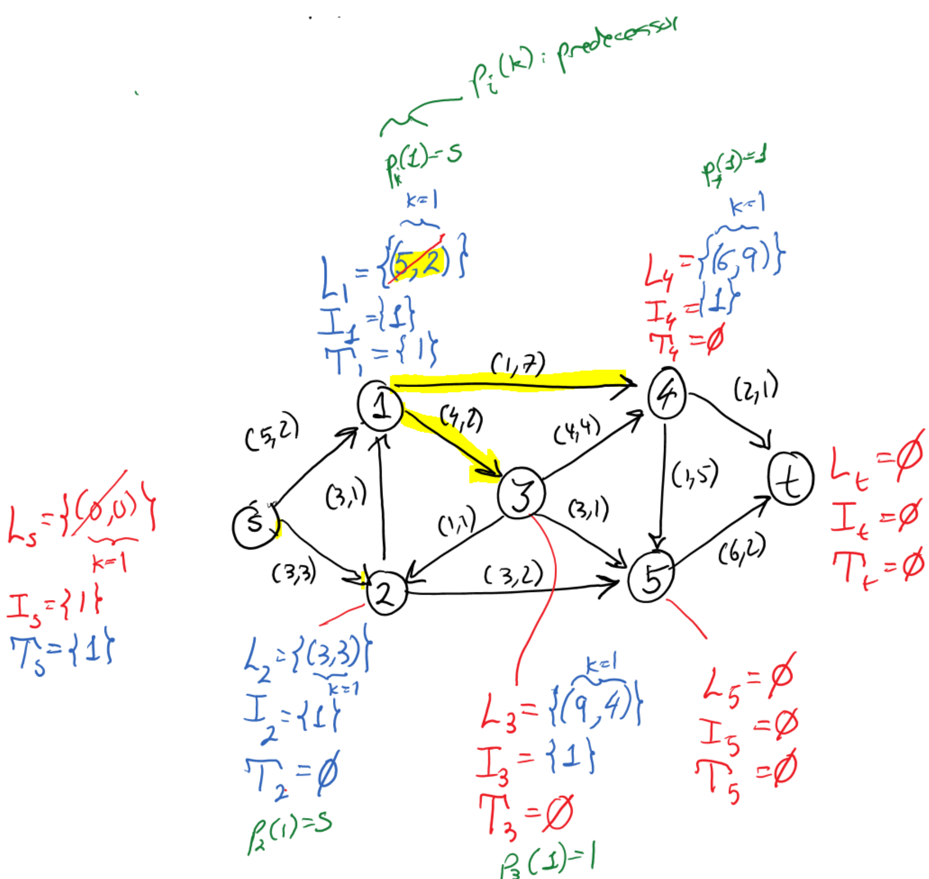
$$(1,1) \leftarrow \arg \min_{i \in V,\, k \in I_i \backslash T_i} \{ \underbrace{2}_{\substack{i=1 \\ k=1}} , \underbrace{3}_{\substack{i=2 \\ k=1}} \}$$

Labeling algorithms

WCSPP - example

Step 2: *Treatment of label* (W_i^k, C_i^k)
for all $(i, j) \in \delta^+(i)$ with $W_i^k + w_{ij} \leqslant W$ **do**
  if $(W_i^k + w_{ij}, C_i^k + c_{ij})$ is not dominated
    by (W_j^l, C_j^l) for any $l \in I_j$
  then
    Set $L_j = L_j \cup \{(W_i^k + w_{ij}, C_i^k + c_{ij})\}$.
    Update I_j accordingly.
Set $T_i := T_i \cup \{k\}$.
go to Step 1.

Step 2:



$T=9$

Labeling algorithms

WCSPP - example

Step 1: *Selection of the label to be treated*

if $\bigcup_{i \in V}(I_i \setminus T_i) = \varnothing$ **then** STOP; all efficient labels have been generated

else choose $i \in V$ and $k \in I_i \setminus T_i$ so that W_i^k is minimal.

