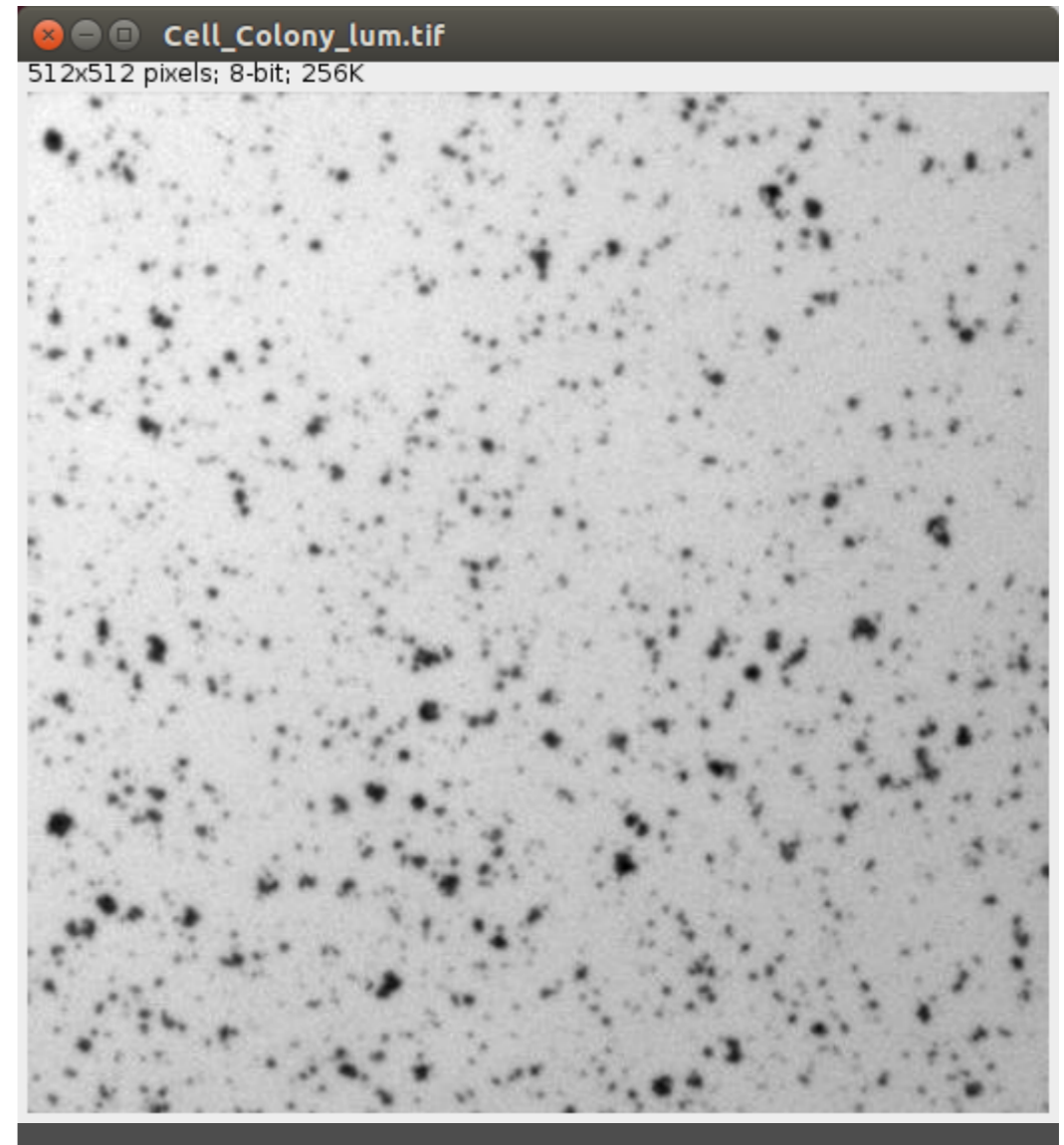


Análisis en frecuencia

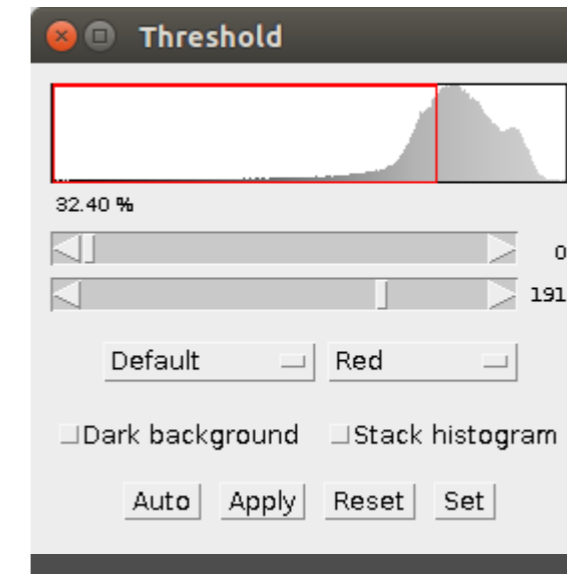
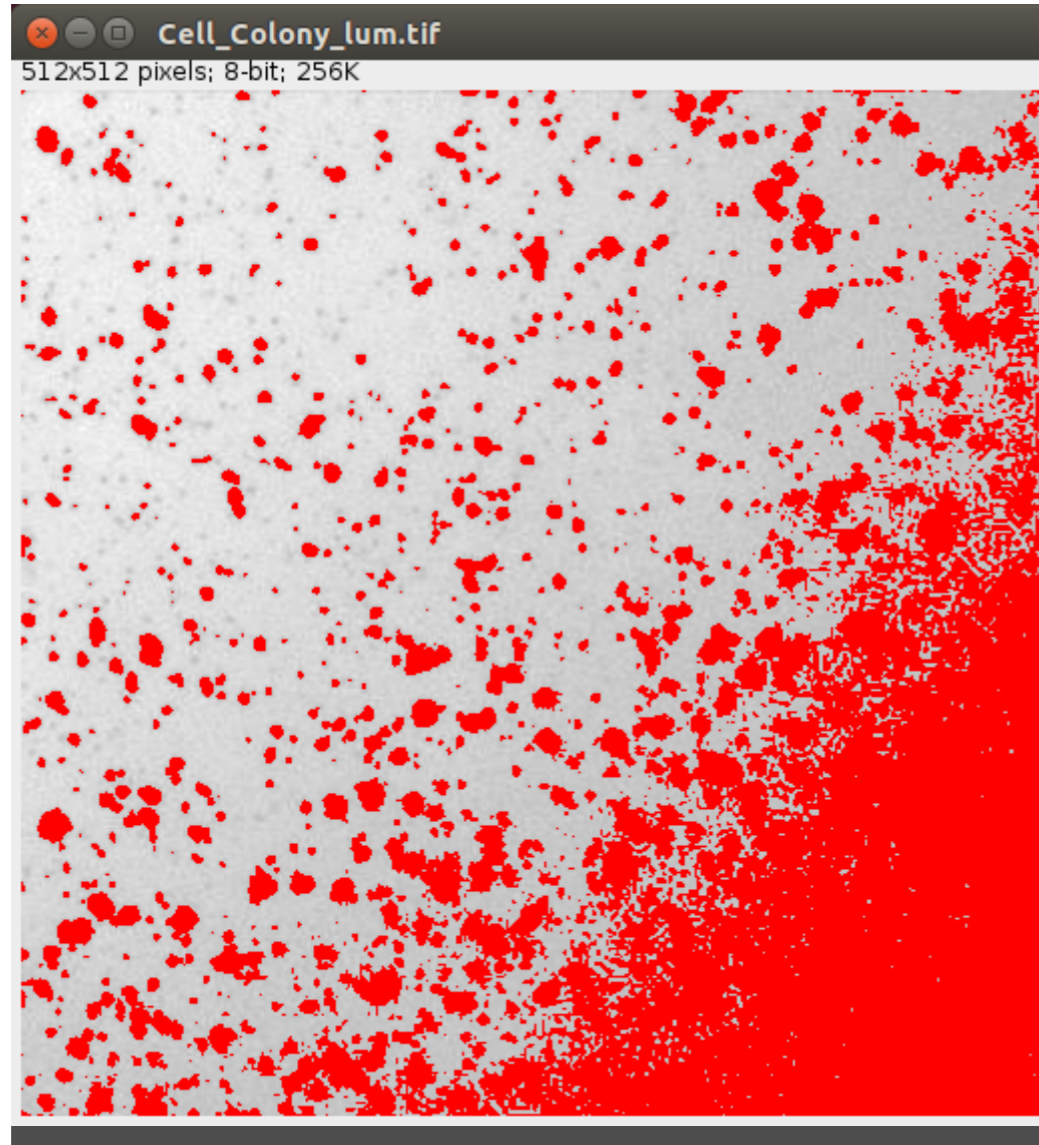
- Digital Image Processing, An Algorithm Introduction to Java
Wilhelm Burger & Mark J. Burge. Springer, ISBN 978-1-84628-379-6
Las transparencias usan material del libro disponible en
www.imagingbook.com
- Digital Image Processing
Gonzalez & Woods
Las transparencias usan material del libro
<http://www.imageprocessingplace.com/>

- Queremos detectar partículas en esta imagen
- Idea:
 - Umbralizar
 - Quedarnos con las partículas de cierto tamaño
- Pero ...



