

# Métodos de Compensación en Frecuencia

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Rev 1.0

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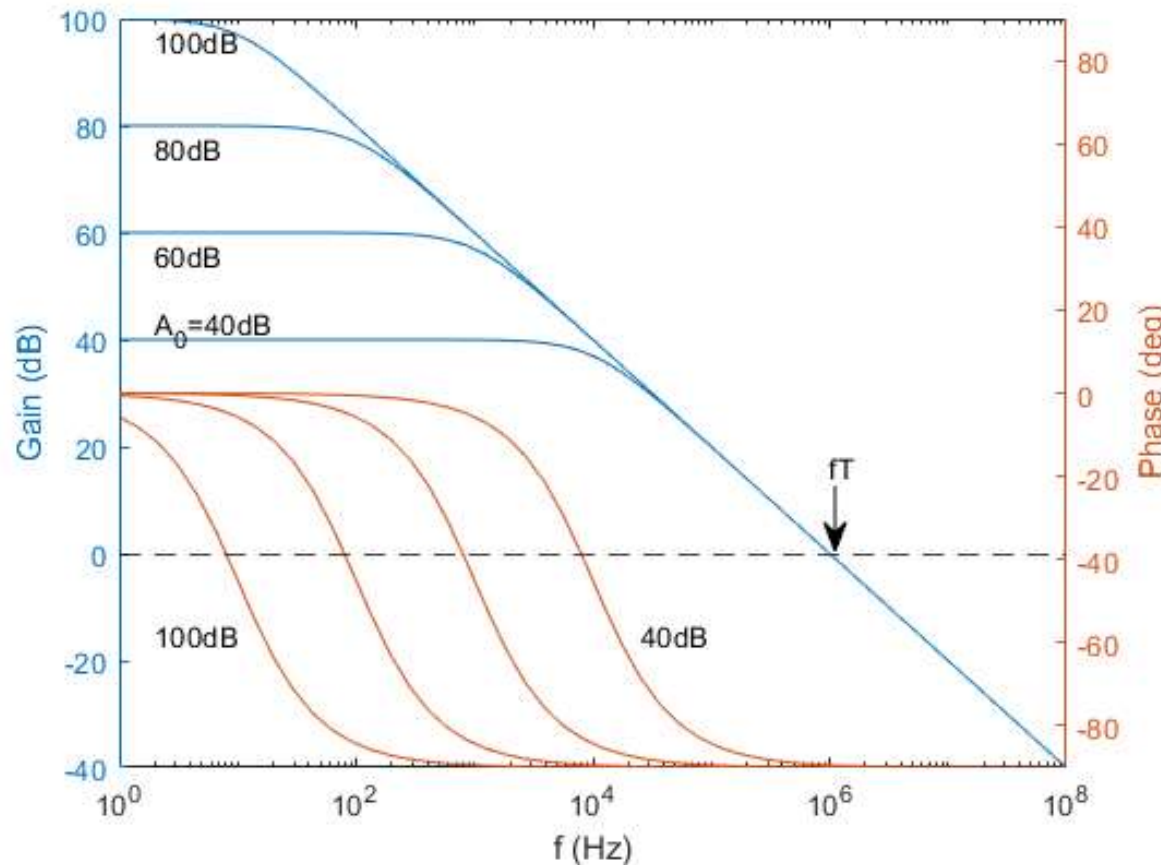
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  - ❑ Sistemas de 2do orden
- ❑ Márgenes de Estabilidad
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# Sistemas de 1er orden (I)

Lazo Abierto:  $A(\omega) = \frac{A_0}{1 + j \frac{\omega}{\omega_p}}$

$$\Rightarrow A(\omega) = \frac{A_0}{1 + j \frac{\omega A_0}{\omega_T}}$$



$$\omega_T = A_0 \omega_p = GBW$$

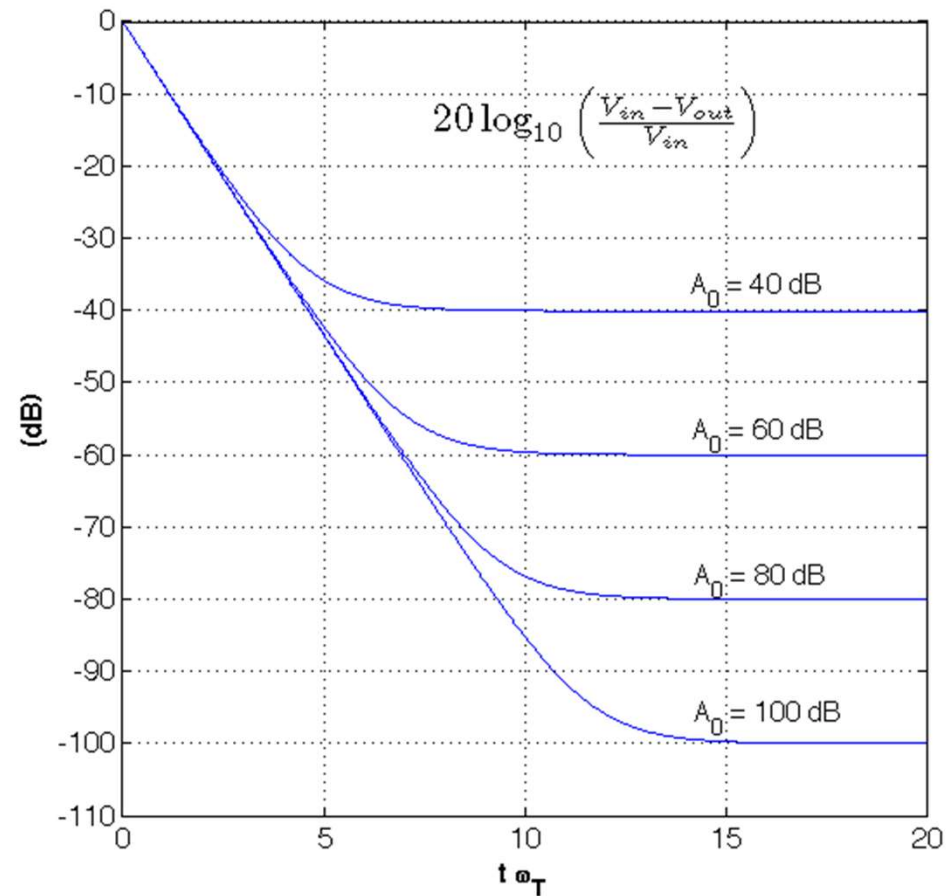
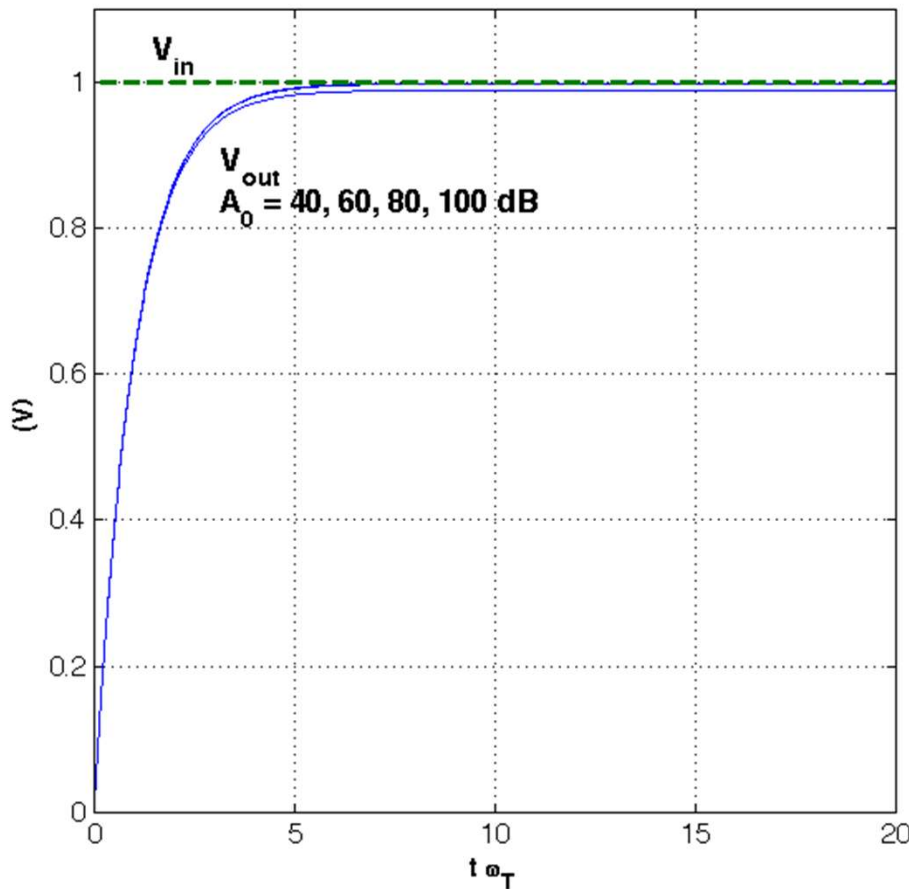
Es la constante de tiempo del sistema, determina la velocidad de respuesta del sistema.

