Theoretical Analysis of a Left-Ventricular Pumping Model Based on the Systolic Time-Varying Pressure/Volume Ratio

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Abstract—A left-ventricular pumping model based on a left-ventricular systolic time-varying pressure/volume ratio $e(t)$ has been proposed on the basis of the physiological studies indicating that the parameter $e(t)$ is a function of time in systole, almost independent from left-ventricular volume and the arterial-loading conditions, and is a good index of myocardial inotropism. The model has been formulated by the following two equations:

$$p(t) = e(t)\left\{v(0) - \int_0^t i(t)dt\right\}$$

$$p(t) = f\{i(t)\}$$

where $p(t)$ is the systolic left-ventricular pressure, $v(o)$ is the left-ventricular end-diastolic volume (at the onset of systole), $i(t)$ is the blood-flow rate ejected from the left ventricle, t is the time from the onset of systole, and f is a function of the hydraulic impedance of the load against the ventricular ejection (including the aortic valve). Some theoretical analyses of this model are attempted, first analytically with simplifications of the time course of $e(t)$ and the hydraulic impedance of the load, and then by an analog computer with $e(t)$ approximated to the physiological data. Quantitative relationships among various hemodynamic parameters are obtained, and these appear to be in good agreement with the physiological characteristics of the left ventricle.

I. Introduction

The time courses of pressure and volume of the left ventricle change in a complicated manner when the ventricle of a given inotropic state pumps blood under various alterations in the end-diastolic volume and the arterial-loading conditions. This complicated pressure–volume relationship has been successfully analyzed physiologically from the viewpoint of the myocardial tension–velocity relationship [1] and also from the ventricular time-varying pressure/volume ratio [2]–[4]. The myocardial tension–velocity relationship combined with a ventricular geometrical model has been used for simulation of the whole circulatory system, and some of the fundamental hemodynamic characteristics of the heart have been analyzed [5]. Ventricular-pumping models based on the time-varying pressure/volume ratio connected to various types of arterial-loading models have been used in simulation of the cardiovascular dynamics [6]–[9]. The time courses of the ratio in these simulations, however, have not been based on physiological data. Moreover, fundamental hemodynamic characteristics of the heart have not been analyzed systematically, although time courses of various hemodynamic parameters are in good agreement with the physiological data.

The purpose of the present investigations is to analyze some fundamental hemodynamic characteristics of a left-ventricular pumping model based on the left-ventricular systolic time-varying pressure/volume ratio $e(t)$ and to compare the results of this analysis with the established physiological properties of the left ventricle.

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II. Historical Reviews

Some of the systolic time courses of the left-ventricular pressure/volume ratio $e(t)$ used by previous investigators are as follows:

$$e(t) = \text{constant}, \qquad \text{(Warner [6])}$$

$$e(t) = e_{ed} + \{g(v_{ed})\}t, \qquad \text{(Defares [7])}$$

where e_{ed} is $e(t)$ at the end of diastole ($t=0$), g is the function of left-ventricular end-diastolic volume v_{ed} and is numerically calculated from physiological data, and t is the time in systole. $e(t)$ is a function of time similar to a half-wave-rectified sine wave (Beneken [8]), or it is determined in order to obtain reasonable ventricular pressure and volume curves (Snyder *et al.* [9]). The left-ventricular pumping models based on these time courses of $e(t)$ were used in the simulation of the whole circulatory system on an analog computer connected to detailed arterial models where various hemodynamic parameters were shown in most cases as functions of time. The relationships among these parameters however, were not systematically discussed. Moreover, these $e(t)$ were not induced from physiological data on time courses of left-ventricular pressure and volume in various circulatory conditions.

Recently, the time course of left-ventricular systolic pressure/volume ratio $e(t)$ has been investigated in detail in anesthetized, vagotomized, and stellectomized

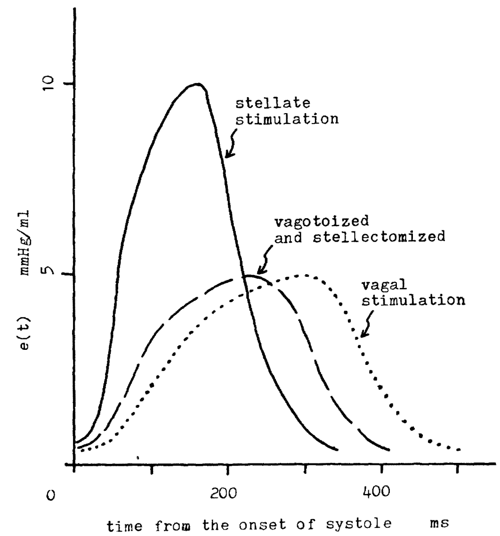


Fig. 1. Typical examples of time course of $e(t)$ of the dog.