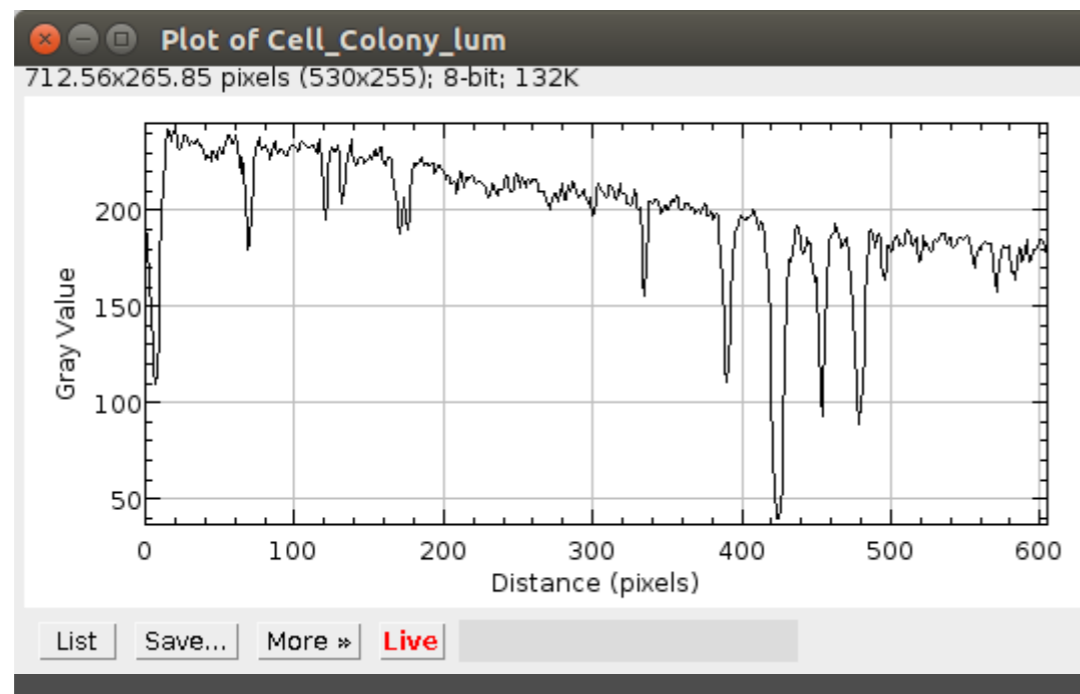
¿Para qué analizar en frecuencia?

- No es posible encontrar un umbral global



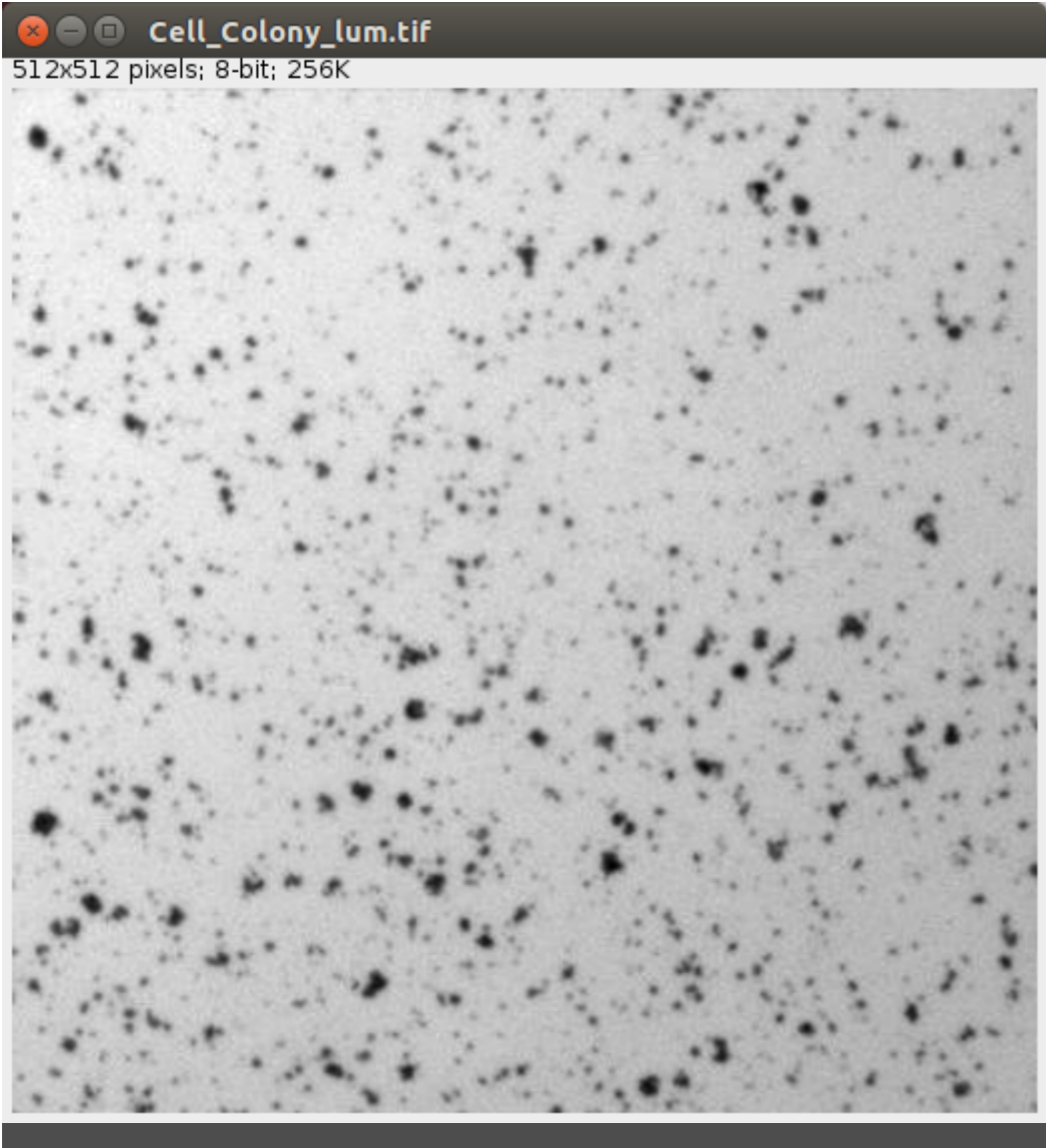
¿Para qué analizar en frecuencia?

- La imagen tiene:
 - iluminación no uniforme (variación suave)
 - ruido (variación rápida)



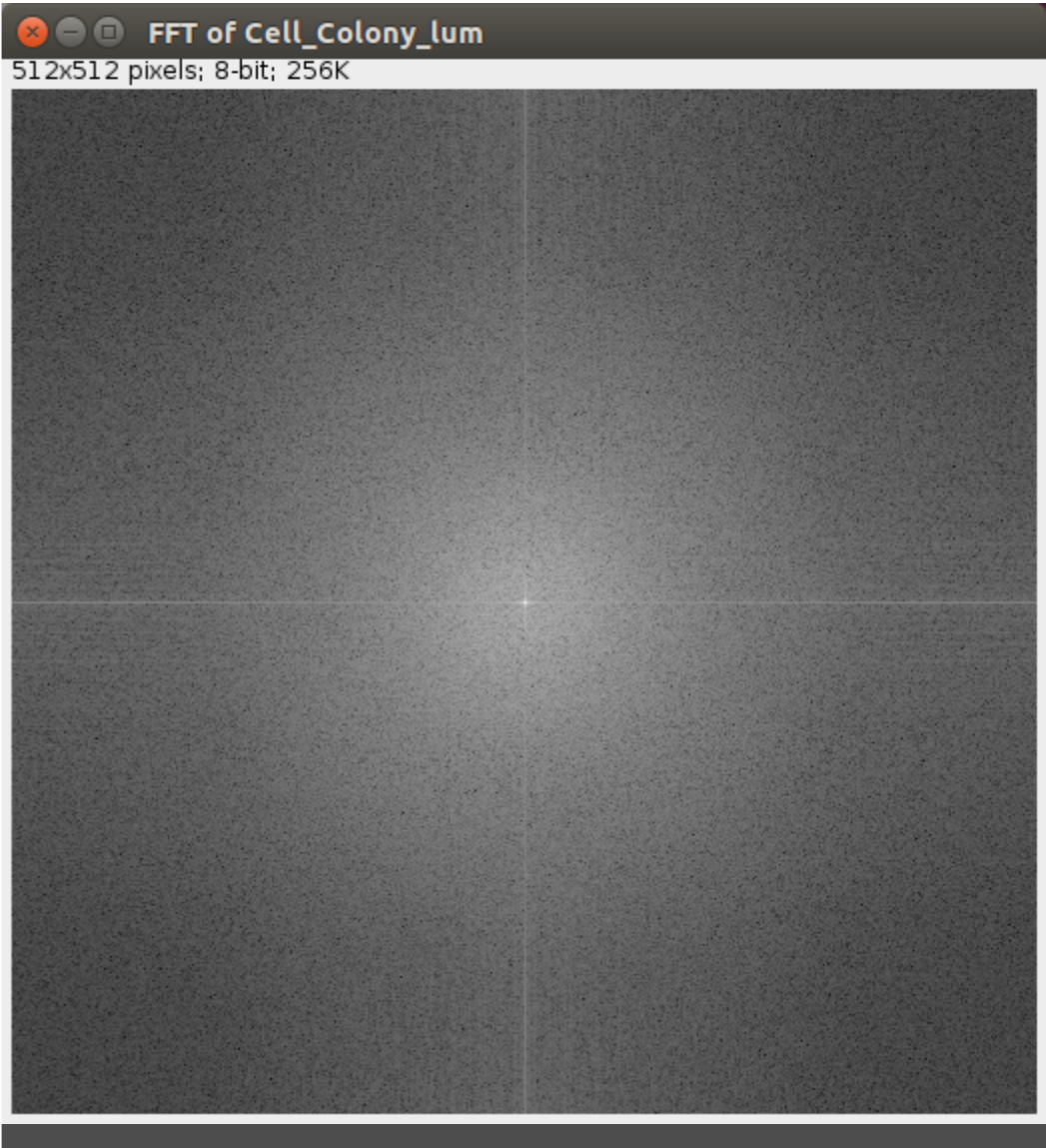
- Nos interesa en la imagen
 - Ver qué cosas varían lentamente (frecuencia baja)
 - Ver qué cosas varían rápidamente (frecuencia alta)
- Describir la imagen en función de estas componentes de distinta frecuencia
- Nos va a permitir por ejemplo filtrar componentes no deseadas.

Transformada de Fourier (FFT)



directa

inversa



- Funciones periódicas
 - ¿Puedo representar una función periódica como suma de sinusoides?
 - ¿Qué frecuencias van a tener las sinusoides?
 - ¿Cuánto de cada senoide tengo que poner para formar mi función?