Step 1:

$$(2,1) \leftarrow \arg \min_{i \in V,\ k \in I_i \setminus T_i} \left\{ \underbrace{3}_{\substack{i=2 \\ k=1}}, \underbrace{9}_{\substack{i=4 \\ k=1}}, \underbrace{4}_{\substack{i=3 \\ k=1}} \right\}$$

Labeling algorithms

WCSPP - example

Step 2: *Treatment of label* (W_i^k, C_i^k)

for all $(i, j) \in \delta^+(i)$ with $W_i^k + w_{ij} \leqslant W$ **do**

if $(W_i^k + w_{ij}, C_i^k + c_{ij})$ is not dominated by (W_j^l, C_j^l) for any $l \in I_j$

then

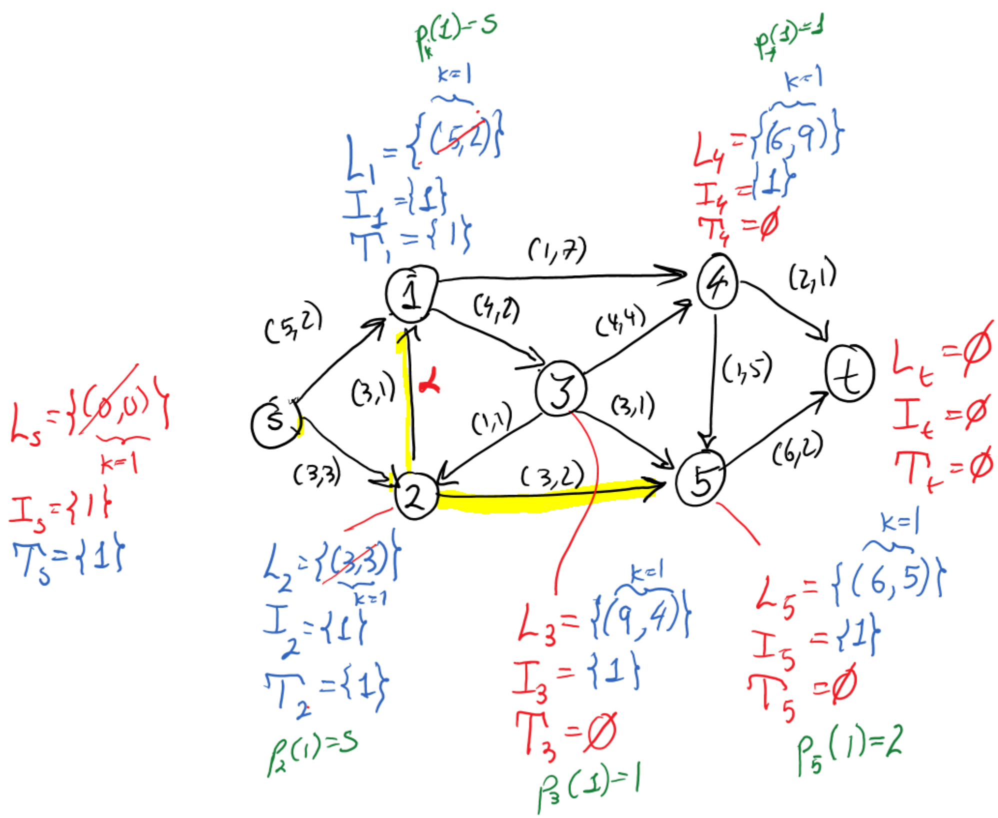
Set $L_j = L_j \cup \{(W_i^k + w_{ij}, C_i^k + c_{ij})\}$.

Update I_j accordingly.

Set $T_i := T_i \cup \{k\}$.

go to Step 1.

Step 2:



$T=9$

Labeling algorithms

WCSPP - example

Step 1: *Selection of the label to be treated*

if $\bigcup_{i \in V}(I_i \backslash T_i) = \varnothing$ **then** STOP; all efficient labels have been generated

else choose $i \in V$ and $k \in I_i \backslash T_i$ so that W_i^k is minimal.

Step 1:

$$(3,1) \leftarrow \arg\min_{i \in V,\, k \in I_i \backslash T_i} \left\{ \underbrace{4}_{\substack{i=3\\k=1}}, \underbrace{9}_{\substack{i=4\\k=1}}, \underbrace{5}_{\substack{i=5\\k=1}} \right\}$$