# Sistemas de 1er orden (II)

Lazo Cerrado:  $H(\omega) = \frac{A(\omega)}{1+A(\omega)} = \frac{1}{1+j\frac{\omega}{\omega_T}}$

Realimentación Unitaria

Respuesta al escalón: Dos formas de verlo



# Sistemas de 2do orden (I)

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Transferencia en  
Lazo Abierto:

$$A(\omega) = \frac{A_0}{\left(1 + j \frac{\omega}{\omega_{dp}}\right) \left(1 + j \frac{\omega}{\omega_{ndp}}\right)}$$

Producto Ganancia por  
Ancho de Banda:

$$GBW = A_0 \omega_{dp}$$

Posición relativa del  
polo no dominante:

$$NDP = \frac{\omega_{ndp}}{GBW}$$

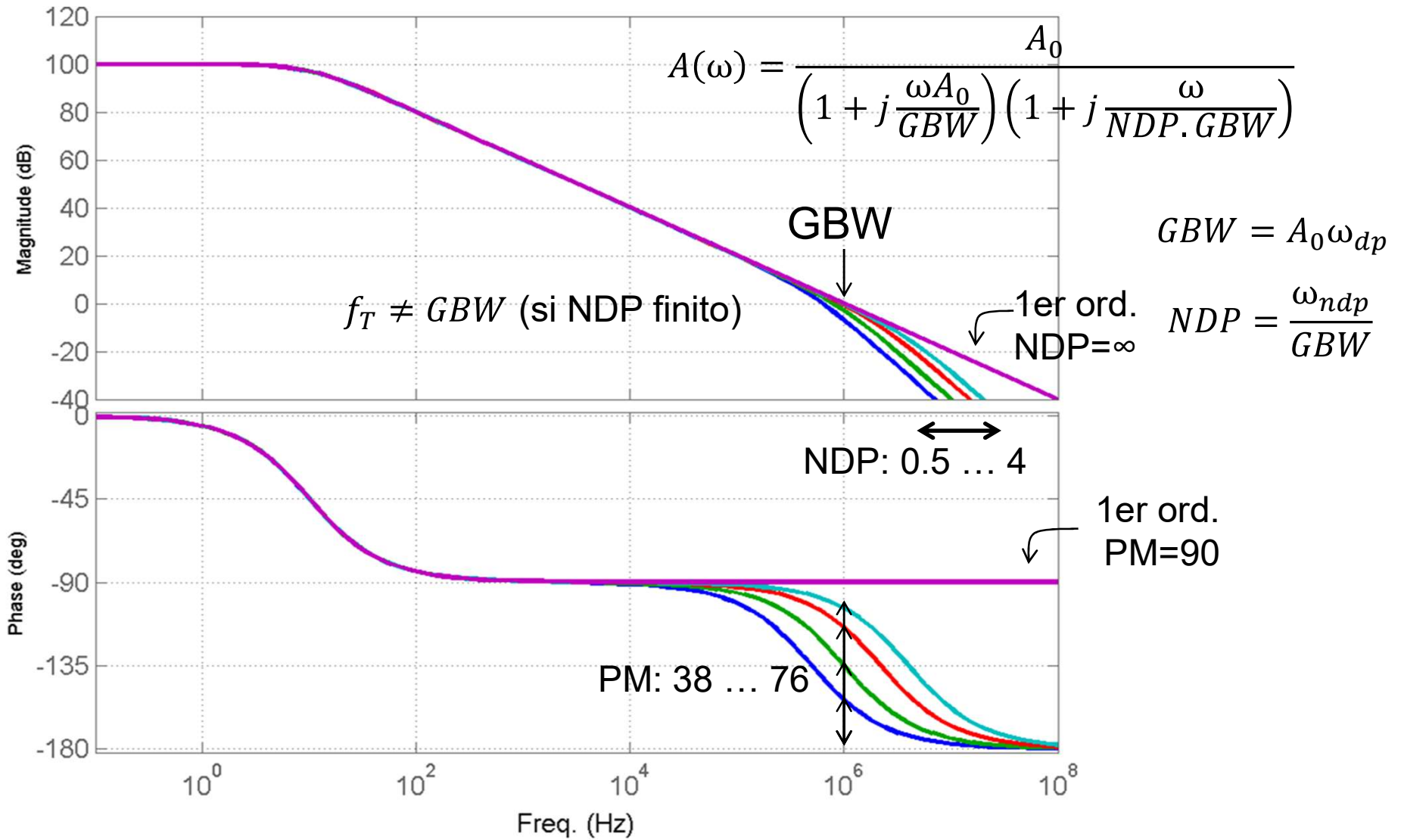
Polo no dominante

Polo dominante

$$A(\omega) = \frac{A_0}{\left(1 + j \frac{\omega A_0}{GBW}\right) \left(1 + j \frac{\omega}{NDP \cdot GBW}\right)}$$

# Sistemas de 2do orden (II)

## Lazo Abierto:

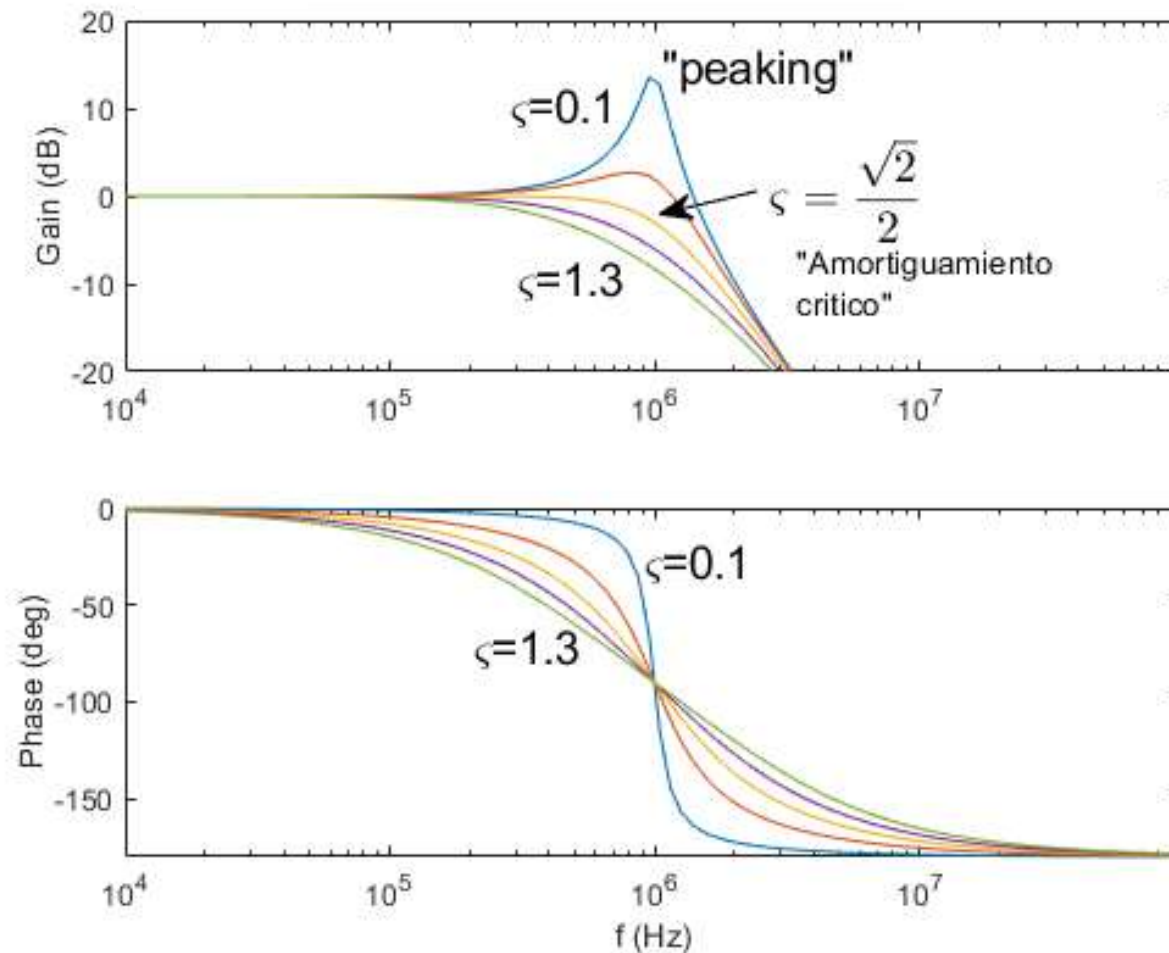


# Sistemas de 2do orden (III)

## Lazo Cerado:

$$A(\omega) = \frac{1}{1 + j2\zeta \frac{\omega}{\omega_n} + \left(j \frac{\omega}{\omega_n}\right)^2}$$