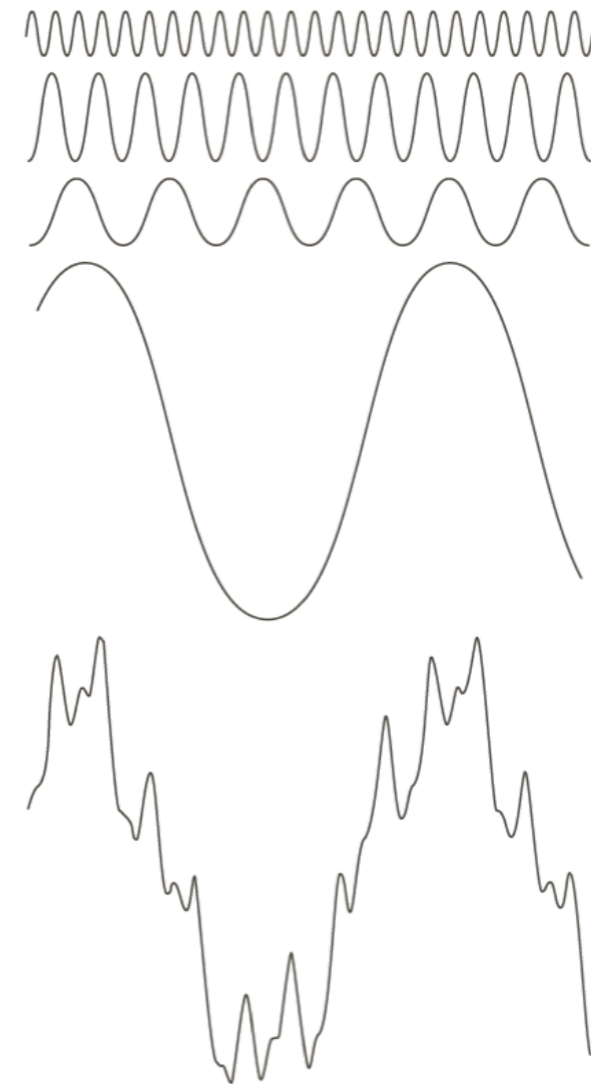


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

- Funciones periódicas
 - ¿Puedo representar una función periódica como suma de sinusoides?
 - ¿Qué frecuencias van a tener las sinusoides?
 - ¿Cuánto de cada senoide tengo que poner para formar mi función?



Si! Con las Series de Fourier.

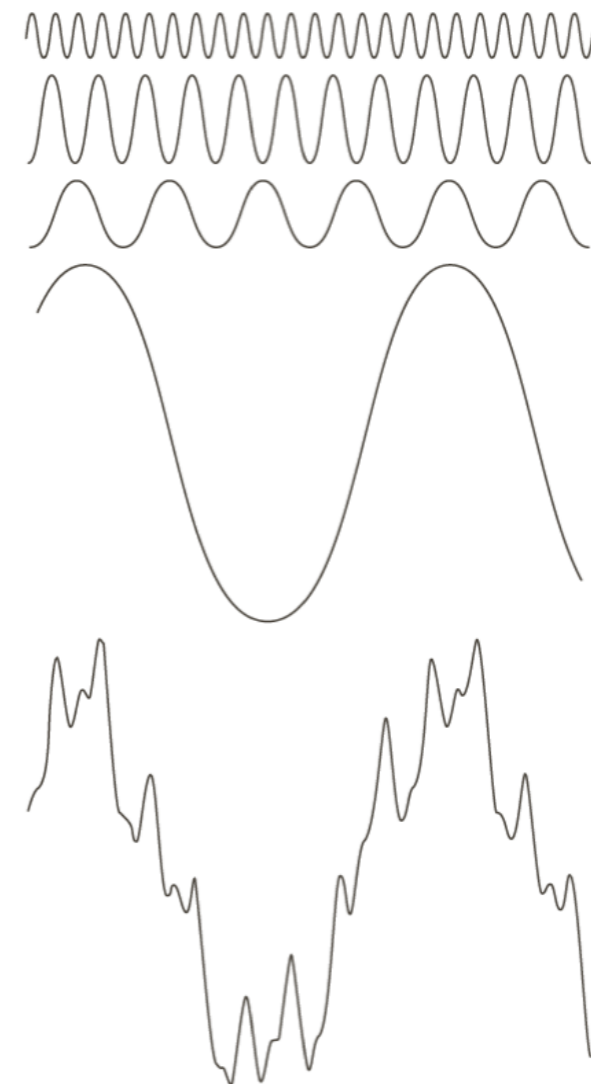


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

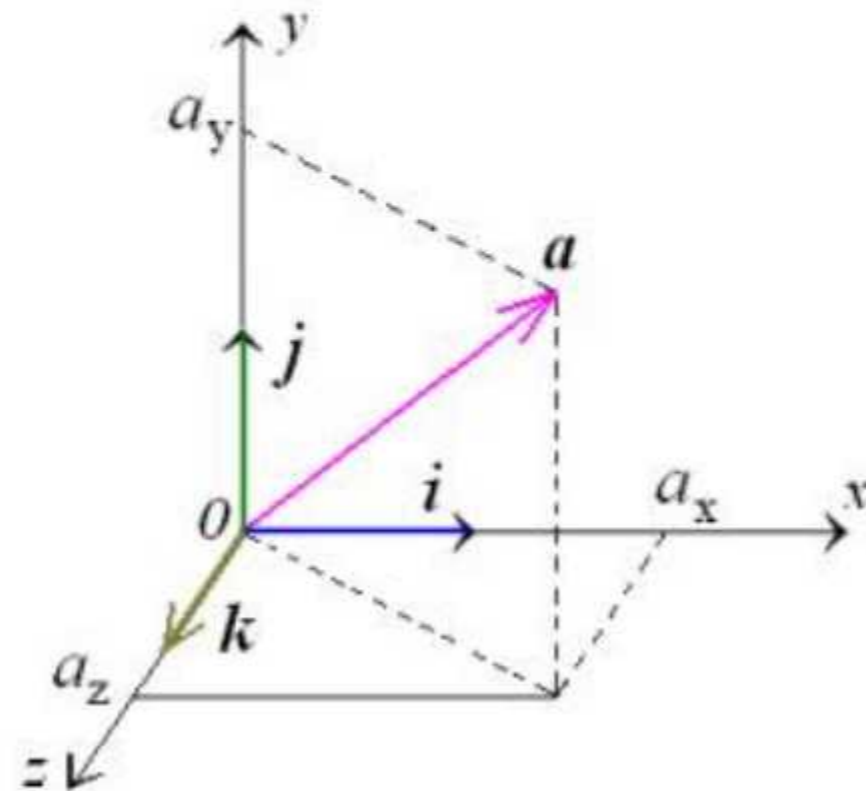
- Funciones periódicas
 - Podemos poner una función periódica como la suma de senos y cosenos (Serie de Fourier)

$$g(x) = \sum_{k=0}^{\infty} [A_k \cos(k\omega_0 x) + B_k \sin(k\omega_0 x)]$$

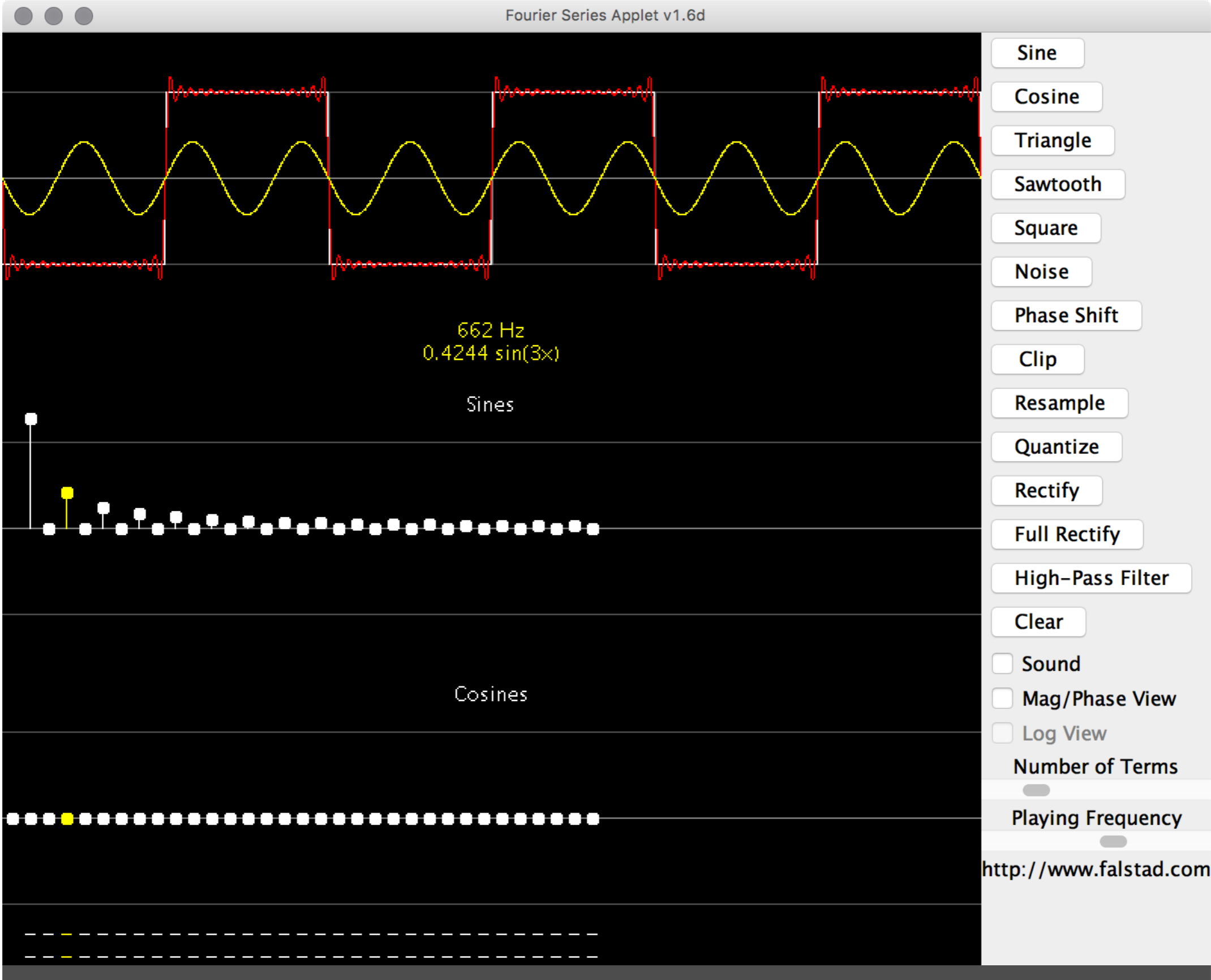
- Componentes con frecuencias múltiplos de la frecuencia de la función periódica
- ¿Cuanto valen A_k y B_k ?
 - La idea es comparar (“correlacionar”) la señal $g(x)$ con las sinusoides de manera de ver cuánto se parecen.