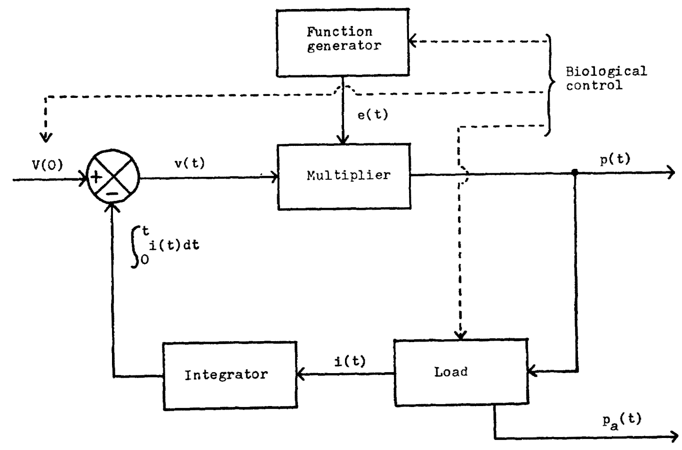


Fig. 2. Block diagram of left-ventricular pumping model based on systolic ventricular pressure/volume ratio $e(t)$. Integral of $i(t)$ is set to zero at onset of every systole.

dogs [2]–[4]. These experimental results show that $e(t)$ has an approximately unchanged tendency of the time course in systole without changes in the dog and $e(t)$ is nearly unchanged by alterations in the left-ventricular end-diastolic volume and arterial conditions. However, the cardiac–sympathetic stimulation increases the magnitude of $e(t)$ significantly and shortens the systolic duration of $e(t)$, but the cardiac vagal stimulation does not alter $e(t)$ at a fixed heart rate (Fig. 1) [2]–[4]. On the basis of these fundamental properties of the ventircular pressure–volume relationship, a left-ventricular pumping model has been proposed with the ratio $e(t)$ [2].

III. Mathematical Analysis

The left-ventricular pumping model proposed in the previous paper is formulated as follows [2]:

$$p(t) = e(t)v(t) \tag{1}$$

$$v(t) = v(0) - \int_0^t i(t)\, dt \tag{2}$$

$$p(t) = f\{i(t)\} \tag{3}$$

where $p(t)$ is the systolic left-ventricular pressure, $v(t)$ is systolic left-ventricular volume, $v(0)$ is $v(t)$ at $t=0$, or the left-ventricular end-diastolic volume, $i(t)$ is the blood-flow rate ejected out of the ventricle, and f is a function of the hydraulic impedance of the load against the ejection. Fig. 2 shows the block diagram of this model, in which $e(t)$, heart rate, $v(0)$, and the loading properties are biologically controlled parameters. As mentioned, $e(t)$ is unchanged in its shape and magnitude by alterations in $v(0)$ and the loading conditions, but is altered by stimulation of the cardiac sympathetic nerves and changes in heart rate.

It is almost impossible to solve (1)–(3) analytically with the same $e(t)$ as obtained in experiments on the dog, since they are rearranged into a higher order nonlinear differential equation with time-varying parameters. Therefore, they are solved in this paper first analytically with some assumptions of $e(t)$ and the loading properties, and then they are solved by an analog computer with $e(t)$ approximated to that obtained in the experiments. In the following analysis, the hemodynamic properties of the aortic valve and the arterial load are oversimplified in order that the analysis may emphasize influences on ventricular pumping properties of various important hemodynamic parameters and also clarify fundamental characteristics of the left ventricle itself.

A. Model Analysis

The first model analysis is made with the assumptions that $e(t)$ is constant in systole and that the load is a constant aortic pressure with a resistive aortic valve. In order to solve (1)–(3), the following assumptions are are made: 1) $e(t)$ is a constant e_s in systole ($0 \leqq t \leqq t_s$) and zero in diastole (the same assumption as by Warner [6]); 2) the load against ventricular ejection is a constant-pressure (p_a) arterial system (which will be approximately realized by connecting a large compliance to the arterial system or by using a pressure clamp system) connected in series with the aortic valve with a small forward resistance r and no back flow, making and braking by the pressure difference. With these assumptions, the previously mentioned equations are rewritten as follows. In systole,

$$p(t) = e_s v(t) \tag{4}$$

$$v(t) = v(0) - \int_0^t i(t)\, dt \tag{5}$$

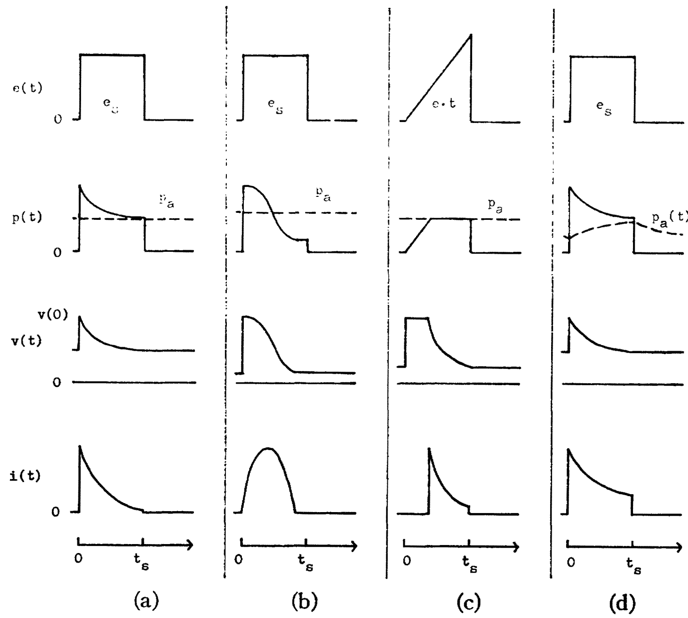


Fig. 3. Time courses of left ventricular $e(t)$, pressure $p(t)$, volume $v(t)$, and blood-flow rate ejected out of ventricle (from above down) in model analyses. (a) From the model in Section III-A. (b) From the model in Section III-B. (c) From the model in Section III-C. (d) From the model in Section III-D. Broken lines show arterial pressure p_a or $p_a(t)$.