Bode:



$$\zeta = \frac{\sqrt{NDP}}{2}$$

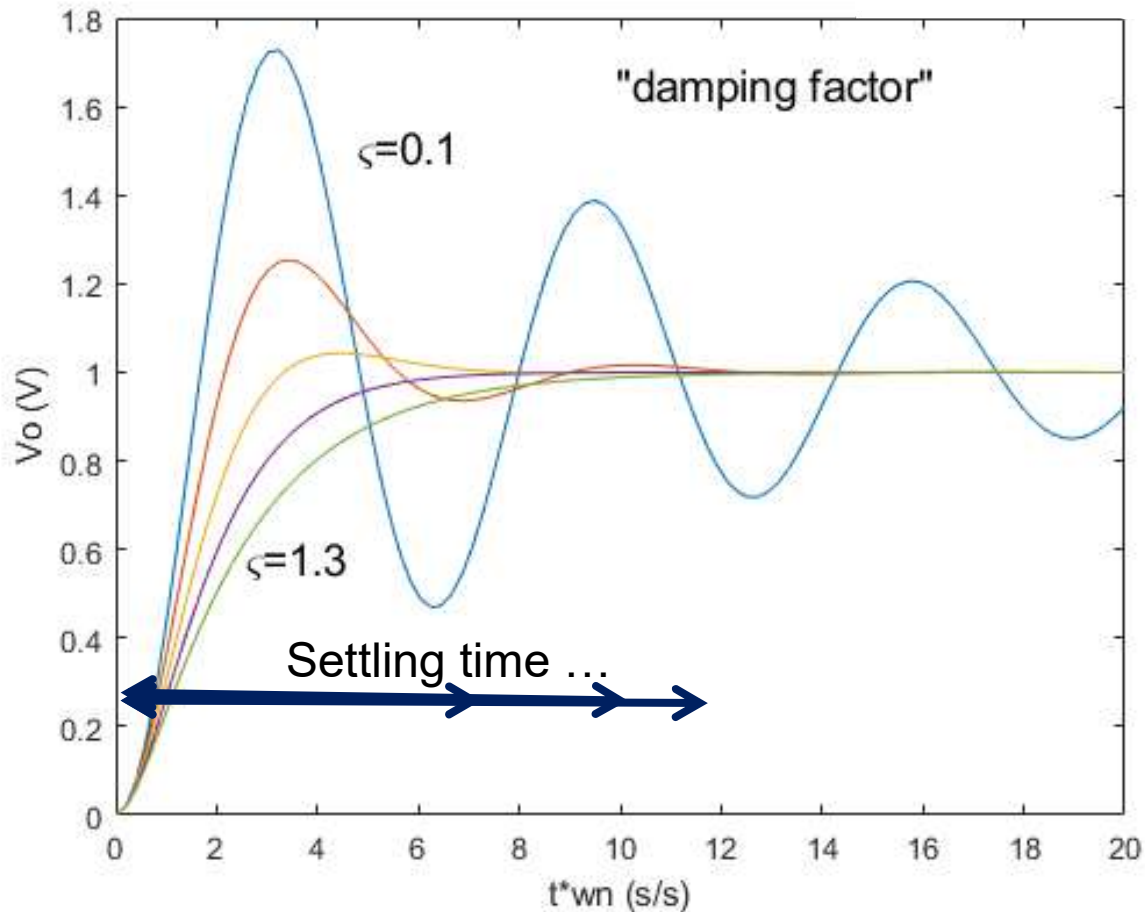
$$\omega_n = GBW \sqrt{NDP}$$

# Sistemas de 2do orden (IV)

Lazo Cerado:

$$A(\omega) = \frac{1}{1 + j2\zeta \frac{\omega}{\omega_n} + \left(j \frac{\omega}{\omega_n}\right)^2}$$

Respuesta al escalón:



$$\zeta = \frac{\sqrt{NDP}}{2}$$

$$\omega_n = GBW \sqrt{NDP}$$



# Márgenes de Estabilidad

NDP	PM (°)	$\zeta$	Peaking (dB)	Sobre Tiro (%)	Sett. Time x GBW	
					err: 5%	err: 1%
0.5	38.7	0.35	3.6	30.5	11.2	16.3
1	51.8	0.5	1.25	16.3	5.3	8.8
2	65.5	0.71	0	4.3	2.1	4.7
2.2	67.3	0.74	-	3.1	2.1	4.5
4	76.3	1	-	0	2.4	3.3
Inf	90	inf	-	0	3.0	4.6

1er ord.

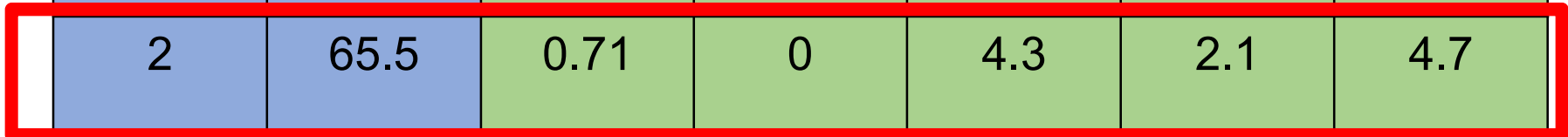
Open Loop

Closed Loop

# Márgenes de Estabilidad

NDP	PM (°)	$\zeta$	Peaking (dB)	Sobre Tiro (%)	Sett. Time x GBW	
					err: 5%	err: 1%
0.5	38.7	0.35	3.6	30.5	11.2	16.3
1	51.8	0.5	1.25	16.3	5.3	8.8
2	65.5	0.71	0	4.3	2.1	4.7
2.2	67.3	0.74	-	3.1	2.1	4.5
4	76.3	1	-	0	2.4	3.3
Inf	90	inf	-	0	3.0	4.6

Amortiguamiento crítico



1er ord.

Open Loop

Closed Loop

# Márgenes de Estabilidad

NDP	PM (°)	$\zeta$	Peaking (dB)	Sobre Tiro (%)	Sett. Time x GBW	
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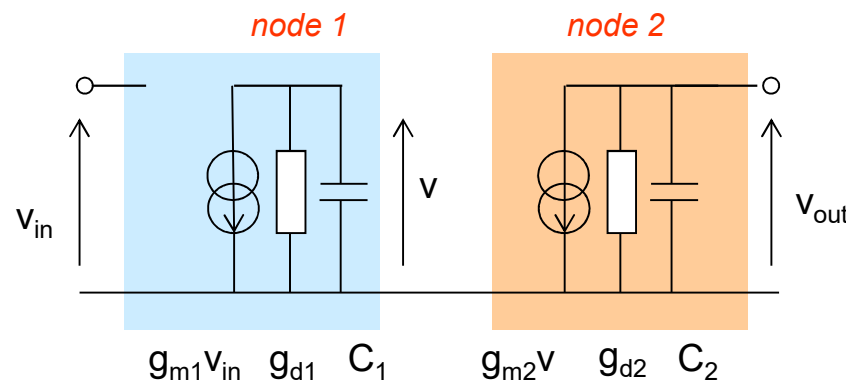
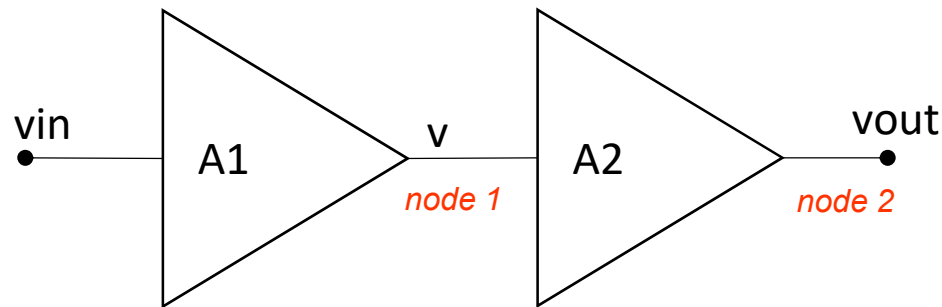
**Elección usual de diseño:**

1er ord.

Open Loop

Closed Loop

# Amplificadores 2 etapas: Modelo Norton



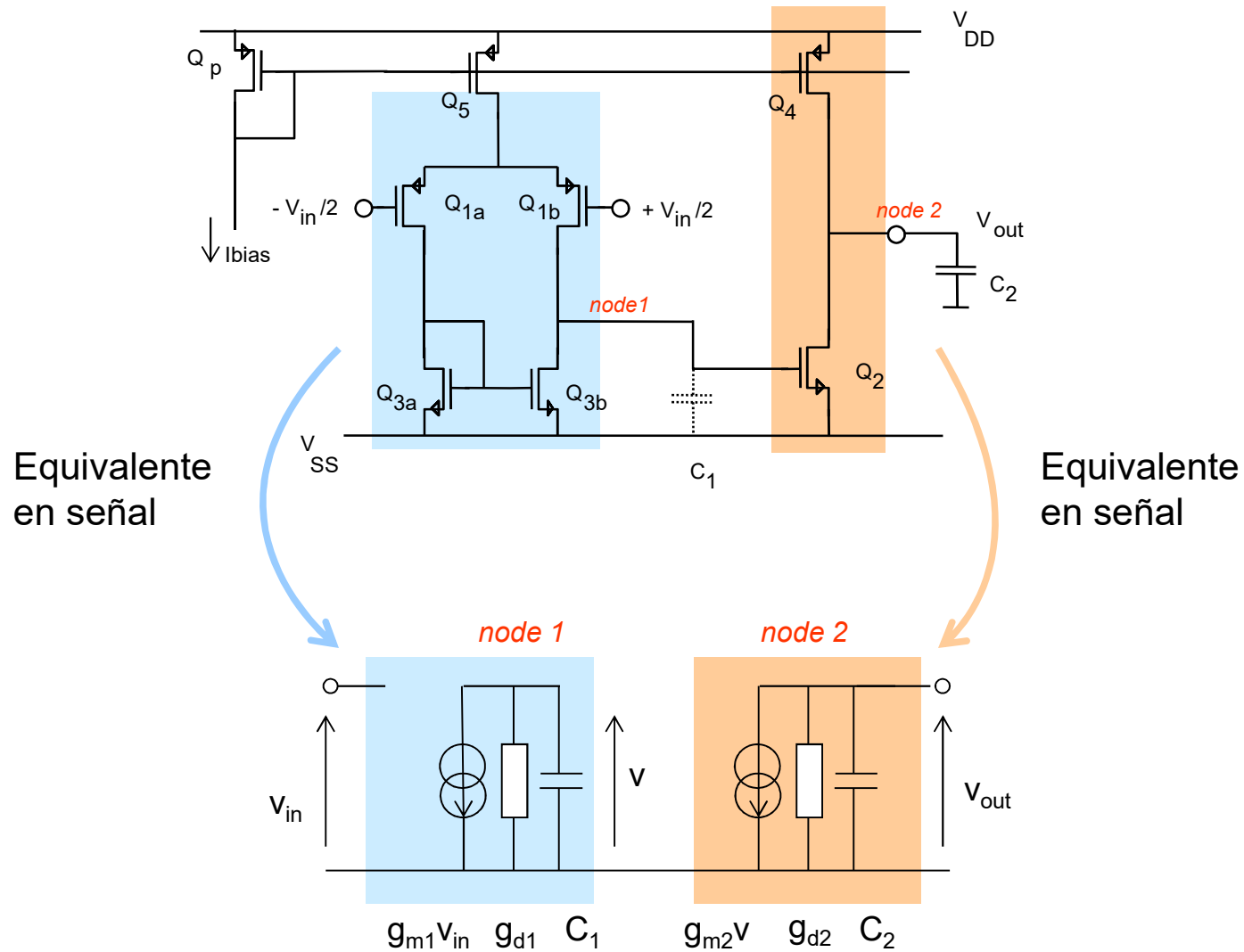
Equivalente Norton de **cualquier** amplificador de 2 etapas