- Expresamos una función periódica como una suma ponderada de sinusoides
- Sinusoides = Funciones “base”

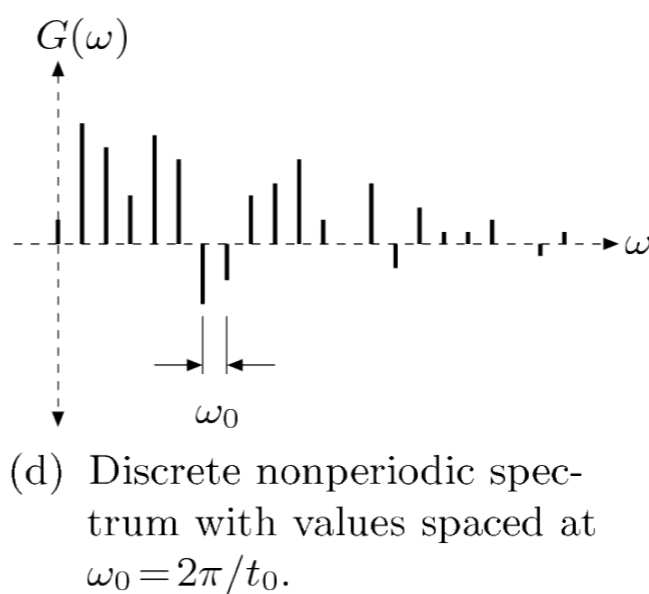
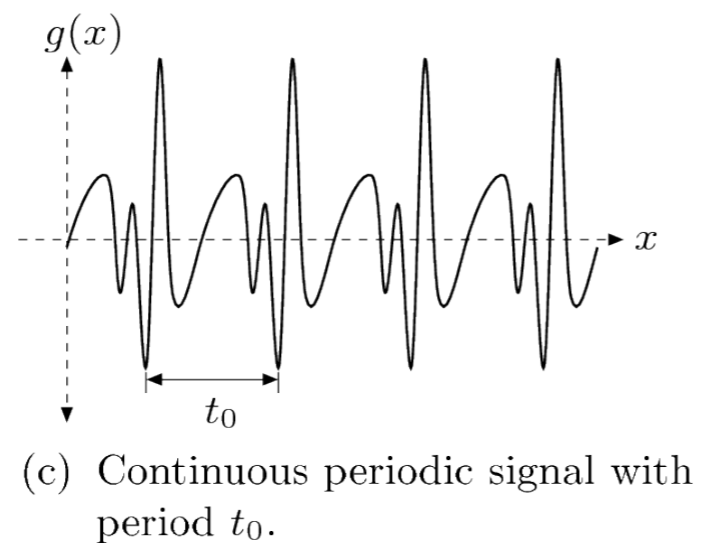
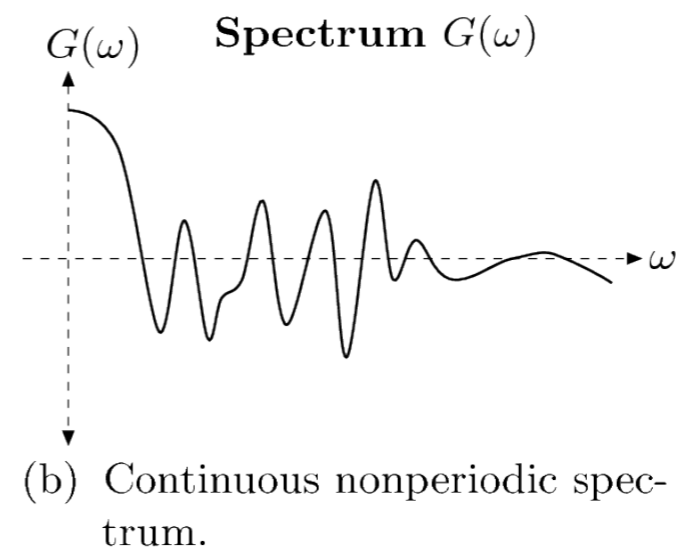
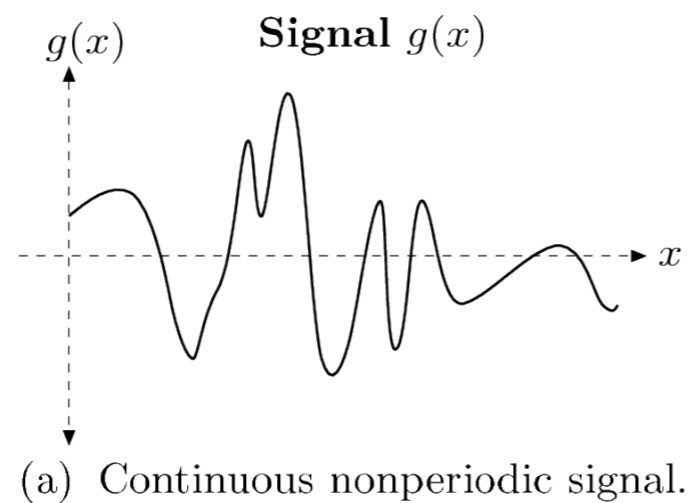
$$\begin{aligned}\mathbf{a} = \{a_x, a_y, a_z\} &= a_x \{1, 0, 0\} + a_y \{0, 1, 0\} + a_z \{0, 0, 1\} \\ &= a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k}\end{aligned}$$



Análisis de frecuencia en 1D

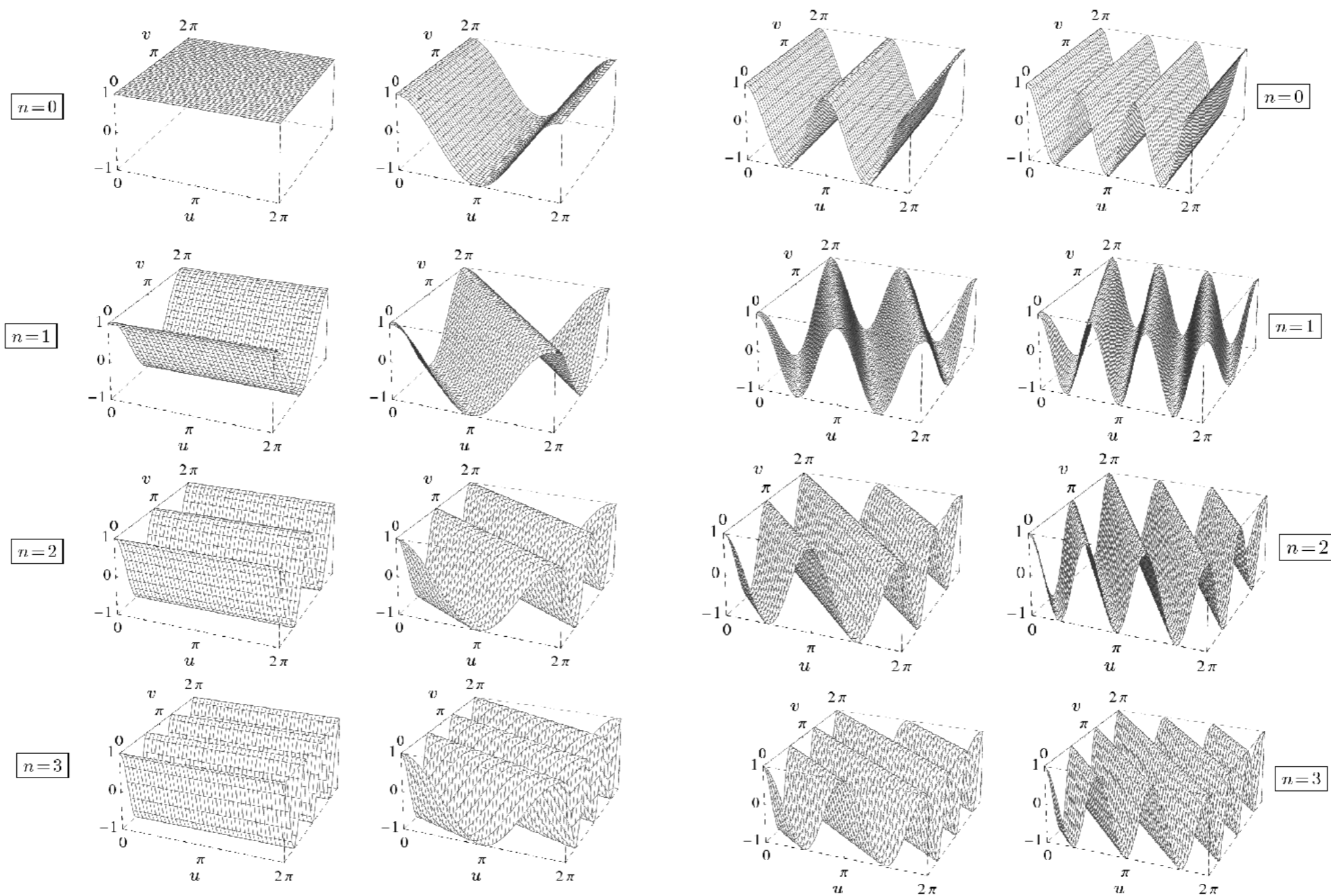


- ¿Se puede extender la idea a funciones no periódicas ?
- Si, pero ahora en lugar de tener frecuencias discretas tenemos un continuo