$$p(t) = p_a + ri(t) \tag{6}$$

and in diastole, $p(t)$ and $i(t)$ are zero, besides which $v(t)$ is set to $v(0)$ at the end of diastole. Then the following differential equation is obtained in systole:

$$(r/e_s)dp(t)/dt + p(t) = p_a. \tag{7}$$

The solutions are as follows in the case that p_a is smaller than $e_sv(0)$:

$$p(t) = p_a + \{v(0)e_s - p_a\} \exp(-e_st/r) \tag{8}$$

$$v(t) = p_a/e_s + \{v(0) - p_a/e_s\} \exp(-e_st/r) \tag{9}$$

and

$$i(t) = \{v(0)e_s - p_a\} \exp(-e_st/r)/r. \tag{10}$$

These parameters are shown as functions of time in Fig. 3(a). Various important hemodynamic parameters are formulated according to their definitions from these solutions; stroke volume is

$$v_s = \{v(0) - p_a/e_s\}\{1 - \exp(-e_st_s/r)\} \tag{11}$$

residual volume,

$$v_r = p_a/e_s + \{v(0) - p_a/e_s\} \exp(-e_st_s/r) \tag{12}$$

ejection ratio (equal to the ratio of stroke volume to end-diastolic volume),

$$q = [1 - p_a/\{e_sv(0)\}]\{1 - \exp(-e_st_s/r)\} \tag{13}$$

and the total mechanical energy generated by the ventricle in a cardiac cycle (equal to $\int_0^{t_s} p(t)i(t)\,dt$) is

$$w_t = p_a\{v(0)e_s - p_a\}\{1 - \exp(-e_st_s/r)\}/e_s + \{v(0)e_s - p_a\}^2\{1 - \exp(-2e_st_s/r)\}/2e_s \tag{14}$$

where the first term is the mechanical energy consumed by the arterial load (w_l) and the second is that consumed by the aortic valve (w_v). Then w_l is rewritten as follows:

$$w_l = [v(0)^2e_s^2/4 - \{v(0)e_s/2 - p_a\}^2] \cdot \{1 - \exp(-e_st_s/r)\}/e_s. \tag{15}$$

The definition of the total peripheral resistance R is $p_a/(\mathrm{HR}\ v_s)$; thus some of the previously mentioned parameters are rewritten by the use of R as follows:

$$p_a = \frac{\mathrm{HR}\ Rv(0)\{1 - \exp(-e_st_s/r)\}}{1 + \mathrm{HR}\ R\{1 - \exp(-e_st_s/r)\}/e_s} \tag{16}$$

$$v_s = \frac{v(0)\{1 - \exp(-e_st_s/r)\}}{1 + \mathrm{HR}\ R\{1 - \exp(-e_st_s/r)\}/e_s} \tag{17}$$

$$w_l = e_sv(0)^2\{1 - \exp(-e_st_s/r)\} \cdot \left\{\frac{1}{4} - \left(\frac{1}{2} - \frac{\mathrm{HR}\ R\{1 - \exp(-e_st_s/r)\}}{e_s + \mathrm{HR}\ R\{1 - \exp(-e_st_s/r)\}}\right)^2\right\}. \tag{18}$$

These relationships are shown graphically in Figs. 4–7. If the aortic valvular resistance is negligibly small, $\exp(-e_st_s/r)$ in these equations is approximately zero.

B. Model Analysis

A second model analysis is made with the assumptions that $e(t)$ is constant in systole and that the load is a constant aortic pressure with blood inertia at the aortic valve. Inertia of the blood at the aortic valve is not negligible and is more important than blood viscosity [10]. The previously mentioned aortic valvular resistance is replaced here by inertia of the blood; (6) is then replaced by the following equation. In systole,

$$p(t) = p_a + Ldi(t)/dt \tag{19}$$

where L is the blood inertia at the aortic valve and $i(t)$ is not less than zero in systole and is zero in diastole. Therefore, the following differential equation is derived from (4), (5), and (19):

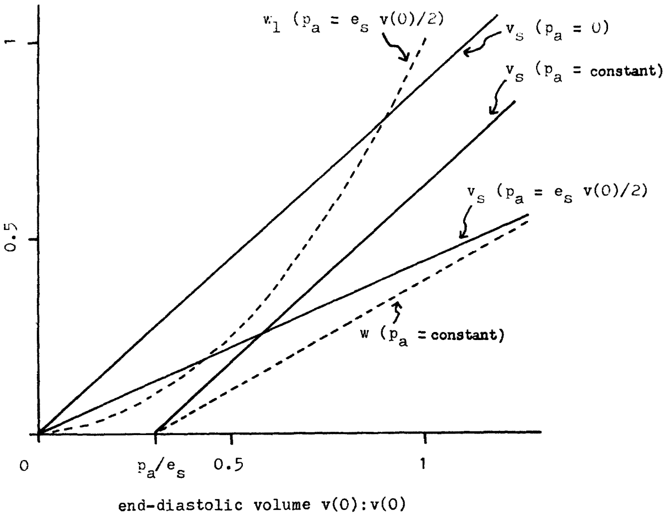
$$(L/e_s)d^2p(t)/dt^2 + p(t) = p_a. \tag{20}$$