Labeling algorithms

WCSPP - example

Step 2: *Treatment of label* (W_i^k, C_i^k)

  for all $(i, j) \in \delta^+(i)$ with $W_i^k + w_{ij} \leqslant W$ **do**

    if $(W_i^k + w_{ij}, C_i^k + c_{ij})$ is not dominated

      by (W_j^l, C_j^l) for any $l \in I_j$

    then

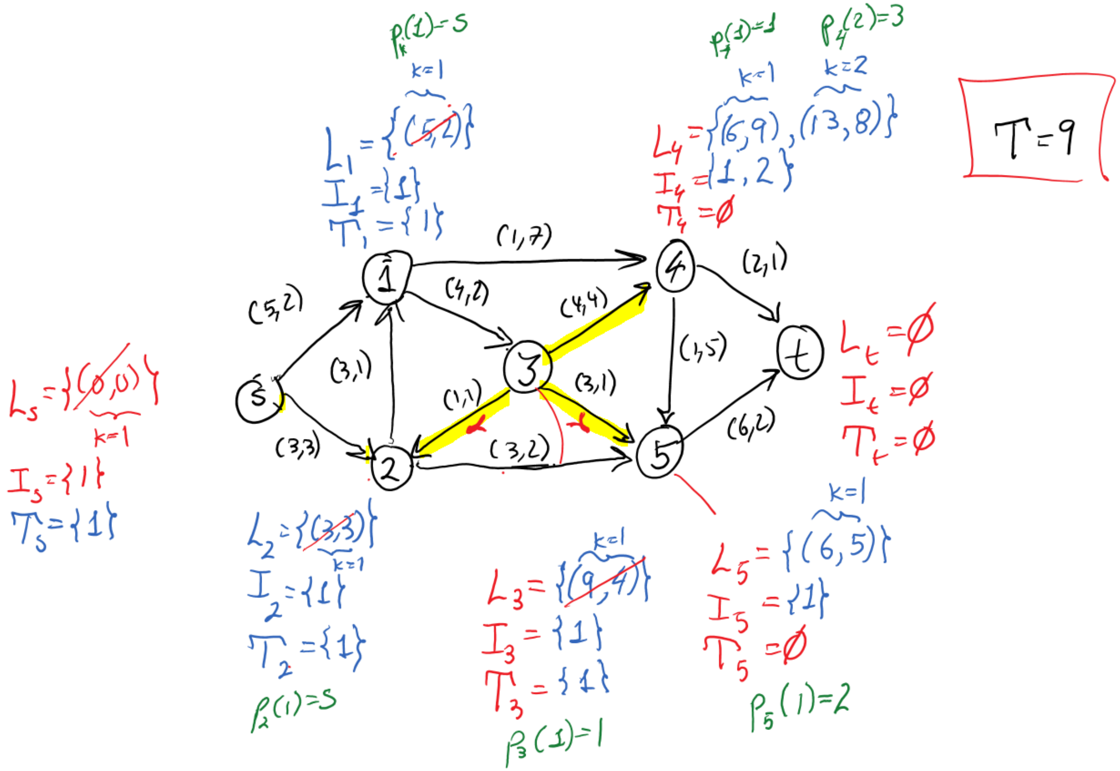
      Set $L_j = L_j \cup \{(W_i^k + w_{ij}, C_i^k + c_{ij})\}$.

      Update I_j accordingly.

  Set $T_i := T_i \cup \{k\}$.

  go to Step 1.

Step2:



Labeling algorithms

WCSPP - example

Step 1: *Selection of the label to be treated*

if $\bigcup_{i \in V}(I_i \setminus T_i) = \emptyset$ **then** STOP; all efficient labels have been generated

else choose $i \in V$ and $k \in I_i \setminus T_i$ so that W_i^k is minimal.

Step 1:

$$(5,1) \leftarrow \arg \min_{i \in V,\, k \in I_i \setminus T_i} \left\{ \underbrace{8}_{\substack{i=4 \\ k=2}}, \underbrace{5}_{\substack{i=5 \\ k=1}} \right\}$$

Labeling algorithms

WCSPP - example

Step 2: *Treatment of label* (W_i^k, C_i^k)

for all $(i, j) \in \delta^+(i)$ with $W_i^k + w_{ij} \leqslant W$ **do**

if $(W_i^k + w_{ij}, C_i^k + c_{ij})$ is not dominated by (W_j^l, C_j^l) for any $l \in I_j$