- ¿Qué quiere decir transformar la señal?
 - Transformamos la señal para verla / analizarla de otra manera
 - Expresamos la señal en términos de sus componentes de frecuencia
 - Podemos movernos en los dos sentidos sin perder información
 - Transformada directa: del espacio a frecuencia
 - Transformada inversa: de frecuencia al espacio

- Ahora nuestras funciones base son sinusoides en 2D
- Vamos a comparar nuestra señal con sinusoides espaciales con distintas frecuencias y orientaciones



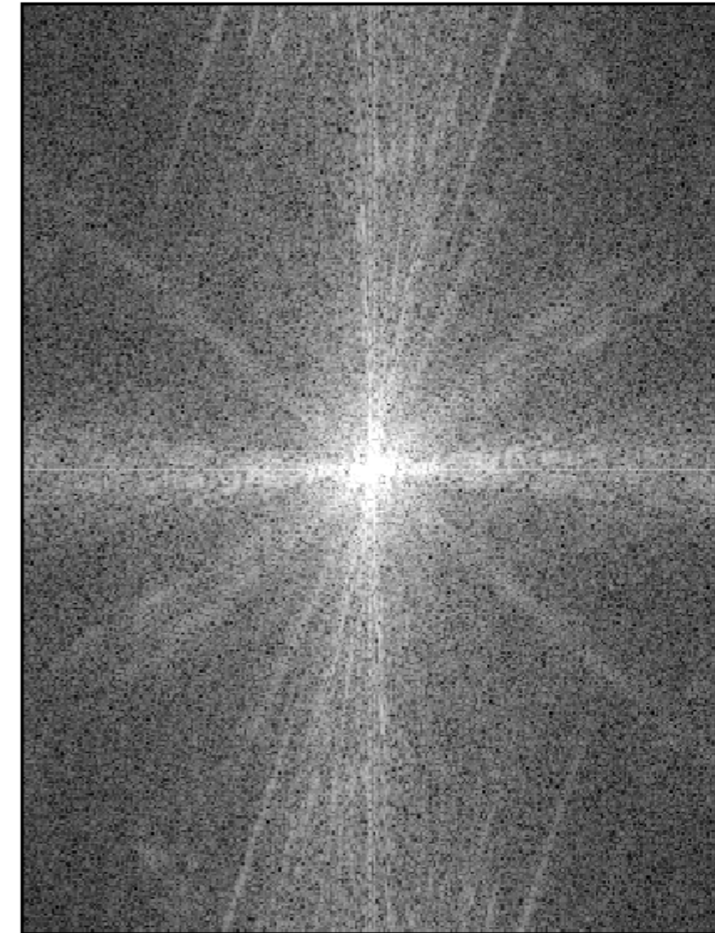
- Visualización de la transformada: espectro



(a)

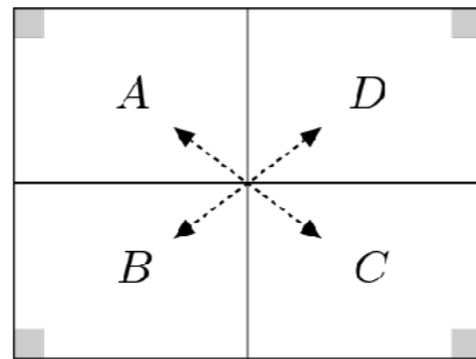


(b)

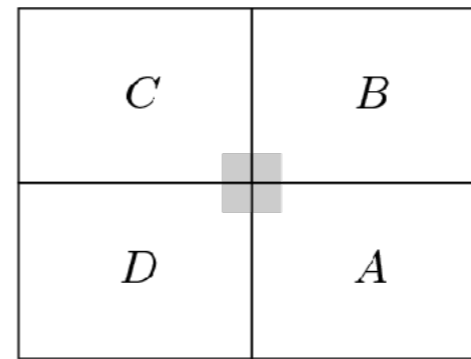


(c)

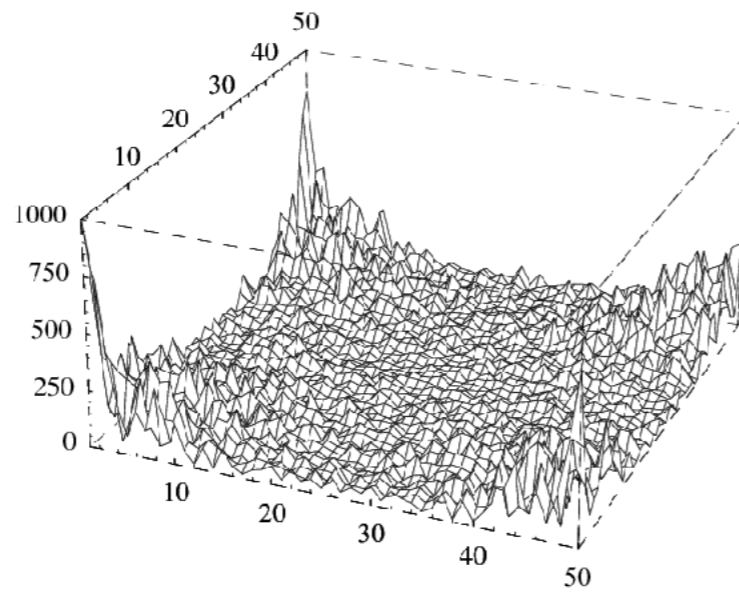
- Visualización de la transformada



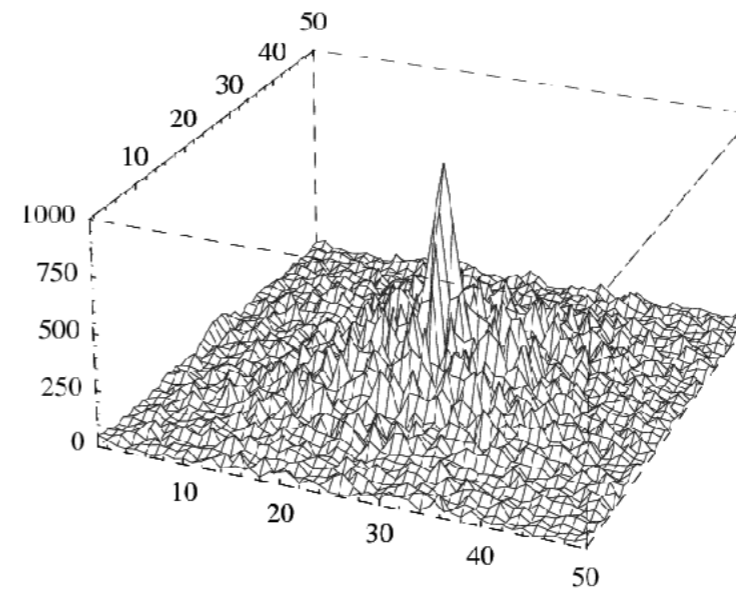
(a)