Es un Sistema Lineal de 2do Orden:

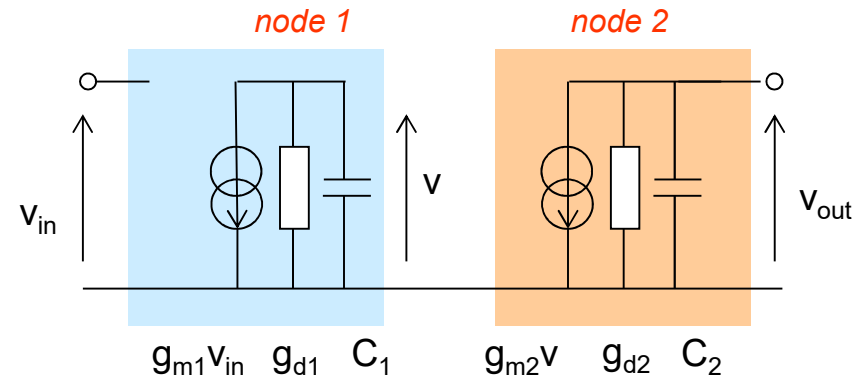
$$A(\omega) = \frac{A_0}{\left(1 + j \frac{\omega}{\omega_{p1}}\right) \left(1 + j \frac{\omega}{\omega_{p2}}\right)}$$

$$A_0 = \frac{g_{m1}g_{m2}}{g_{d1}g_{d2}}, \omega_{p1} = \frac{g_{d1}}{C_1}, \omega_{p2} = \frac{g_{d2}}{C_2}$$

# Ejemplo: Amplificador MOS



# Estabilidad de un Amp. de 2 Etapas



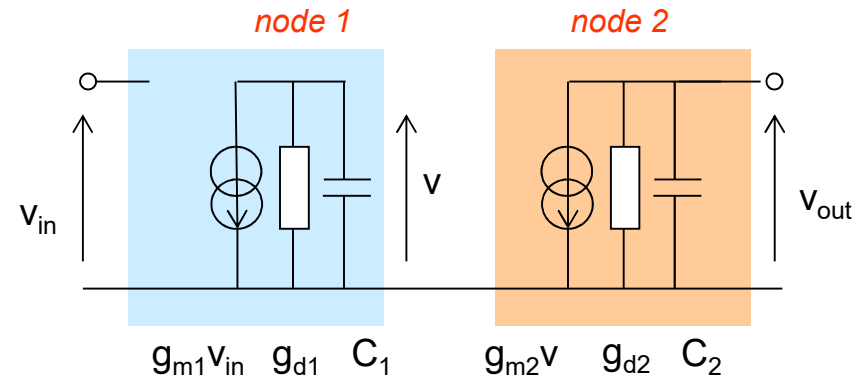
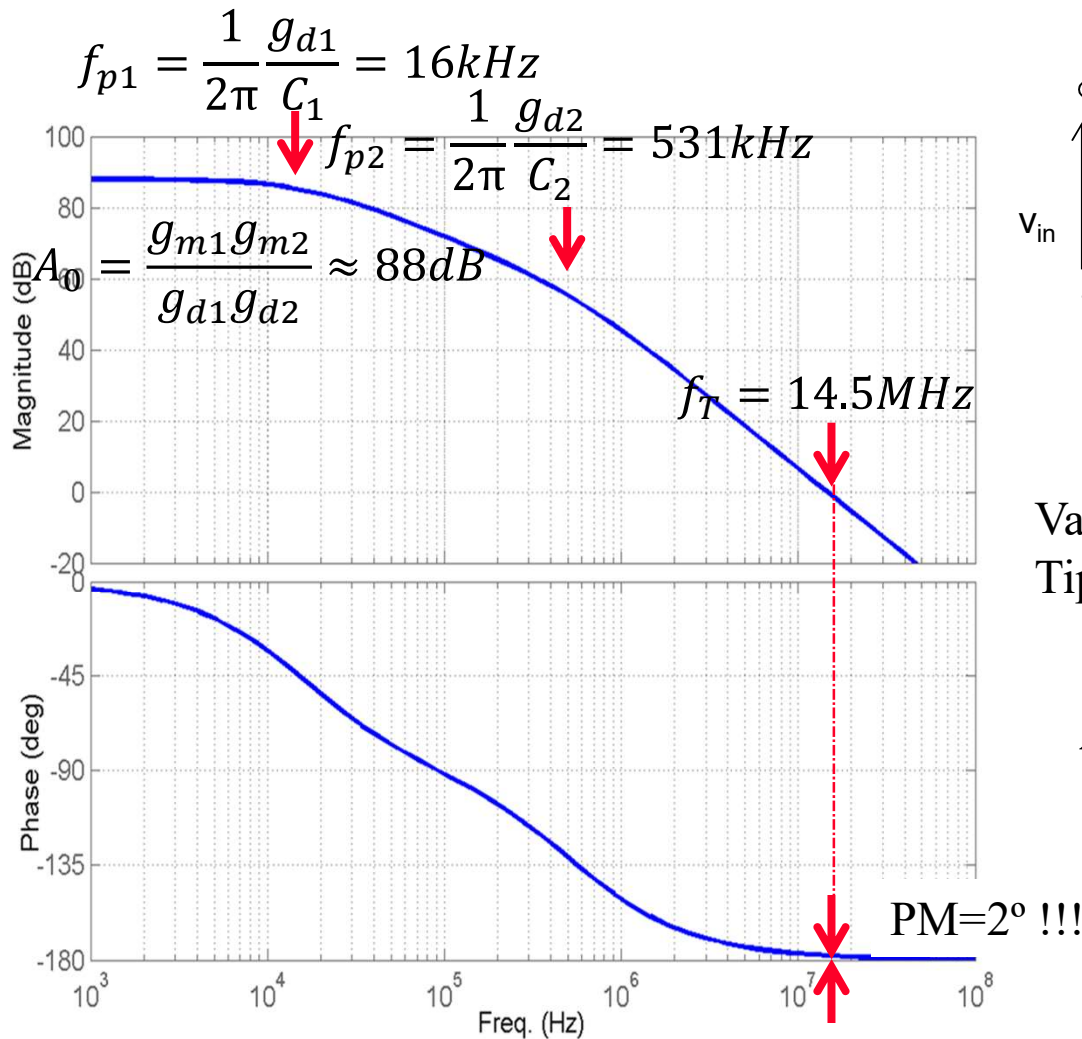
Valores  
 Tipicos:

$g_{m1} = 50\mu A/V$	$g_{m2} = 500\mu A/V$
$g_{d1} = 10^{-7}\Omega^{-1}$	$g_{d2} = 10^{-5}\Omega^{-1}$
$C_1 = 1pF$	$C_2 = 3pF$

$$A(\omega) = \frac{A_0}{\left(1 + j\frac{\omega}{\omega_{p1}}\right)\left(1 + j\frac{\omega}{\omega_{p2}}\right)}$$

$$A_0 = \frac{g_{m1}g_{m2}}{g_{d1}g_{d2}}, \omega_{p1} = \frac{g_{d1}}{C_1}, \omega_{p2} = \frac{g_{d2}}{C_2}$$

# Estabilidad de un Amp. de 2 Etapas



Valores

Tipicos:

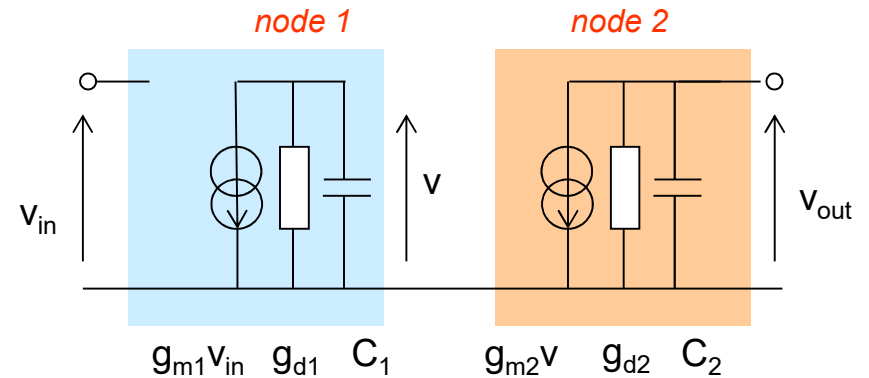
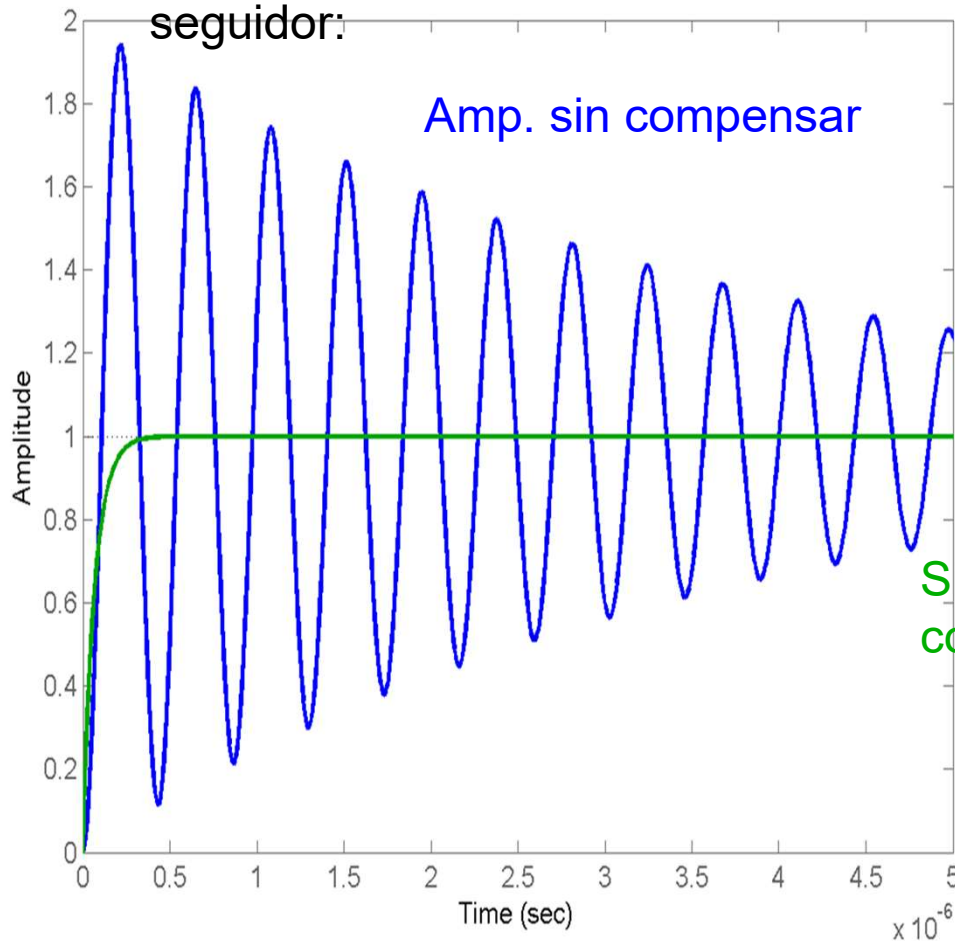
$g_{m1} = 50\mu\text{A}/\text{V}$	$g_{m2} = 500\mu\text{A}/\text{V}$
$g_{d1} = 10^{-7}\Omega^{-1}$	$g_{d2} = 10^{-5}\Omega^{-1}$
$C_1 = 1\text{pF}$	$C_2 = 3\text{pF}$

$$A(\omega) = \frac{A_0}{\left(1 + j\frac{\omega}{\omega_{p1}}\right)\left(1 + j\frac{\omega}{\omega_{p2}}\right)}$$

$$f_T \neq GBW (A_0 f_{p1} = 400\text{MHz})$$

# Estabilidad de un Amp. de 2 Etapas

Respuesta al escalón del amp. en configuración seguidor:

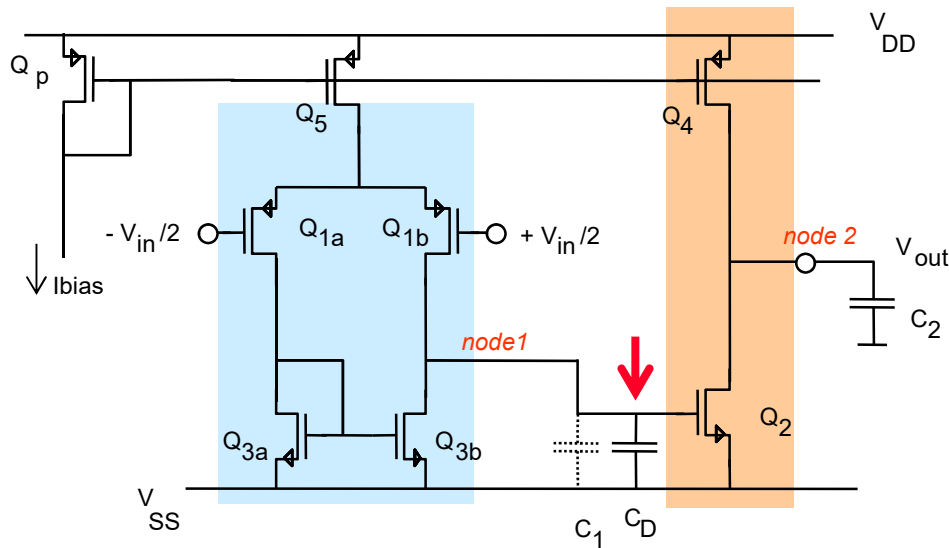


$$\begin{aligned}
 g_{m1} &= 50 \mu A/V & g_{m2} &= 500 \mu A/V \\
 g_{d1} &= 10^{-7} \Omega^{-1} & g_{d2} &= 10^{-5} \Omega^{-1} \\
 C_1 &= 1 pF & C_2 &= 3 pF
 \end{aligned}$$

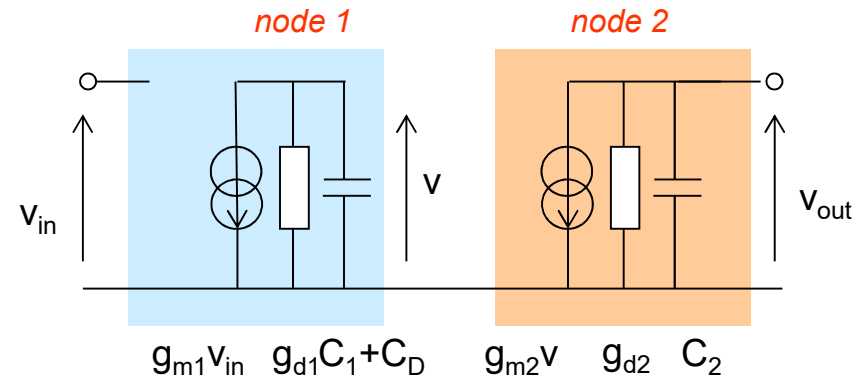
Sist. de 1er orden con el mismo  $f_T$  y  $A_0$ .