then

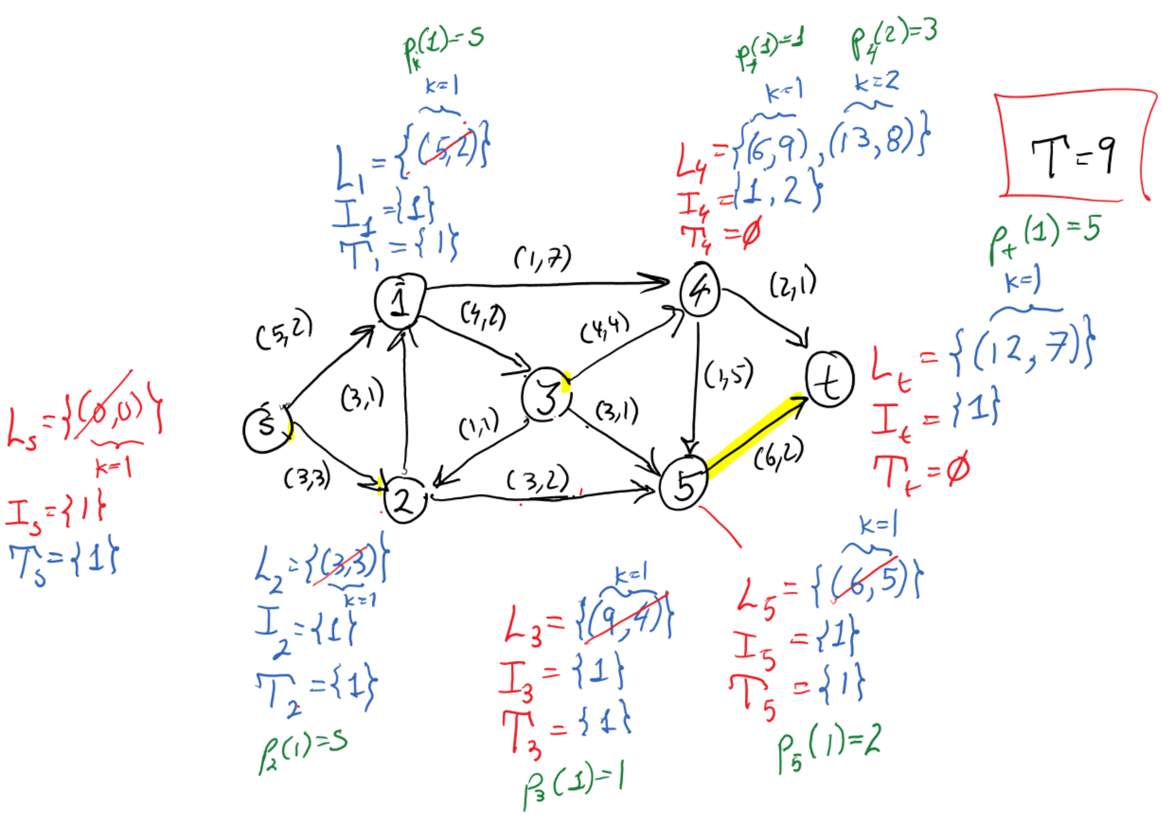
Set $L_j = L_j \cup \{(W_i^k + w_{ij}, C_i^k + c_{ij})\}$.

Update I_j accordingly.

Set $T_i := T_i \cup \{k\}$.

go to Step 1.

Step 2:



Labeling algorithms

WCSPP - example

Step 1: *Selection of the label to be treated*
if $\bigcup_{i \in V}(I_i \setminus T_i) = \varnothing$ **then** STOP; all efficient labels have been generated
else choose $i \in V$ and $k \in I_i \setminus T_i$ so that W_i^k is minimal.

Step 1:

$$(t,1) \leftarrow \arg \min_{i \in V,\, k \in I_i \setminus T_i} \Big\{ \underbrace{8}_{\substack{i=4 \\ k=2}}, \underbrace{7}_{\substack{i=t \\ k=1}} \Big\}$$

Labeling algorithms

WCSPP - example

Step 2: *Treatment of label* (W_i^k, C_i^k)

for all $(i, j) \in \delta^+(i)$ with $W_i^k + w_{ij} \leqslant W$ **do**

if $(W_i^k + w_{ij}, C_i^k + c_{ij})$ is not dominated by (W_j^l, C_j^l) for any $l \in I_j$