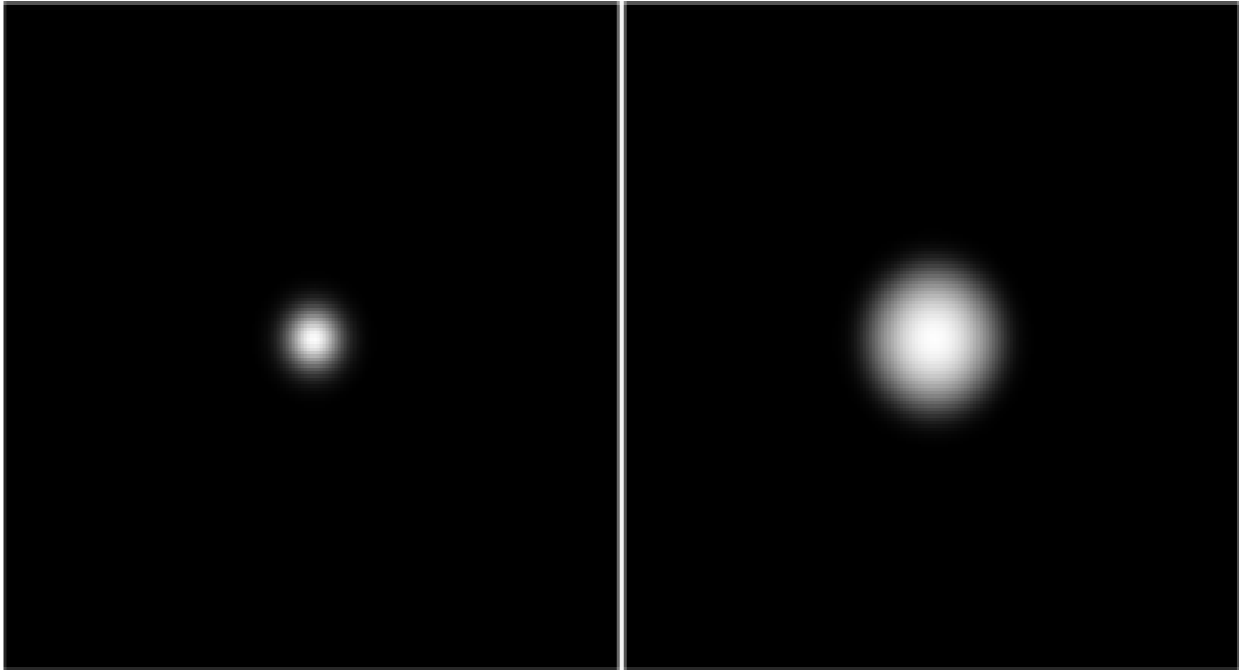
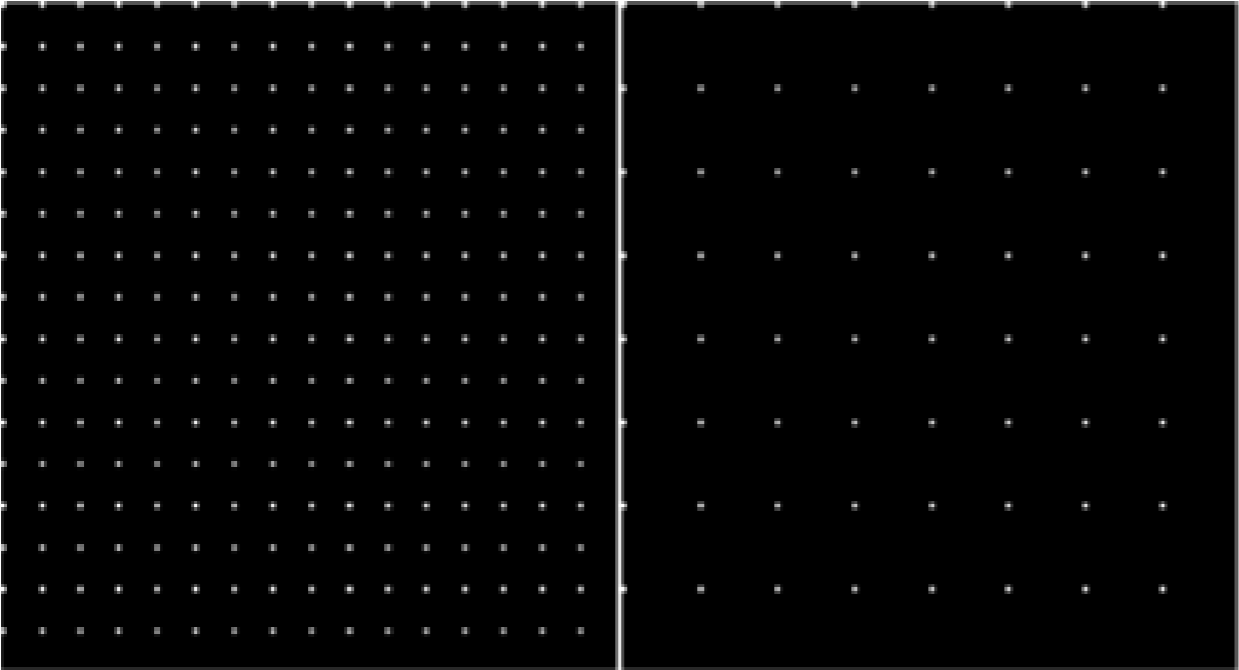
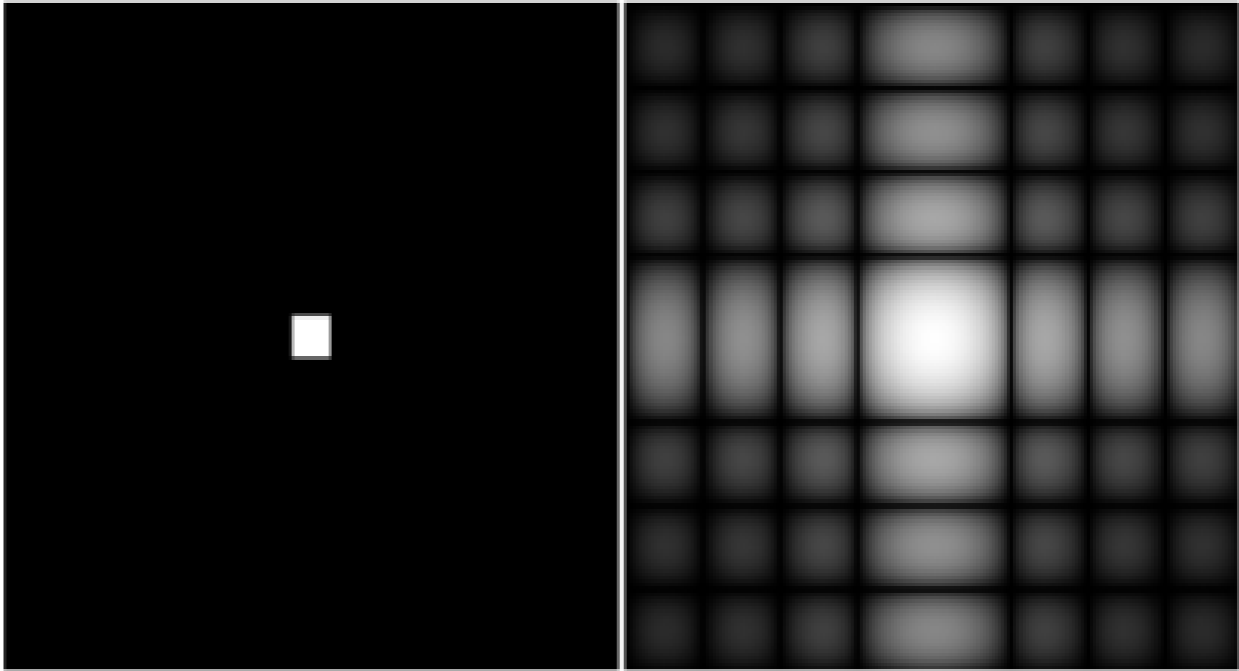
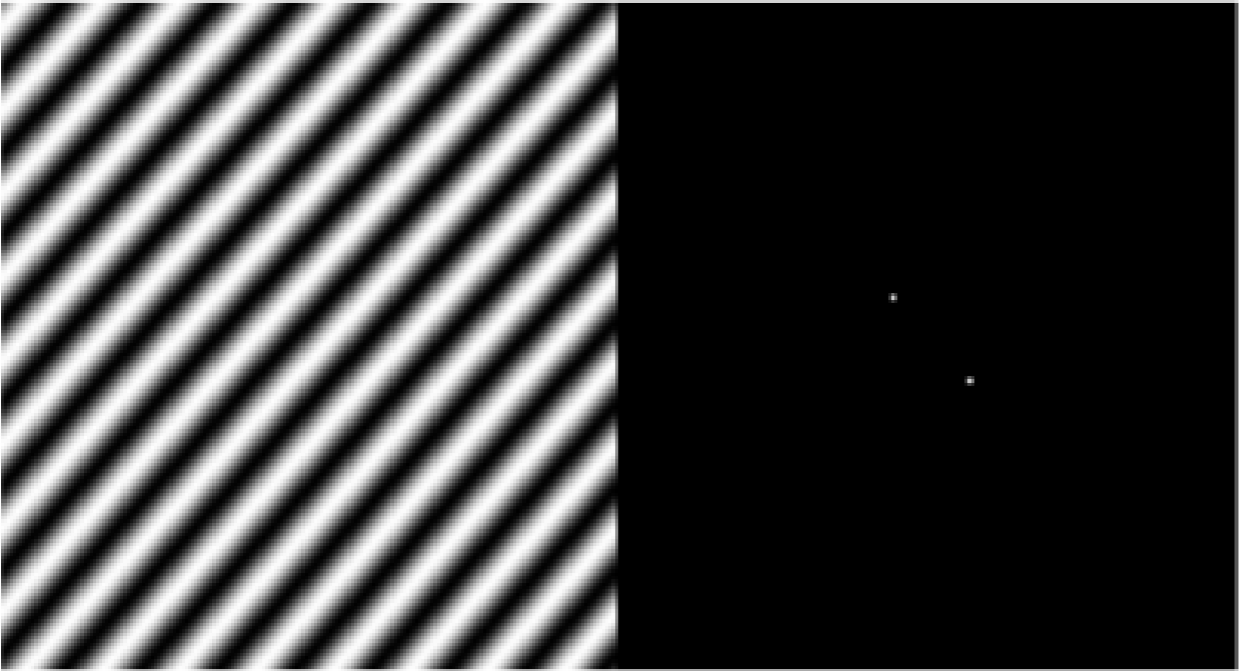
(b)

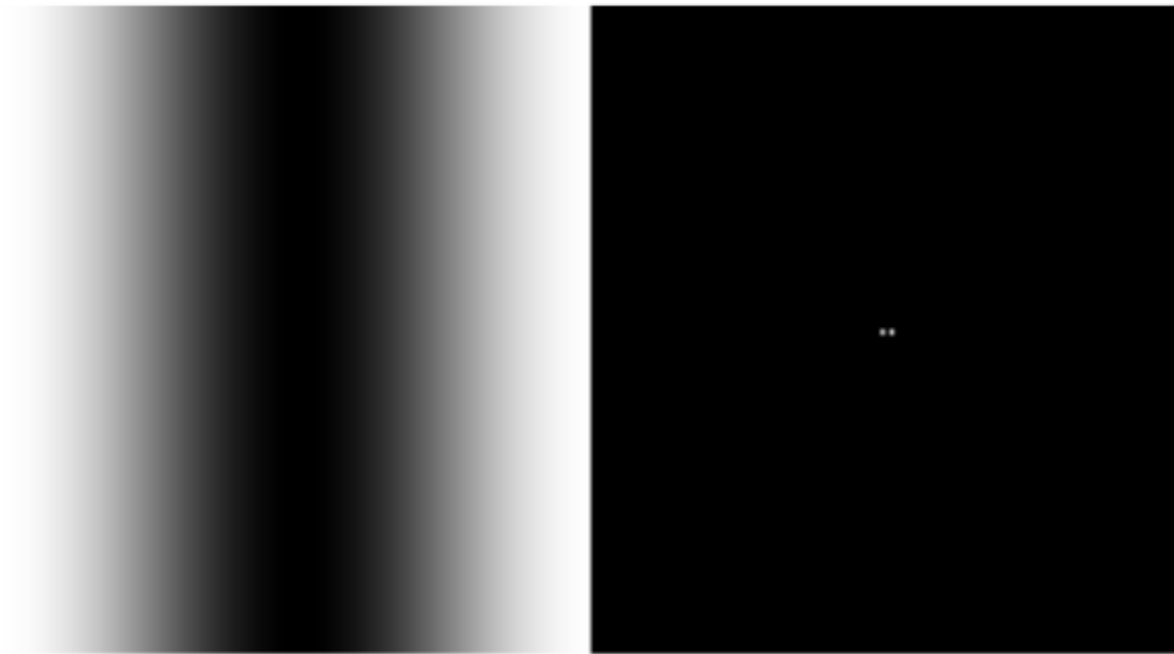


(c)