The solutions are as follows in the case where p_a is smaller than $e_sv(0)$. In systole, for t less than or equal to $\pi\sqrt{L/e_s}$,

$$p(t) = \{v(0)e_s - p_a\} \cos(t\sqrt{e_s/L}) + p_a \tag{21}$$

$$v(t) = \{v(0) - p_a/e_s\} \cos(t\sqrt{e_s/L}) + p_a/e_s \tag{22}$$

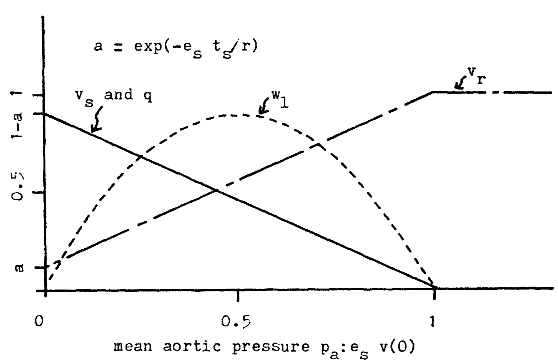
$$i(t) = 1/\sqrt{e_sL}\{v(0)e_s - p_a\} \sin(t\sqrt{e_s/L}) \tag{23}$$



Units on y axis

stroke work $w_1 : e_s v(O)^2/4$
stroke volume $v_s : v(O)$

Fig. 4. Model analysis of relationships of stroke volume and stroke work to left-ventricular end-diastolic volume with mean aortic pressure as a parameter. $v(O)$, v_s, and w_1 are normalized to the respective values shown in the legend following the colon.



Units on y axis

stroke work $w_1 : e_s v(O)^2/4$
residual volume $v_r : v(O)$
stroke volume $v_s : v(O)$
ejection ratio $q : 1$

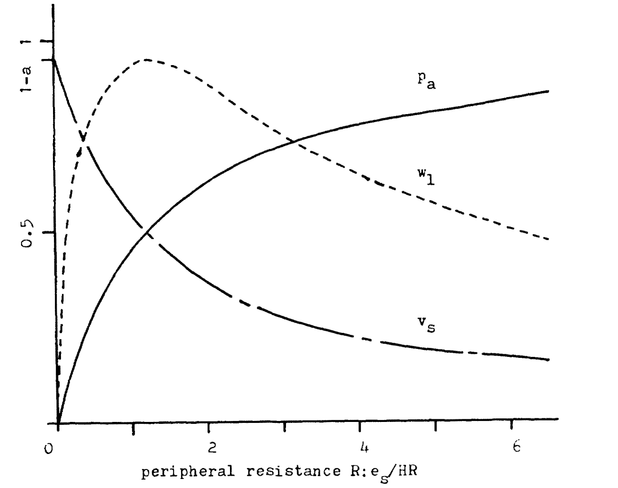
Fig. 5. Model analysis of relationships of ejection ratio, stroke volume, residual volume, and stroke work to mean aortic pressure. All variables are normalized to respective values shown in legend. a is a constant decided by aortic valvular properties.

and for t greater than $\pi\sqrt{L/e_s}$, $p(t)$, $v(t)$, and $i(t)$ are equal to those at $t=\pi\sqrt{L/e_s}$. These parameters are shown as function of time in Fig. 3(b). As in the previous section, various important hemodynamic parameters are formulated as follows. For the end of systole, with t_s not greater than $\pi\sqrt{L/e_s}$,

$$v_s = \{v(0) - p_a/e_s\}\{1 - \cos(t_s\sqrt{e_s/L})\} \tag{24}$$

$$v_r = p_a/e_s + \{v(0) - p_a/e_s\} \cos(t_s\sqrt{e_s/L}) \tag{25}$$

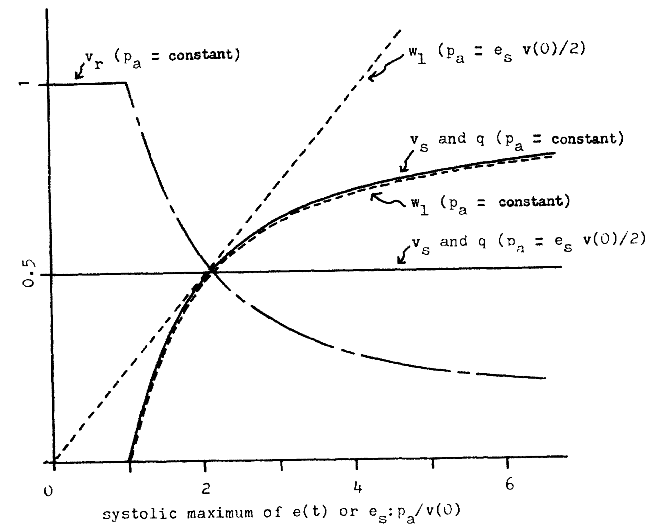
$$q = [1 - p_a/\{e_s v(0)\}]\{1 - \cos t_s(\sqrt{e_s/L})\} \tag{26}$$



Units on y axis

stroke work $w_1 : e_s v(O)^2/4$
stroke volume $v_s : v(O)$
mean aortic pressure $p_a : e_s v(O)$

Fig. 6. Model analysis of relationships of mean aortic pressure, stroke volume, and stroke work to the total peripheral resistance. All variables are normalized to the respective values shown in lengend. a is the constant in the previous figure.