then

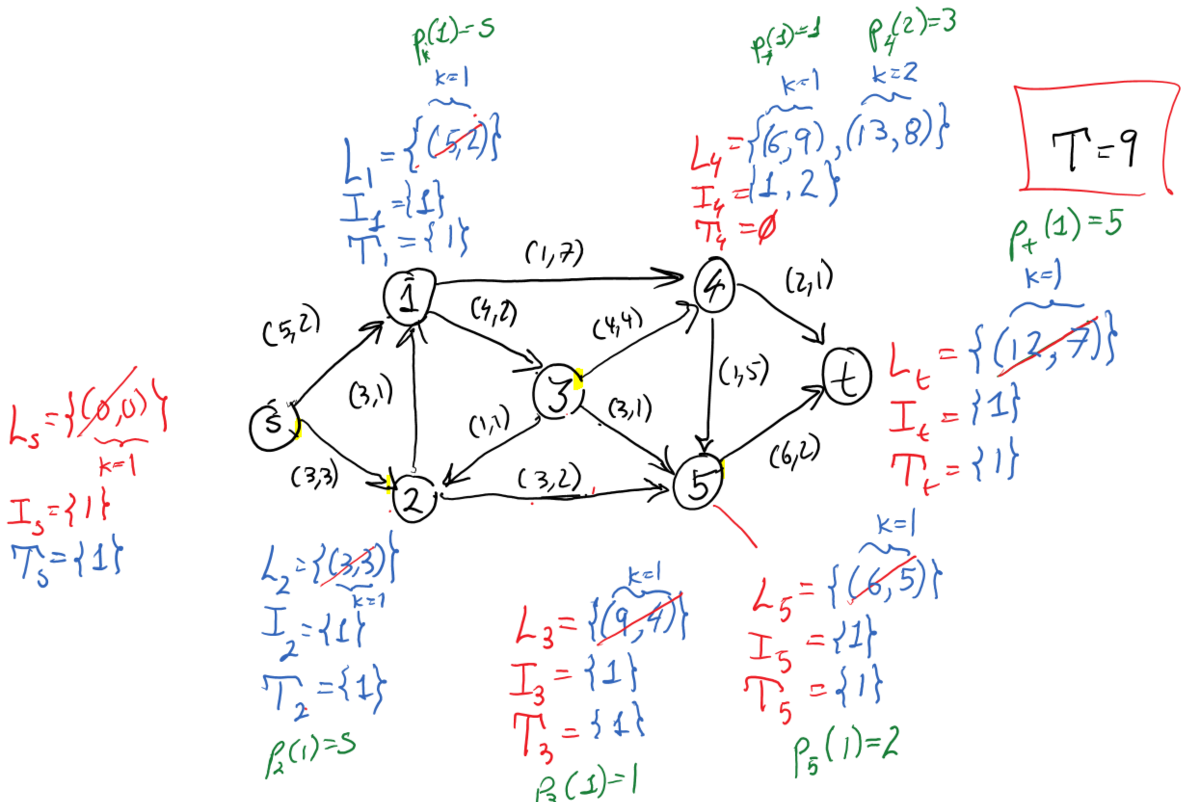
Set $L_j = L_j \cup \{(W_i^k + w_{ij}, C_i^k + c_{ij})\}$.

Update I_j accordingly.

Set $T_i := T_i \cup \{k\}$.

go to Step 1.

Step2:



Labeling algorithms

WCSPP - example

Step 1: *Selection of the label to be treated*
if $\bigcup_{i\in V}(I_i\setminus T_i)=\varnothing$ **then** STOP; all efficient labels have been generated
else choose $i\in V$ and $k\in I_i\setminus T_i$ so that W_i^k is minimal.

Step 1:

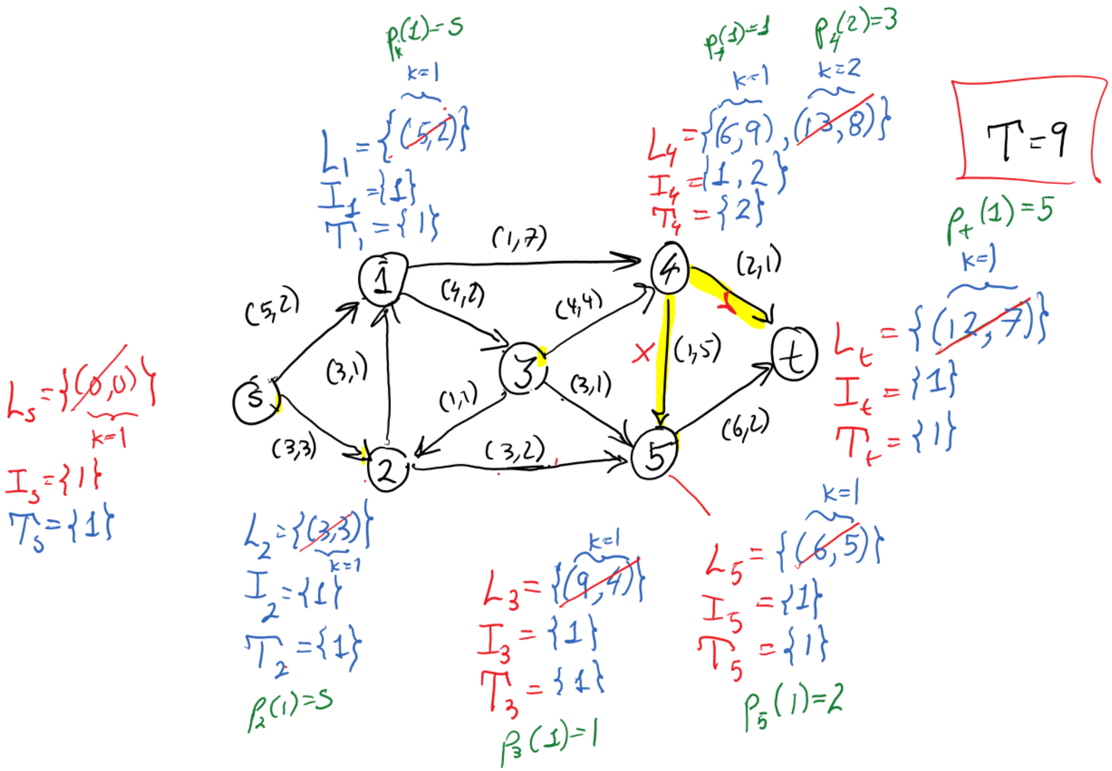
$$(4,2)\leftarrow \arg\min_{i\in V,\, k\in I_i\setminus T_i}\left\{\underbrace{8}_{\substack{i=4\\k=2}},\ \underbrace{9}_{\substack{i=4\\k=1}}\right\}$$

Labeling algorithms

WCSPP - example

Step 2: *Treatment of label* (W_i^k, C_i^k)
for all $(i, j) \in \delta^+(i)$ with $W_i^k + w_{ij} \leqslant W$ **do**
if $(W_i^k + w_{ij}, C_i^k + c_{ij})$ is not dominated by (W_j^l, C_j^l) for any $l \in I_j$
then
Set $L_j = L_j \cup \{(W_i^k + w_{ij}, C_i^k + c_{ij})\}$.
Update I_j accordingly.
Set $T_i := T_i \cup \{k\}$.
go to Step 1.

Step 2:



Labeling algorithms

WCSPP - example

Step 1: *Selection of the label to be treated*

if $\bigcup_{i \in V}(I_i \setminus T_i) = \emptyset$ **then** STOP; all efficient labels have been generated

else choose $i \in V$ and $k \in I_i \setminus T_i$ so that W_i^k is minimal.

Step 1:

$$(4,1) \leftarrow \arg\min_{i \in V,\, k \in I_i \setminus T_i} \Big\{ \underbrace{9}_{\substack{i=4 \\ k=1}} \Big\}$$

Labeling algorithms

WCSPP - example

Step 2: *Treatment of label* (W_i^k, C_i^k)

for all $(i, j) \in \delta^+(i)$ with $W_i^k + w_{ij} \leqslant W$ **do**

if $(W_i^k + w_{ij}, C_i^k + c_{ij})$ is not dominated by (W_j^l, C_j^l) for any $l \in I_j$