# Met. de Compensación: Compensación Directa



Ejemplo Amp. MOS



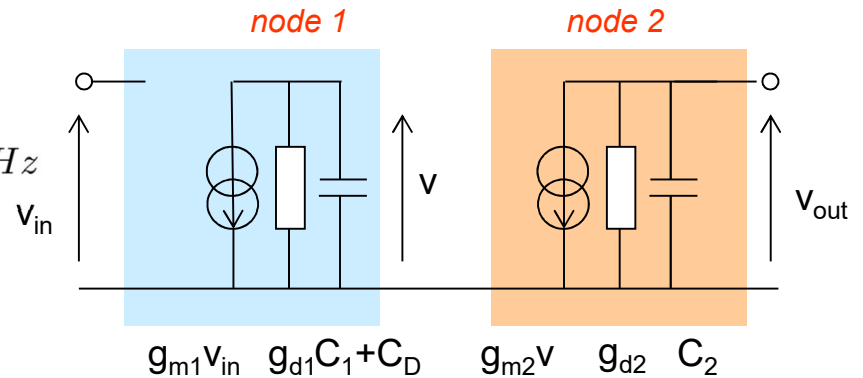
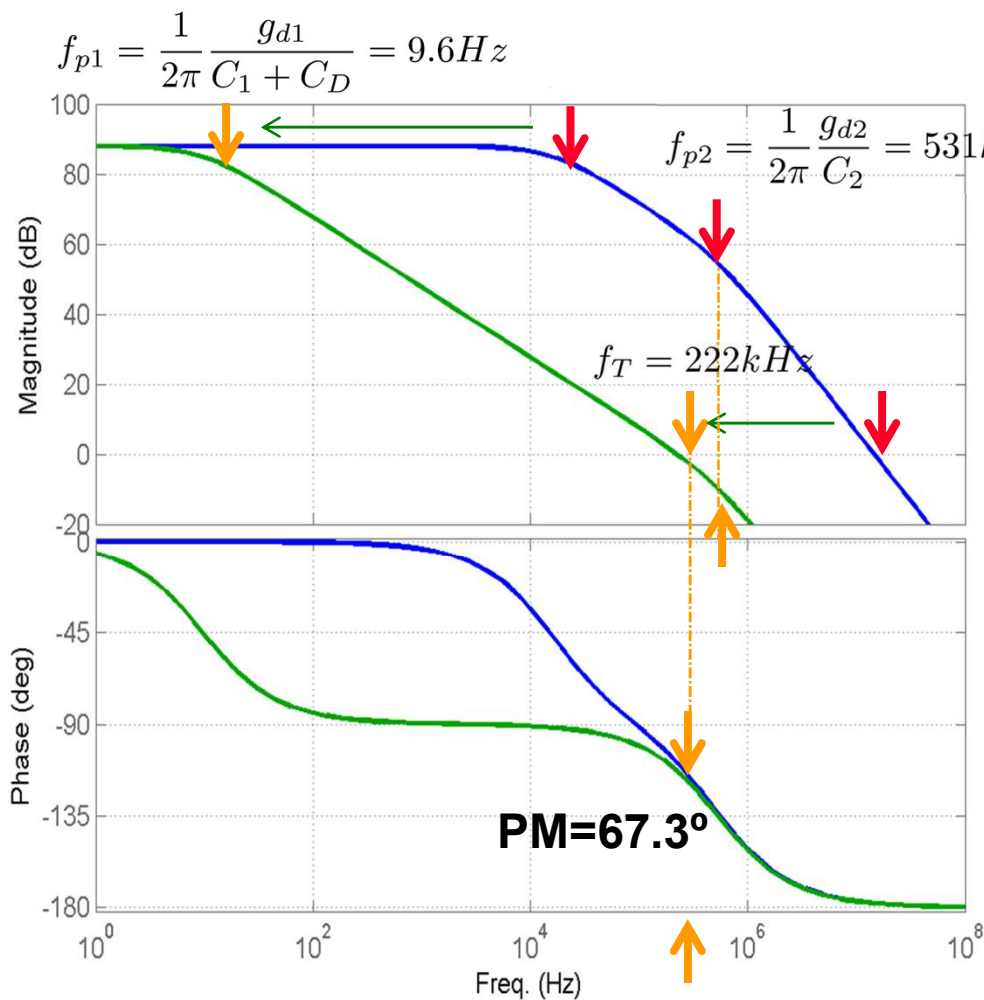
Valores  
 Tipicos:

$g_{m1} = 50 \mu A/V$	$g_{m2} = 500 \mu A/V$
$g_{d1} = 10^{-7} \Omega^{-1}$	$g_{d2} = 10^{-5} \Omega^{-1}$
$C_1 = 1 pF$	$C_2 = 3 pF$

Si agrego  
 $C_D$

$$f_{p1} \downarrow \Rightarrow f_T \downarrow \Rightarrow PM \uparrow$$

# Met. de Compensación: Compensación Directa



**Valores**  
**Típicos:**

$g_{m1} = 50\mu A/V$	$g_{m2} = 500\mu A/V$
$g_{d1} = 10^{-7}\Omega^{-1}$	$g_{d2} = 10^{-5}\Omega^{-1}$
$C_1 = 1pF$	$C_2 = 3pF$

Criterio usual: NDP=2.2

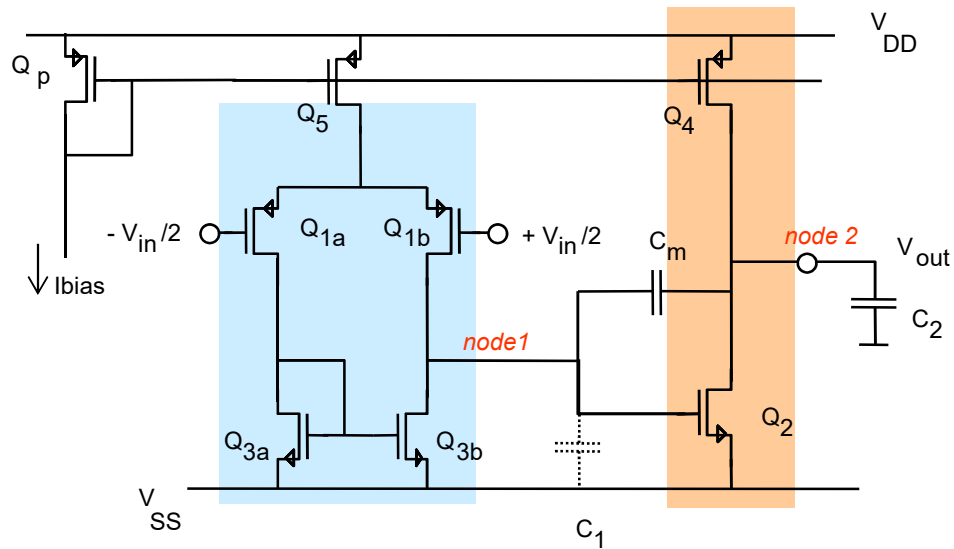
$$f_{p2} = 2.2f_T$$

$$\Rightarrow f_{p2} = 2.2A_0f_{p1}$$

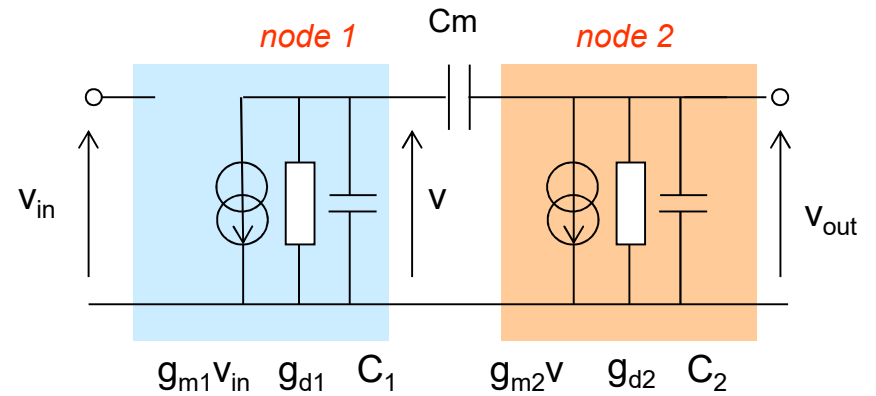
$$\Rightarrow \frac{g_{d2}}{C_2} = 2.2 \frac{g_{m1}g_{m2}}{g_{d1}g_{d2}} \frac{g_{d1}}{C_1 + C_D}$$

$\Rightarrow C_D = 1.65nF!!!$  **Imposible de integrar!!!**

# Met. de Compensación: Compensación Miller

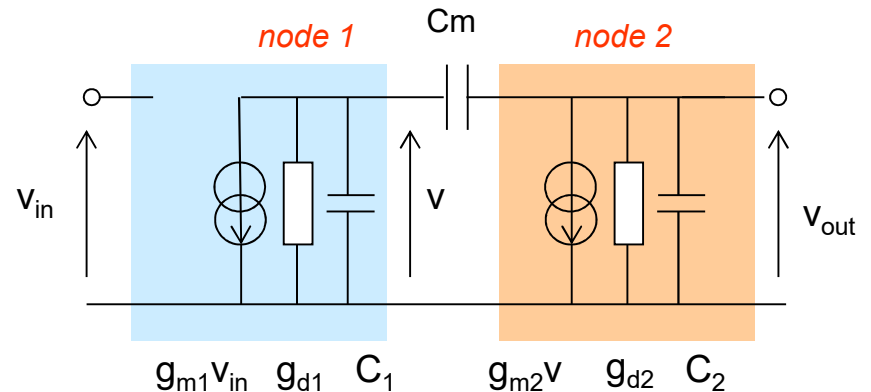


Ejemplo Amp. MOS



# Met. de Compensación: Compensación Miller

$$A(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{dp}}\right) \left(1 + \frac{s}{\omega_{ndp}}\right)}$$



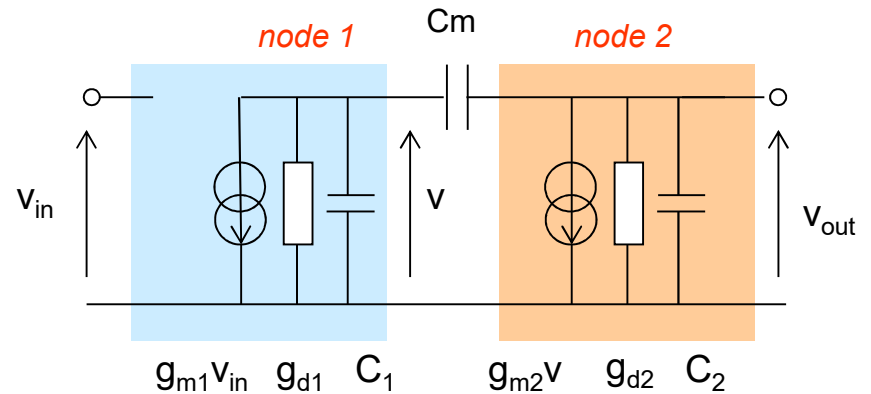
$$A(s) = \frac{\frac{g_{m1}g_{m2}}{g_{d1}g_{d2}} \left(1 - s \frac{C_m}{g_{m2}}\right)}{1 + s \left(\frac{C_1}{g_{d1}} + \frac{C_2}{g_{d2}} + \frac{C_m}{g_{d1}g_{d2}} (g_{m2} + g_{d2} + g_{d1})\right) + s^2 \left(\frac{C_1C_2 + C_m(C_1 + C_2)}{g_{d1}g_{d2}}\right)}$$