Units on y axis

stroke work $w_1 : e_s v(O)^2$
residual volume $v_r : v(O)$
stroke volume $v_s : v(O)$
ejection ratio $q : 1$

Fig. 7. Model analysis of relationships of ejection ratio, stroke volume, residual volume, and stroke work to systolic maximum of left-ventricular pressure/volume ratio $e(t)$ or e_s with mean aortic pressure as a parameter. All variables are normalized to the respective values shown in legend.

$$w_t = p_a\{v(0)e_s - p_a\}\{1 - \cos t_s(\sqrt{e_s/L})\}/e_s + \{v(0)e_s - p_a\}^2\{1 - \cos(2t_s\sqrt{e_s/L})\}/2e_s \tag{27}$$

$$w_1 = [v(0)^2 e_s^2/4 - \{v(0)e_s/2 - p_a\}^2] \cdot \{1 - \cos(t_s\sqrt{e_s/L})\}/e_s \tag{28}$$

and for t_s greater than $\pi\sqrt{L/e_s}$,

$$v_s = 2v(0) - 2p_a/e_s \tag{29}$$

$$v_r = 2p_a/e_s - v(0) \tag{30}$$

$$q = 2 - 2p_a/\{v(0)e_s\} \tag{31}$$

$$w_t = w_1 = 2p_a\{v(0) - p_a/e_s\} \tag{32}$$

$$w_v = 0. \tag{33}$$

All these hemodynamic parameters can be formulated by the corresponding equations in Section III-A with $\exp(-e_st_s/r)$ replaced by $\cos(t_s\sqrt{e_s/L})$. Therefore, the relationships among these hemodynamic parameters in this model with blood inertia are similar to those in Section III-A.

C. Model Analysis

A third model analysis is made with the assumptions that $e(t)$ is increased linearly in systole and that there is no impedance between the ventricle and the constant pressure load. The physiological $e(t)$ obtained in the experiments on the dog is a monotonically increasing function of time in the first two thirds of systole, as shown in Fig. 1; this increasing phase of $e(t)$ is approximated here by a linearly increasing function of time $e(t) = et$. The aortic valvular impedance is neglected for simplification of the analysis. Equations to be used corresponding to (4)–(6) are the following.

For t not greater than $p_a/\{e_sv(0)\}$,

$$p(t) = etv(0), \quad i(t) = 0 \tag{34}$$

and for t greater than $p_a/\{e_sv(0)\}$,

$$p(t) = etv(t) \tag{35}$$

$$v(t) = v(0) - \int_0^t i(t)\,dt \tag{36}$$

$$p(t) = p_a. \tag{37}$$

Hence the solutions are as follows:

$$v(t) = p_a/(et) \tag{38}$$

$$i(t) = p_a/(et^2). \tag{39}$$

The time courses of $p(t)$, $v(t)$, and $i(t)$ are shown in Fig. 3(*c*). From these solutions various hemodynamic parameters are formulated. For t_s greater than $p_a/\{e_sv(0)\}$,

$$v_s = v(0) - p_a/(et_s) \tag{40}$$

$$v_r = p_a/(et_s) \tag{41}$$

$$q = 1 - p_a/\{et_sv(0)\} \tag{42}$$

$$w_t = w_1 = et_sv(0)^2/4 - \{et_sv(0)/2 - p_a\}^2/(et_s). \tag{43}$$

It should be noted that these formulations will be the same as those in Section III-A if $\exp(-e_st_s/r)$ and e_s are replaced by 0 and et_s, respectively; therefore, almost the same types of relationships hold true among the parameters in this model as among those in Section III-A.

D. Model Analysis

A fourth model analysis is made with the assumptions that $e(t)$ is constant in systole and that the load is a Windkessel model with a resistive aortic valve. The input impedance of the arterial system can be approximated by the Windkessel model, where compliance C of the arterial system is lumped and combined in parallel with the total peripheral resistance R. Instead of (6), the following two equations are used. In systole,

$$i(t) = p_a(t)/R + Cdp_a(t)/dt \tag{44}$$

$$p(t) = p_a(t) + ri(t) \tag{45}$$

where $p_a(t)$ is aortic blood pressure and r is the aortic valvular resistance, as shown in Section III-A. In diastole (t is from t_s to t_s+t_d), $p(t)$ and $i(t)$ are zero, and

$$p_a(t) = p_a(t_s)\exp\{-(t - t_s)/(RC)\}. \tag{46}$$

In systole the following second-order differential equation is obtained:

$$(CRr/e_s)d^2p(t)/dt^2 + (CR + R/e_s + r/e_s)dp(t)/dt + p(t) = 0. \tag{47}$$

The solutions in the case of cardiac ejection in a steady condition are as follows, omitting the intermediate transformations:

$$p(t) = v(0)e_s\{D_1(E_d - E_1)F_2 - D_2(E_d - E_2)F_1\}/A \tag{48}$$

$$p_a(t) = v(0)e_sD_1D_2\{(E_d - E_1)F_2 - (E_d - E_2)F_1\}/A \tag{49}$$

$$v(t) = v(0)\{D_1(E_d - E_1)F_2 - D_2(E_d - E_2)F_1\}/A \tag{50}$$

$$i(t) = v(0)\{d_1D_2(E_d - E_2)F_1 - d_2D_1(E_d - E_1)F_2\}/A \tag{51}$$

where d_1 and d_2 are the roots of the characteristic equation of the differential equation (47) and