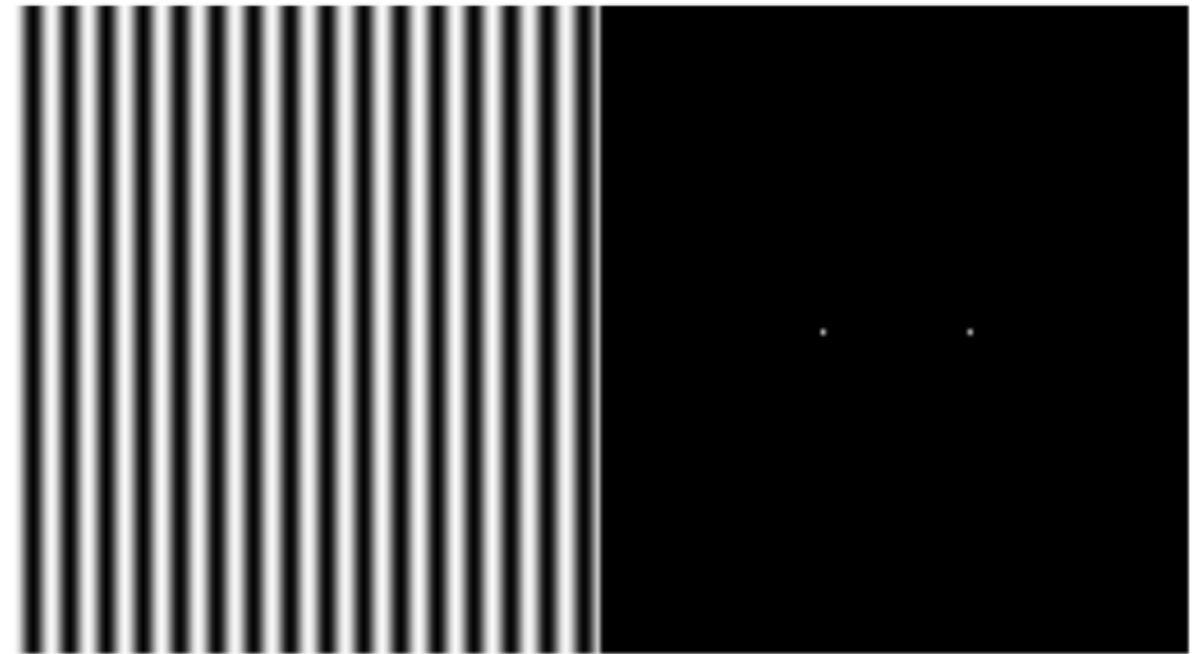


(d)

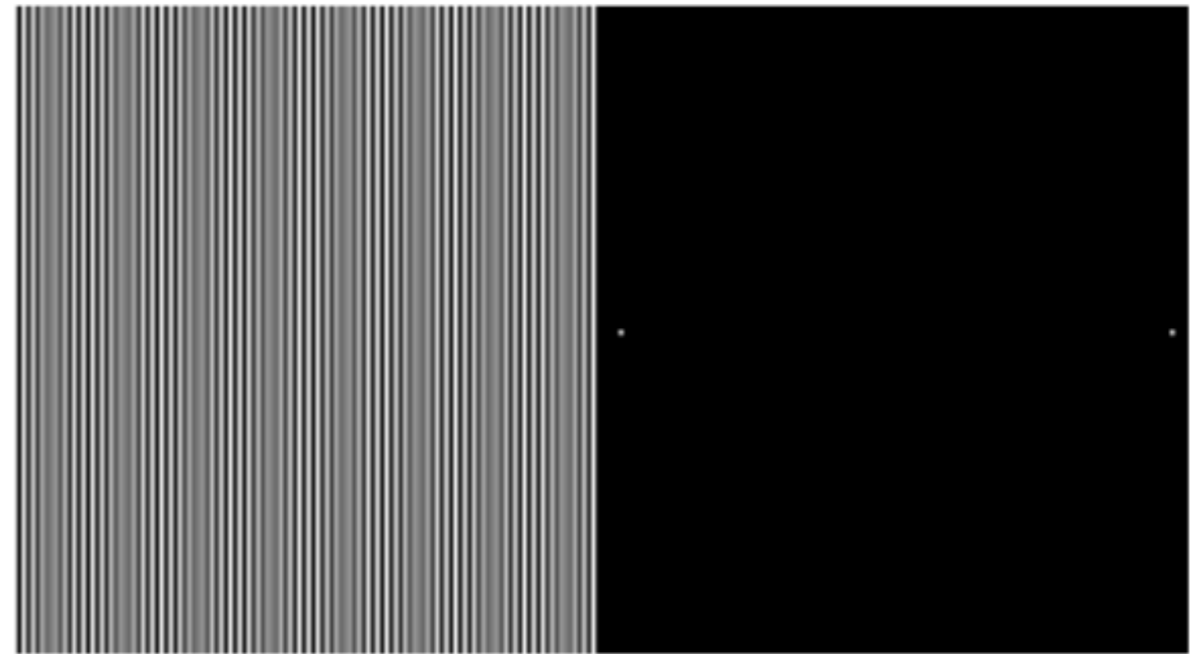
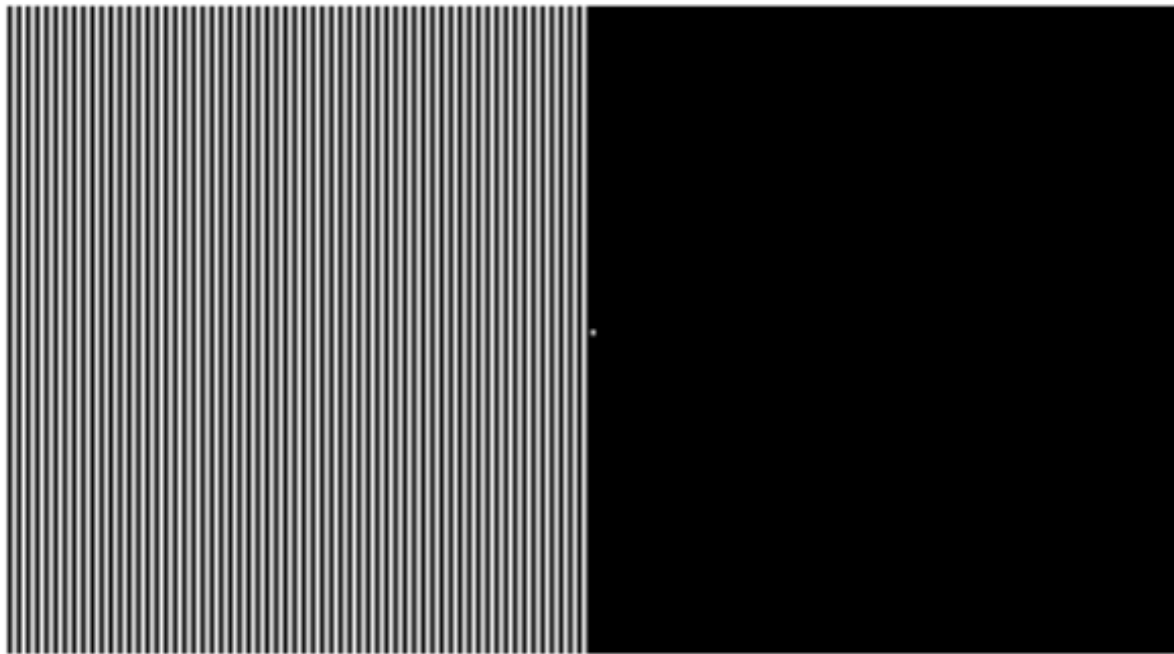