Hipotesis :

- Polo dominante
- Efecto Miller domina

# Met. de Compensación: Compensación Miller

$$A(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{dp}}\right) \left(1 + \frac{s}{\omega_{ndp}}\right)}$$



$$A(s) = \frac{\frac{g_{m1}g_{m2}}{g_{d1}g_{d2}} \left(1 - s \frac{C_m}{g_{m2}}\right)}{1 + s \left(\frac{C_1}{g_{d1}} + \frac{C_2}{g_{d2}} + \frac{C_m}{g_{d1}g_{d2}} (g_{m2} + g_{d2} + g_{d1})\right) + s^2 \left(\frac{C_1C_2 + C_m(C_1 + C_2)}{g_{d1}g_{d2}}\right)}$$

zero RHP :  $\omega_z = \frac{g_{m2}}{C_m}$   
(resta fase)

$$\omega_{p1} \ll \omega_{p2}$$

Hipotesis :

- Polo dominante
- Efecto Miller domina

$$\omega_{p1} \simeq \frac{g_{d1}}{\frac{g_{m2}}{g_{d2}} C_m} \ll \frac{g_{d1}}{C_1} \quad \text{“pole splitting”}$$

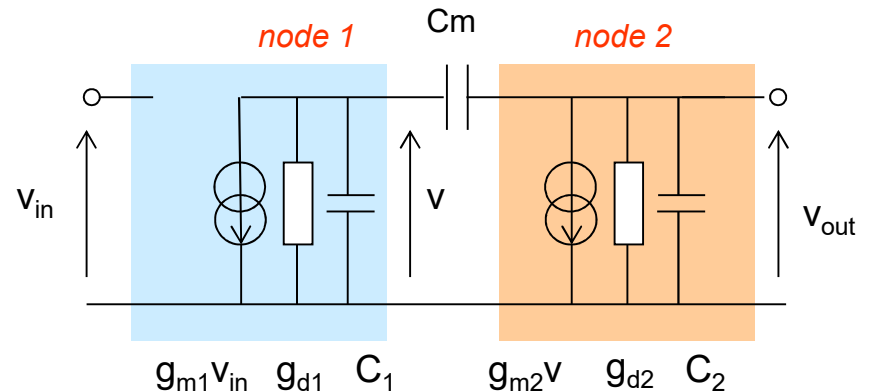
$$\frac{g_{m2}}{g_{d1}g_{d2}} C_m \gg \frac{C_1}{g_{d1}}, \frac{C_2}{g_{d2}}, \frac{C_m}{g_{d1}}, \frac{C_m}{g_{d2}}$$

$$\omega_{p2} \simeq \frac{g_{m2}C_m}{C_1C_2 + C_m(C_1 + C_2)} \gg \frac{g_{d2}}{C_2}$$

Gracias a efecto Miller,  $C_m$  es integrable

# Met. de Compensación: Compensación Miller

$$A(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{dp}}\right) \left(1 + \frac{s}{\omega_{ndp}}\right)}$$