$$D_1 = d_1 + e_s/r, \quad D_2 = d_2 + e_s/r$$

$$E_1 = \exp(d_1t_s), \quad E_2 = \exp(d_2t_s), \quad E_d = \exp\{t_d/(RC)\}$$

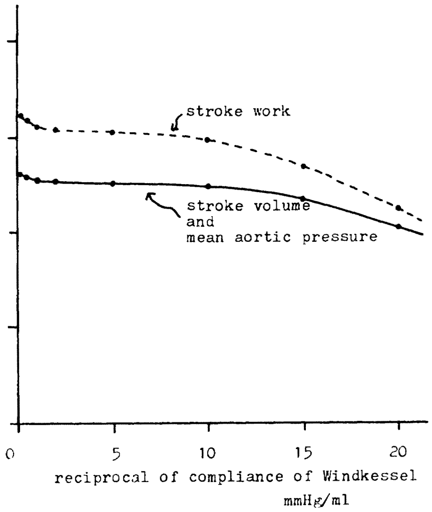
$$F_1 = \exp(d_1t), \quad F_2 = \exp(d_2t)$$

$$A = (d_1 - d_2)E_d + D_2E_2 - D_1E_1.$$

The time courses of $p(t)$, $p_a(t)$, $v(t)$, and $i(t)$ are shown in Fig. 3(d). From these solutions,

$$v_s = v(0)\{D_1(1-E_2)(E_d-E_1) - D_2(1-E_1)\cdot(E_d-E_2)\}/A \tag{52}$$

$$v_r = v(0)\{E_d(D_1E_2 - D_2E_1) + E_1E_2(d_2-d_1)\}/A \tag{53}$$



Units on y axis

mean aortic pressure	×10 mmHg
Stroke work	×100 mmHg·ml
Stroke volume	ml

$v(O)$	25 ml
$e(t)_{max}$	10 mmHg/ml
R	10 mmHg·s/ml
r	0.01 mmHg·s/ml
t_s	0.5 s
HR	60/min

Fig. 8. Model analysis of relationships of stroke volume, stroke work, and mean aortic pressure to reciprocal of arterial compliance with constant $v(O)$ and other parameters set to physiological values of the dog.

$$q=\{D_1(1-E_2)(E_d-E_1)-D_2(1-E_1)(E_d-E_2)\}/A \tag{54}$$

$$\begin{aligned} w_t=v(0)^2 e_s\{&D_1 D_2(E_d-E_1)(E_d-E_2)(E_1 E_2-1)\\ &-D_1^2(E_d-E_1)^2(E_2^2-1)/2\\ &-D_2^2(E_d-E_2)^2(E_1^2-1)/2\}/A^2 \end{aligned} \tag{55}$$

$$\begin{aligned} w_1=v(0)^2 r D_1 D_2\{&(d_1 D_2+d_2 D_1)(E_d-E_2)(E_d-E_1)\\ &\cdot(E_1 E_2-1)/(d_1+d_2)-D_1(E_d-E_1)^2(E_2^2-1)/2\\ &-D_2(E_d-E_2)^2(E_1^2-1)/2\}/A^2. \end{aligned} \tag{56}$$

These equations are then numerically calculated with their parameters set to typical physiological values, and the relationships are in good agreement with those in Figs. 4–7. Moreover, relationships of various hemodynamic parameters to arterial compliance C are formulated, and they are also numerically calculated with their parameters substituted by typical values, an example of which is graphically shown in Fig. 8.

E. Model Analysis

A fifth model analysis was made on an analog computer with $e(t)$ approximated to the experimentally obtained time course. Equations (1)–(3) are computed with $e(t)$ approximated to the physiological data as appear in Fig. 1. The loading properties are in general easily programmed. Fig. 9 is an example of the programs where the cardiac load is the resistive aortic valve and the Windkessel model. Fig. 10 shows some examples of the solutions where the relationship of aortic blood flow

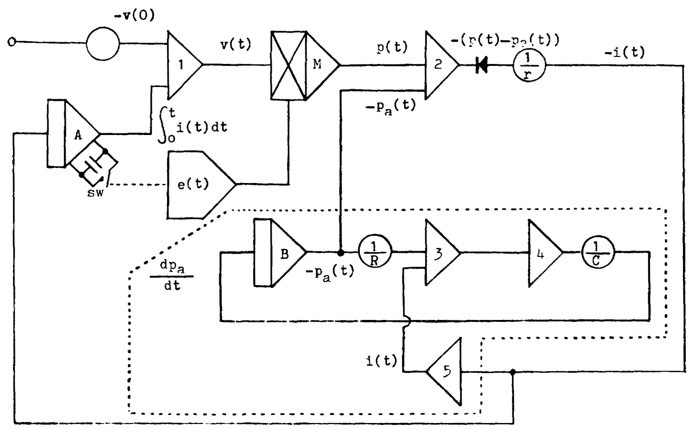


Fig. 9. Analog-computer program of simulation of left-ventricular pumping. Load properties are shown surrounded by dotted line. $e(t)$ is generated periodically by function generator, and its period is equivalent to cardiac cycle. Integrating capacitor of integrator A is short-circuited at the end of each cardiac cycle. M is a multiplier.