then

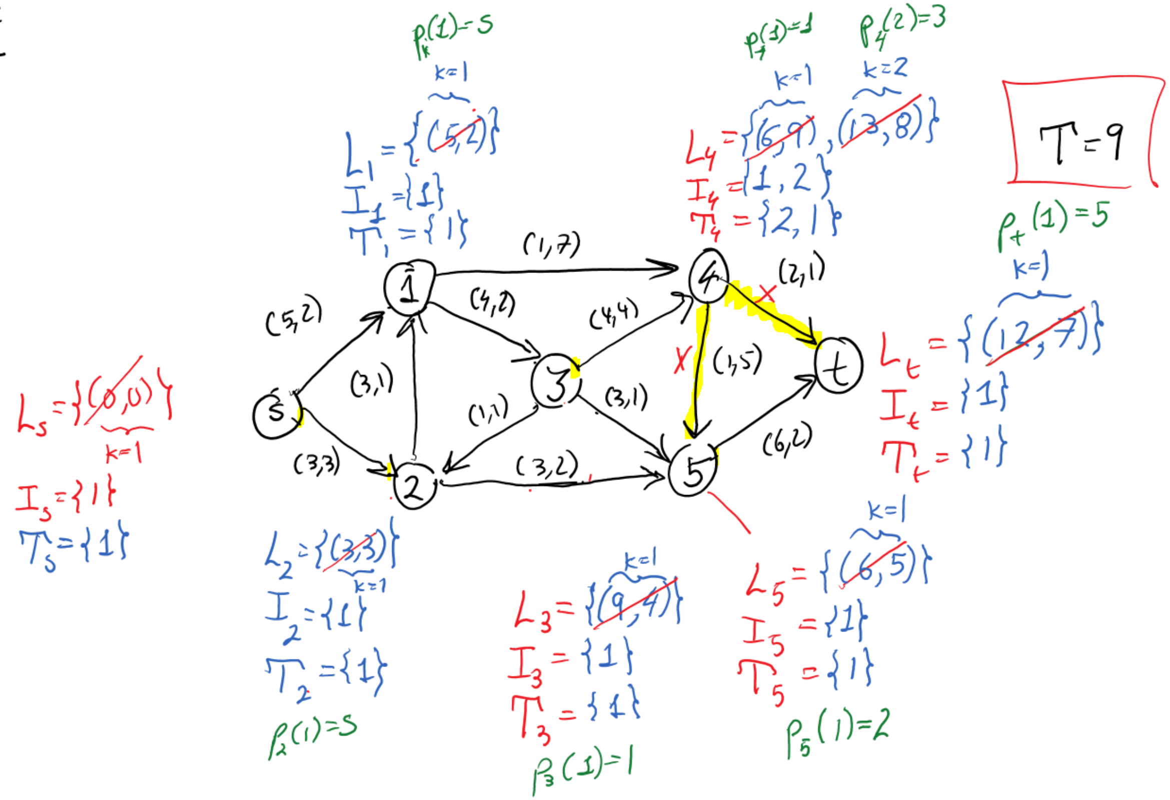
Set $L_j = L_j \cup \{(W_i^k + w_{ij}, C_i^k + c_{ij})\}$.

Update I_j accordingly.

Set $T_i := T_i \cup \{k\}$.

go to Step 1.

Step2:



Labeling algorithms

WCSPP - example

Step 1: *Selection of the label to be treated*

if $\bigcup_{i \in V}(I_i \setminus T_i) = \emptyset$ **then** STOP; all efficient labels have been generated

else choose $i \in V$ and $k \in I_i \setminus T_i$ so that W_i^k is minimal.

Step 1:

$$\bigcup_{i \in V} I_i \setminus T_i = \emptyset \Rightarrow \text{stop}$$

$$L_t = \{(12, 7)\}$$

$$p_t(1) = 5$$

$$L_5 = \{(6, 5)\}$$

$$p_5(1) = 2$$

$$L_2 = \{(3, 3)\}$$

$$p_2(1) = 5$$

$$L_s = \{(0, 0)\}$$

$$p_s(1) = \emptyset$$

$$\boxed{\begin{aligned} P &= (s, 2, 5, t) \\ c(P) &= 12 \\ t(P) &= 7 \end{aligned}}$$

Lecciones aprendidas
Dijkstra

- Formulación de los problemas de ruta más corta como programas matemáticos (optimización lineal)
  - Uno-a-uno
  - Uno-a-todos
- Condiciones de optimalidad de KKT del algoritmo de Dijkstra
  - Factibilidad primal
  - Factibilidad dual (i.e., corrección de etiquetas)
  - Holgura complementaria (propiedades de RMC)
- Complejidad de Dijkstra
  - Escogencia de nodos (extension de etiquetas)
  - Corrección de etiquetas (*scanning*)
- Análisis del comportamiento de la búsqueda expansiva (breadth) de Dijkstra

Lecciones aprendidas

A*, *branch-and-bound, labeling algorithms*

- A* como una especialización de Dijkstra con una meta (dirección)
  - Definición de heurística
  - Propiedades de la heurística
- Principio de *branch & bound* (ramificación y acotamiento)
  - Definición de partición del espacio
  - Uso de cotas para podar el espacio de solución (enumeración implícita)
- Algoritmos de etiquetado para manejar multiples recursos (objetivos)
  - Etiquetas
  - Dominancia
  - Espacio de estados