$$A_0 = \frac{g_{m1}g_{m2}}{g_{d1}g_{d2}}$$

$$\omega_{p1} \simeq \frac{g_{d1}}{\frac{g_{m2}}{g_{d2}}C_m}$$

$$\omega_{p2} \simeq \frac{g_{m2}C_m}{C_1C_2 + C_m(C_1 + C_2)}$$

$$\omega_z = \frac{g_{m2}}{C_m}$$

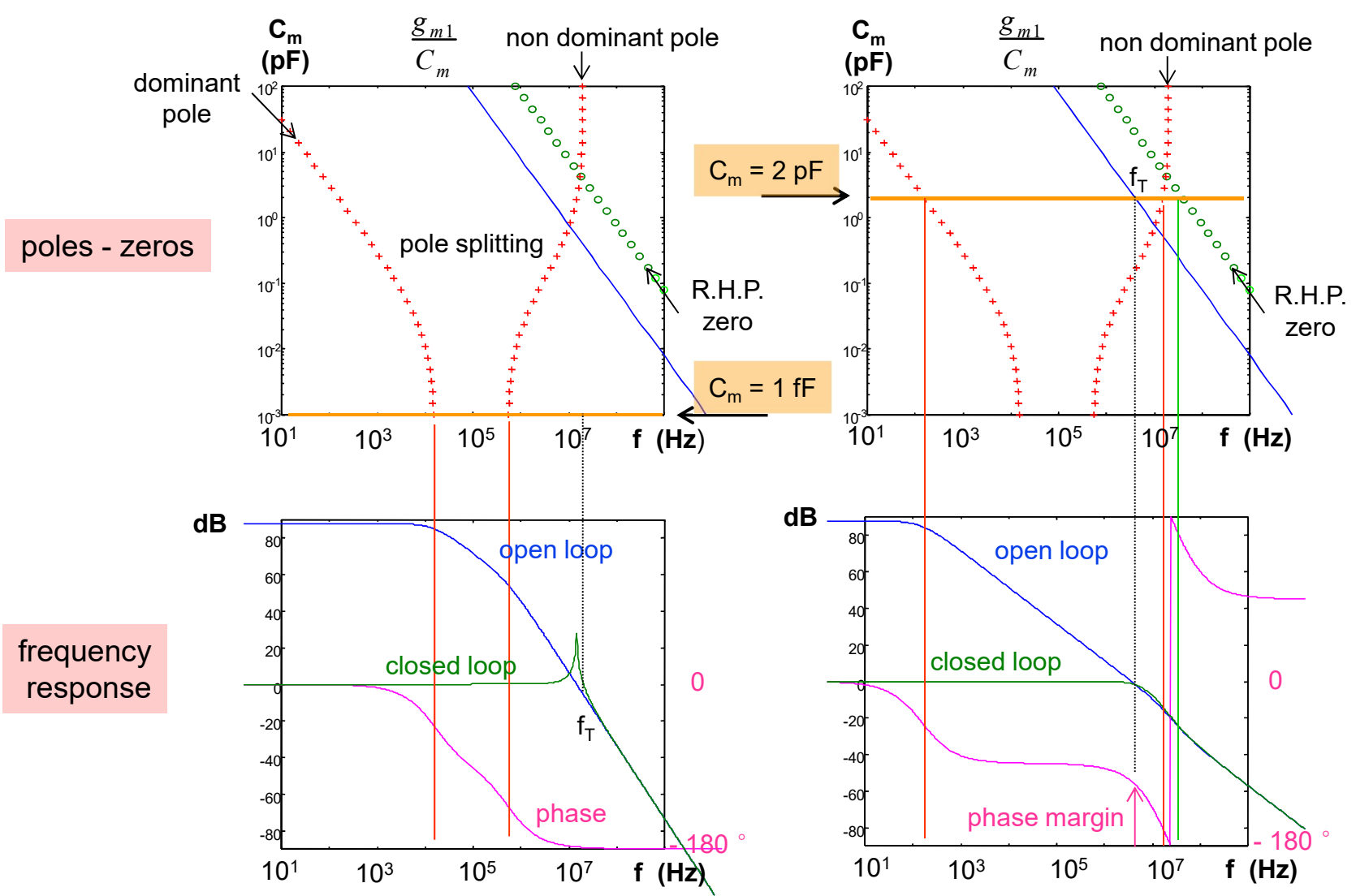
$$GBW = \frac{g_{m1}}{C_m}$$

$$NDP = \frac{g_{m2}}{g_{m1}} \frac{C_m^2}{C_1C_2 + C_m(C_1 + C_2)}$$

$$Z = \frac{g_{m2}}{g_{m1}}$$

$NDP = \frac{\omega_{ndp}}{GBW}$   
 $Z = \frac{\omega_z}{GBW}$

# Met. de Compensación: Compensación Miller

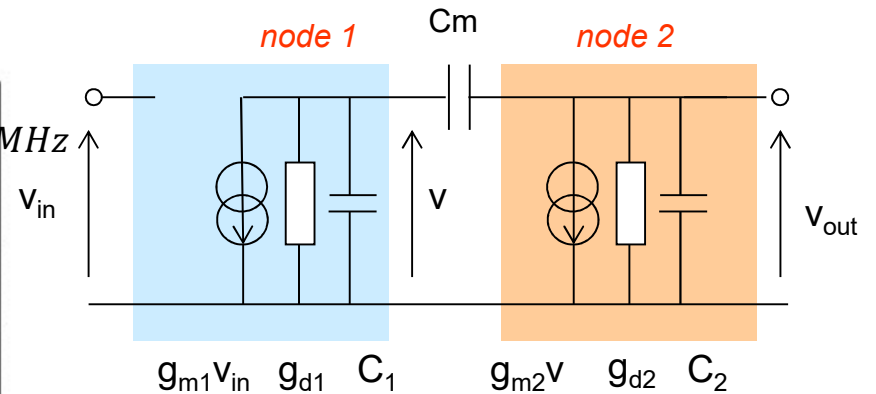
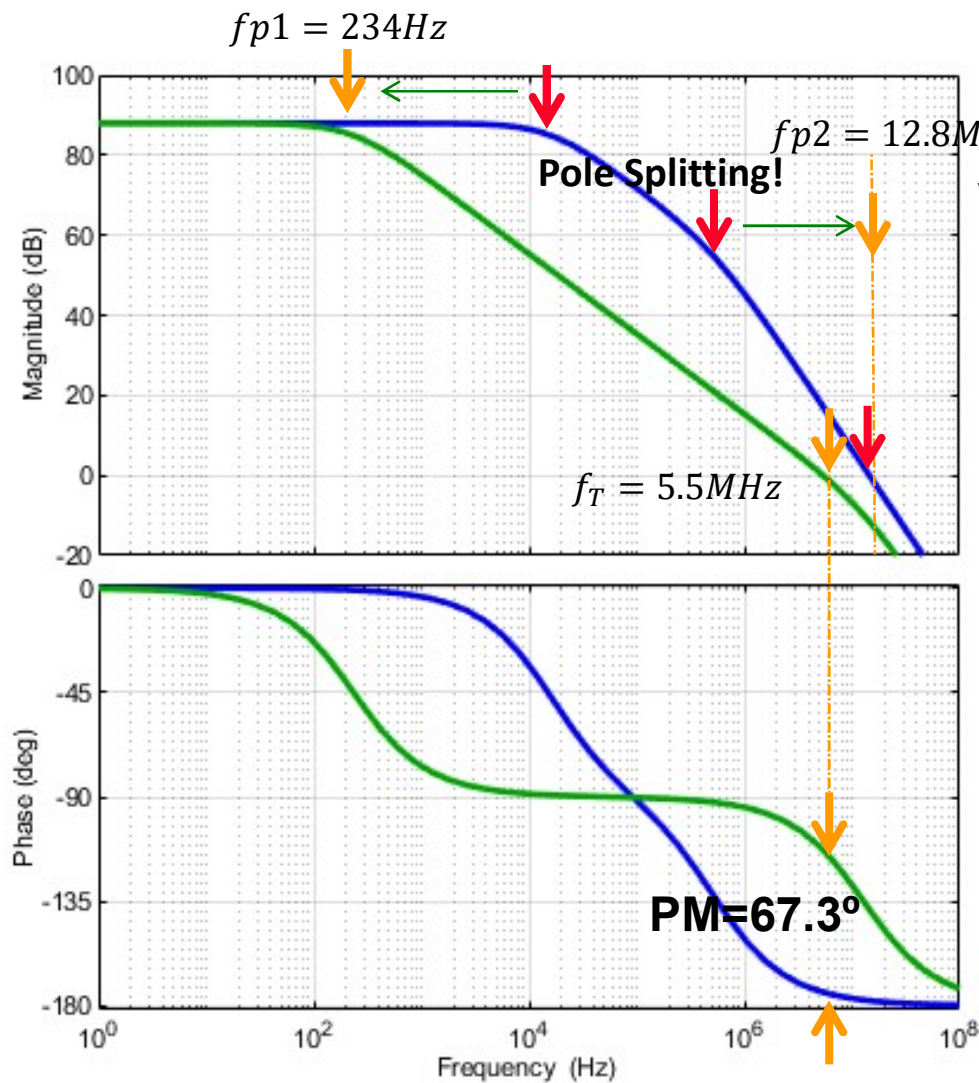


poles - zeros

frequency response

Tomado de P.Jespers – "Interfacing Microsystems" IWS 2001

# Met. de Compensación: Compensación Miller



Valores  
 Tipicos:  $g_{m1} = 50\mu\text{A/V}$      $g_{m2} = 500\mu\text{A/V}$   
 $g_{d1} = 10^{-7}\Omega^{-1}$      $g_{d2} = 10^{-5}\Omega^{-1}$   
 $C_1 = 1\text{pF}$      $C_2 = 3\text{pF}$

Criterio usual: NDP=2.2

$$NDP = \frac{g_{m2}}{g_{m1}} \frac{C_m^2}{C_1 C_2 + C_m(C_1 + C_2)}$$

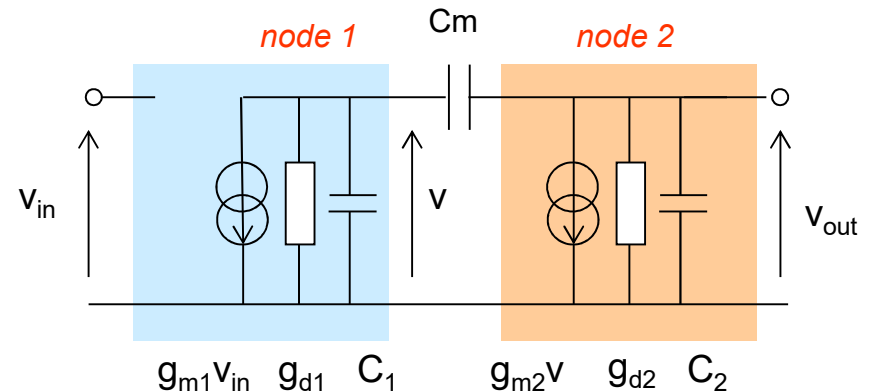
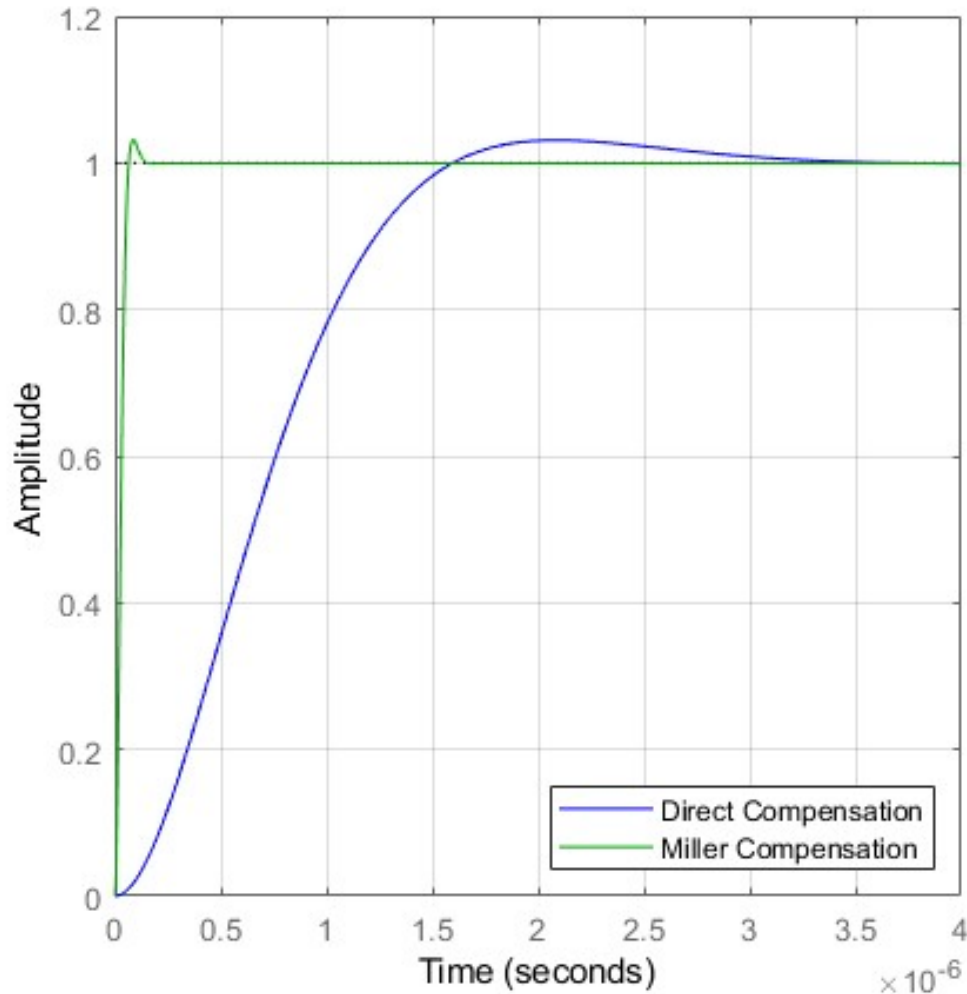
Resuelvo eq. de 2do orden en  $C_m$ :

$$C_m = 1.36\text{pF}$$

**Fácil de integrar** ✓



# Met. de Compensación: Compensación Miller



Valores  $g_{m1} = 50\mu A/V$   $g_{m2} = 500\mu A/V$   
 Tipicos:  $g_{d1} = 10^{-7}\Omega^{-1}$   $g_{d2} = 10^{-5}\Omega^{-1}$   
 $C_1 = 1pF$   $C_2 = 3pF$

Criterio usual:  $NDP=2.2$

$$NDP = \frac{g_{m2}}{g_{m1}} \frac{C_m^2}{C_1 C_2 + C_m(C_1 + C_2)}$$

Resuelvo eq. de 2do orden en  $C_m$ :

$$C_m = 1.36pF$$

Fácil de integrar ✓

## Resumen: Estabilidad en Amps. de 2 Etapas

Compensación	fp1	fp2	fz	fT	PM
Sin compensar	16 kHz	531 kHz	-	14.5 MHz	2°
Directa $C_D = 1650pF$	9.6 Hz	531 kHz	-	222 kHz	67.3°
Miller $C_m = 1.36pF$	228 Hz	20 MHz	58 MHz	5.5 MHz	69.2°

- Ventajas de la compensación Miller:
  - Gracias a efecto Miller,  $C_m$  es pequeña e integrable
  - Gracias a efecto “pole splitting”, no baja mucho el  $f_T$  (buen compromiso velocidad – consumo)