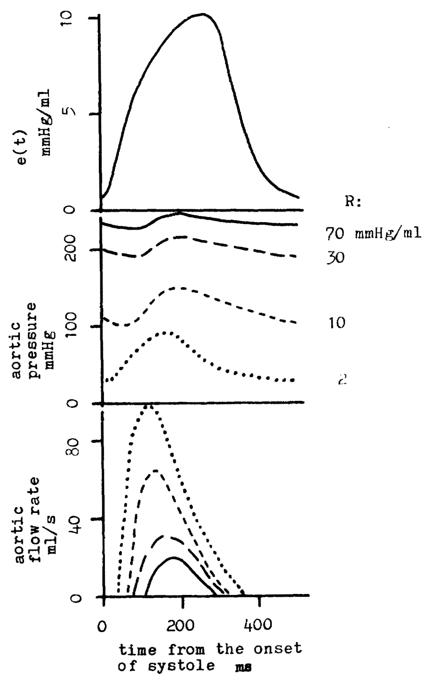


Fig. 10. Some examples of solutions of left-ventricular pumping simulation with $e(t)$ approximated to physiological data. Cardiac load is Windkessel with its compliance of 0.1 ml/mmHg and variable resistance R. Aortic valvular impedance is resistance r of 0.1 mmHg·s/ml.

to arterial pressure is shown under various magnitudes of the total peripheral resistance with fixed $e(t)$. Fig. 11 shows another example of the solutions in which the relationship among the constant loading pressure (p_a) and other hemodynamic parameters is shown with $e(t)$ unchanged. Fig. 12 is an example of the solutions where the effect of blood inertia of a physiological value (0.005 mmHg·s²/ml in the dog) on some hemodynamic parameters is examined; aortic blood flow is smoothed by the inertia, but there are almost no effects on left-ventricular pressure, aortic pressure, and stroke volume. Influences of the time course of $e(t)$ on hemodynamics

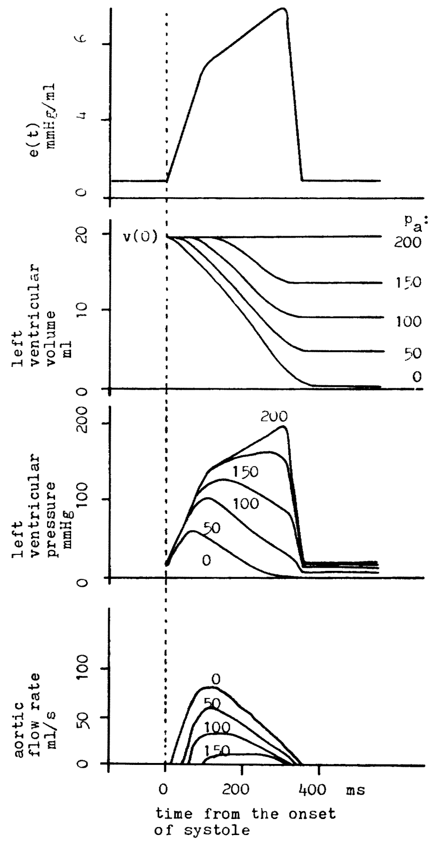


Fig. 11. Some examples of solutions with $e(t)$ approximated to physiological data and with constant aortic pressure load p_a.

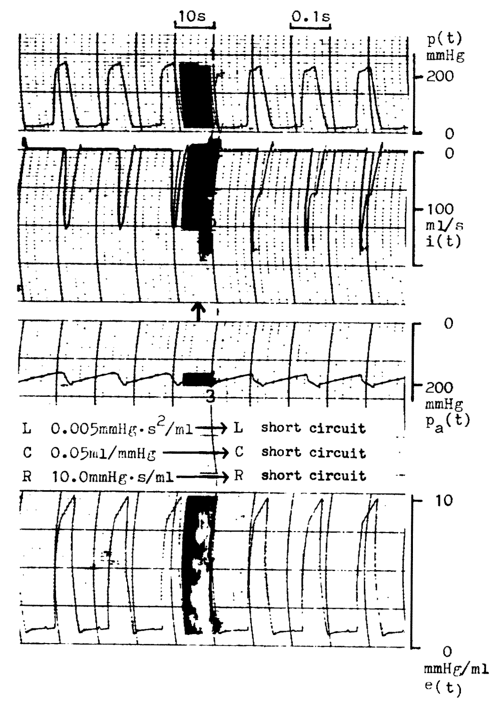


Fig. 12. Effects of blood inertia at aortic valve on various hemodynamic parameters. At time indicated by arrow, blood inertia is omitted with arterial compliance and total peripheral resistance unchanged.

are examined in the pumping model with the values of r, C, R, and heart rate fixed to physiologically typical values ($r=0.1$ mmHg/ml, $C=0.1$ ml/mmHg, $R=10$ mmHg·s/ml, and HR$=2$/s, but blood inertia is neglected). If the maximal value of $e(t)$ is unchanged with various changes in its time course, mean aortic pressure and stroke volume are not altered, although time courses of aortic pressure, intraventricular pressure, intraventricular volume, and aortic blood-flow velocity are changed conspicuously. From these solutions in analog-computer simulation, the relationships among various hemodynamic parameters are obtained, and they are quantitatively in good agreement with those obtained in the previous sections.

IV. Discussions

In the left-ventricular pumping model based on $e(t)$, the left-ventricular systolic time-varying pressure/volume ratio, is analyzed in various ways. The time courses of $e(t)$ used in the analysis are various; the aortic valvular properties are either resistive or inductive; the arterial load is a constant pressure or the Windkessel model. Time courses of various important hemodynamic parameters such as the left-ventricular pressure $p(t)$, volume $v(t)$, and aortic blood-flow velocity $i(t)$ are different from their respective correspondents in different types of model analysis. However, almost the same relationships hold true in these different analyses among mean or integrated values of stroke volume v_s, residual volume v_r, stroke work w_t and w_1, and mean aortic pressure p_a. The main differences in their formulations appear to be caused, at first glance, by the properties of the aortic valve since differences exist in the coefficients $\exp(-e_s t_s/r)$, $\cos(t_s\sqrt{e_s/L})$, etc., where r and L are the characteristic constants of the aortic valve; however, these terms are considered to be negligibly small in the physiological range. The maximal value of $e(t)$ seems to have a much greater influence on the amounts of stroke volume, mean aortic pressure, stroke work, etc., than the whole time course of $e(t)$.

In the physiological system of left-ventricular pumping, the time course of $e(t)$ is like those in Fig. 1, the hemodynamic characteristics of the aortic valve is mainly the impedance due to blood inertia and viscosity in the forward direction without back flow, and the arterial system is a distributed constant system that can be simulated approximately by the Windkessel model and elaborately by more detailed models [9]. The previously mentioned relationships among various hemodynamic parameters also seem approximately to hold true to the physiological system since almost the same relationships are obtained in various types of models in which some of the fundamental properties of the circulatory system are emphasized. Some of these relationships have been already analyzed qualitatively in the previous simulations of other investigators [7] and [9]. It is interesting that the fundamental hemodynamic relationships among mean or integrated parameters are the same in spite of conspicuous differences of respective time courses.

The relationships among the previously mentioned hemodynamic parameters have been investigated physiologically in animal experiments, and it seems necessary to the evaluation of the concept of $e(t)$ to compare the previously mentioned results from the theoretical analysis with the corresponding physiological data. The theoretical relationships of stroke volume and stroke work to end-diastolic volume as in Fig. 4 are qualitatively in good agreement with the correspondents in physiology called either the Frank–Starling law of the heart [11] or Sarnoff's ventricular function curve [12]. Mean aortic pressure is theoretically an important parameter that varies their quantitative relationships, but this has not been studied in detail in animal experiments. Moreover, a detailed comparison of the theoretical curves and the experimental data is difficult since mean left-atrial pressure or left-ventricular end-diastolic pressure has been measured instead of left-ventricular volume in almost all previous works, and there exists some nonlinearity in the left-ventricular pressure–volume relationship in diastole. The theoretical relationships of stroke volume and stroke work to mean arterial pressure with constant left-ventricular end-diastolic volume, as in Fig. 5, or to the total peripheral resistance, as in Fig. 6, are also qualitatively in good agreement with the corresponding physiological data [13] and [14]. The theoretical relationships of various parameters to the maximal value of $e(t)$ as in Fig. 7 has no correspondents in physiological data; nevertheless $e(t)$ is roughly compatible with physiological properties of the left ventricle since the maximum of $e(t)$ is a good index of myocardial inotropism [4]. The theoretical effect of arterial compliance on various hemodynamic parameters, as in Fig. 8, also approximately agrees with physiological data [15].

From these discussions, the theoretical properties of the left-ventricular pumping model based on the systolic pressure/volume ratio are considered to be in satisfactory agreement with the established experimental data on properties of the left ventricle. Consequently, the concept of systolic left-ventricular pressure/volume ratio $e(t)$ appears to explain the basic characteristics of the pumping left ventricle, and the analysis in this investigation corroborates the conclusion of the physiological experimentation, indicating that $e(t)$ is a good index of the pumping left ventricle.

Moreover, the myocardial tension–velocity relationship in physiology has been recently deduced theoretically from $e(t)$ and a ventricular geometrical model [16], indicating that the pumping properties of the left ventricle can be described by $e(t)$ from the viewpoint of the ventricle as a whole and by the tension–velocity relationship from the viewpoint of the myocardium as the ventricular constituent. In this respect, the hemodynamic characteristics of the Beneken's model [5] based on the tension–velocity relationship is considered to be compatible with the models based on $e(t)$.

Nomenclature

C	Compliance of the arterial system (ml/mmHg).
e	Constant in Section III-C.
e_s	Constant in Section III-A and systolic left-ventricular pressure/volume ratio (mmHg/ml).
$e(t)$	Systolic left-ventricular pressure/volume ratio (mmHg/ml).
HR	Heart rate (beats/s).
$i(t)$	Blood-flow rate ejected out of the left ventricle (ml/s).
L	Blood inertia at the aortic value (mmHg·s²/ml).
p_a	Constant aortic pressure (mmHg).
$p_a(t)$	Aortic pressure (mmHg).
$p(t)$	Left ventricular pressure in systole (mmHg).
q	Ejection ratio of the left ventricle [1].
R	Total peripheral resistance (mmHg·s/ml).
r	Aortic valvular resistance (mmHg·s/ml).
t	Time from the onset of systole (s).
t_d	Diastolic interval (s).
t_s	Systolic interval (s).
$v(0)$	Left-ventricular end-diastolic volume, or left-ventricular volume at the onset of systole (ml).
v_r	Residual volume of the left ventricle (ml).
v_s	Stroke volume of the left ventricle (ml).
$v(t)$	Left ventricular volume in systole (ml).
w_1	Mechanical energy consumed by the arterial load in one cardiac cycle (mmHg·ml).
w_t	Total mechanical energy generated by the left ventricle in one cardiac cycle (mmHg·ml).
w_v	Mechanical energy consumed at the aortic valve (mmHg ml).

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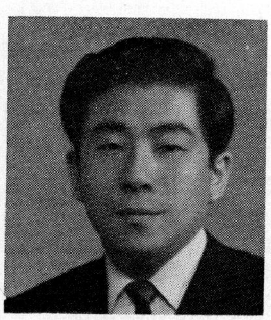

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