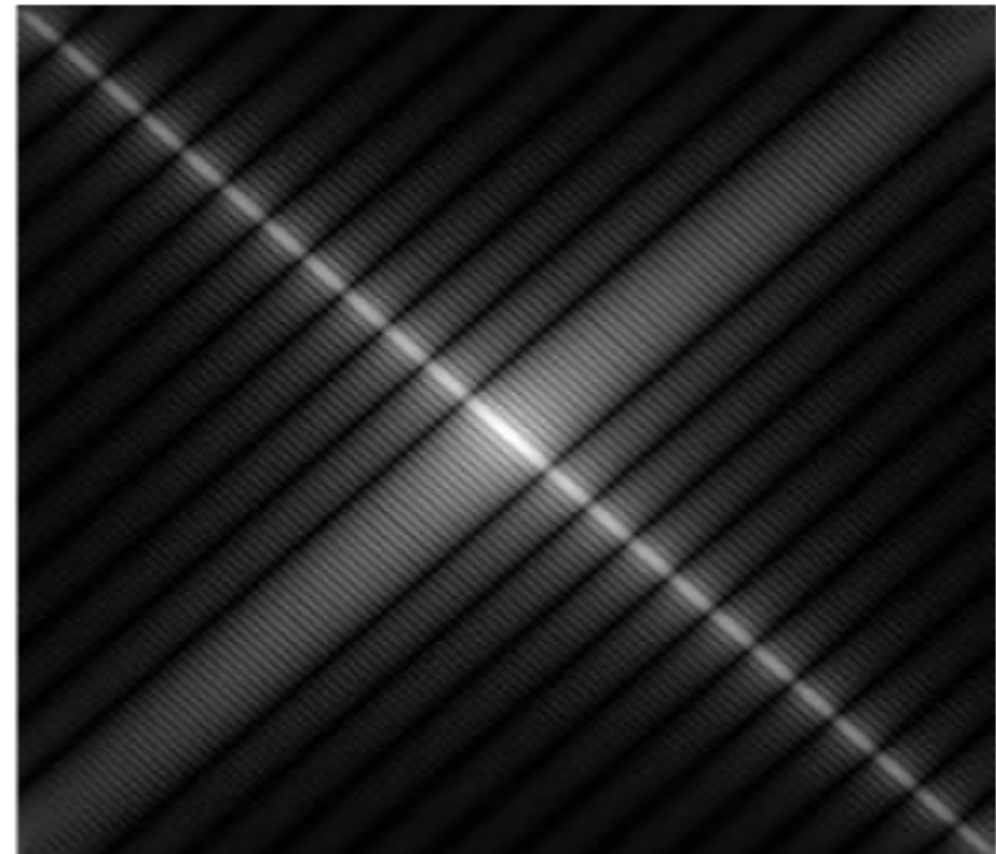
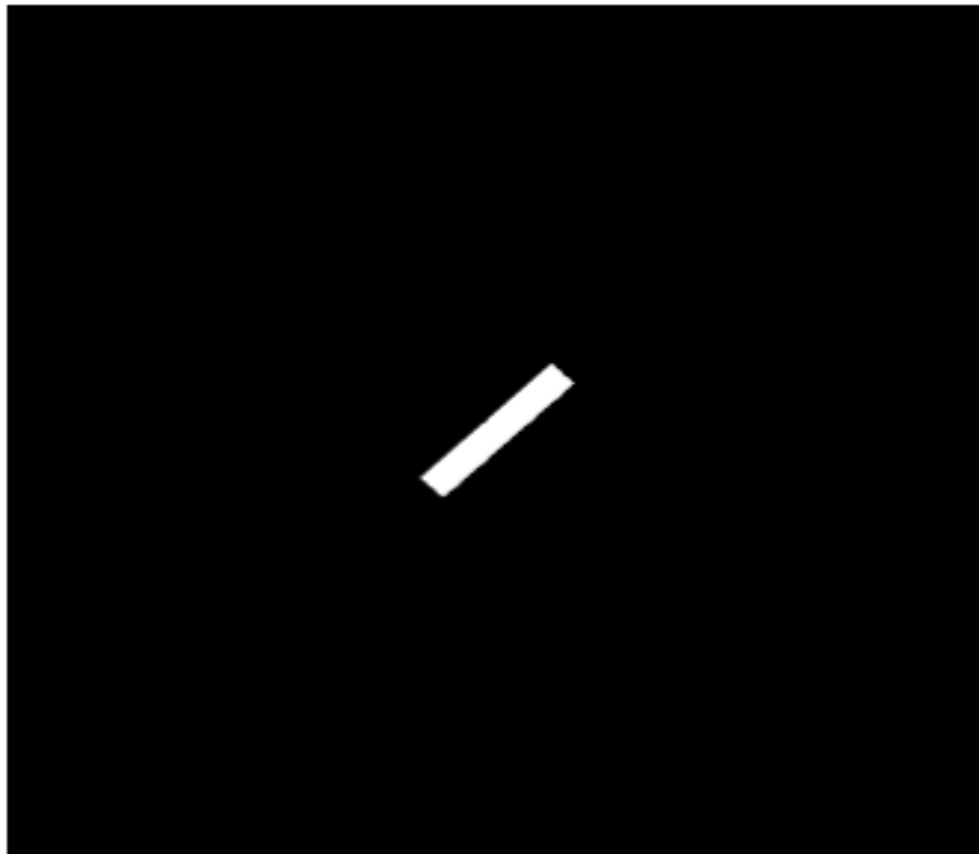
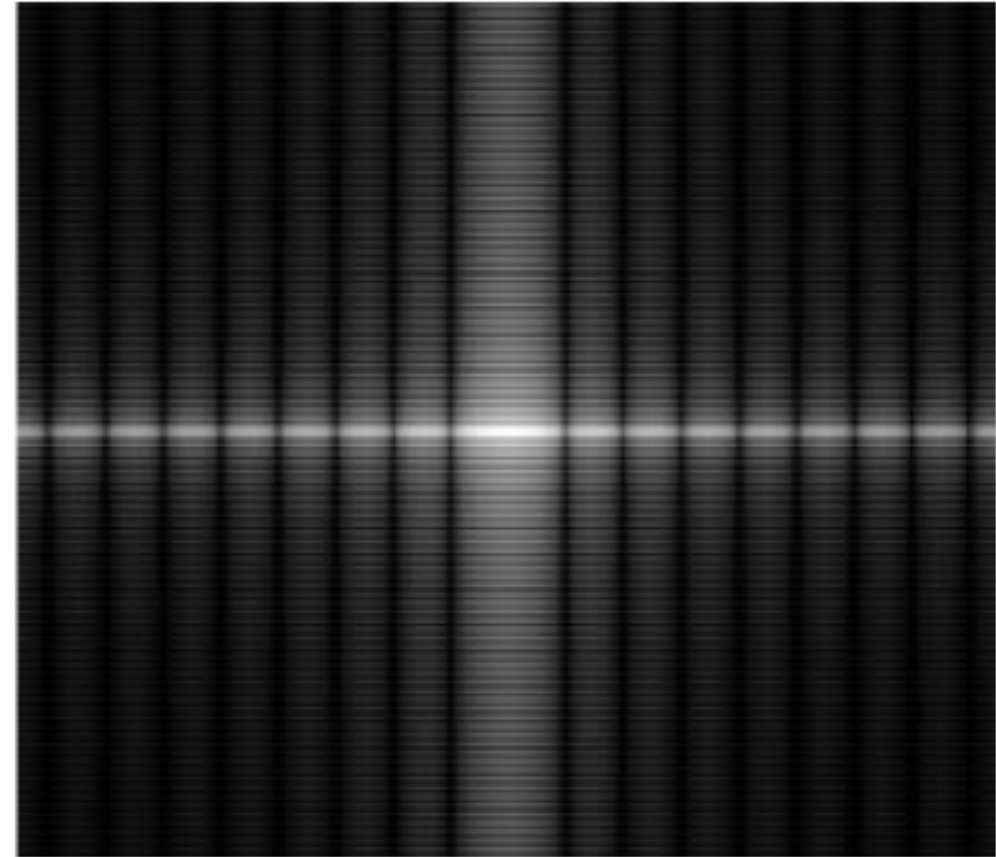
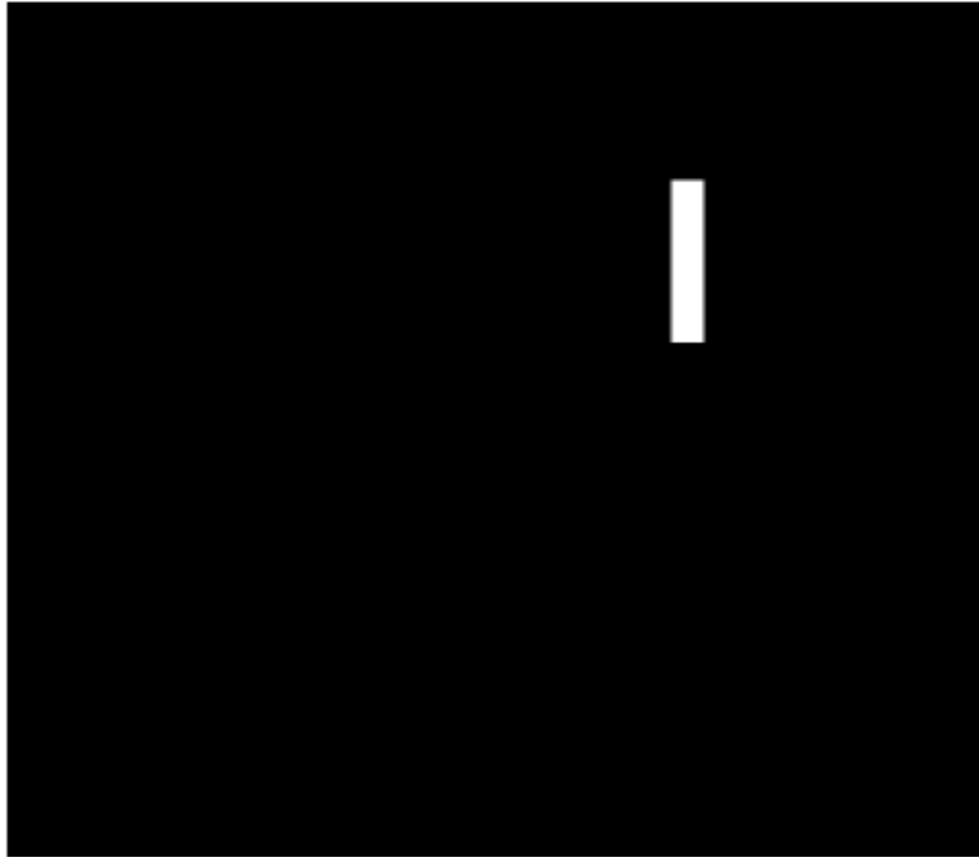


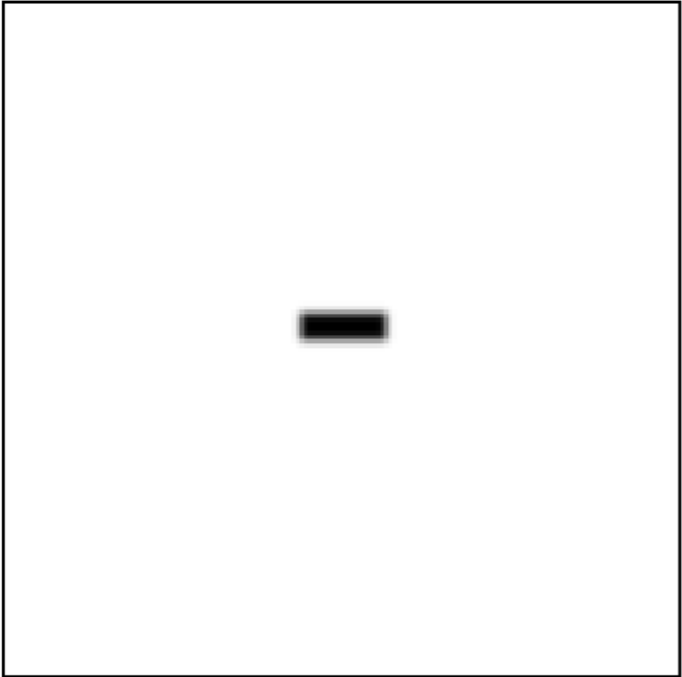
1/128



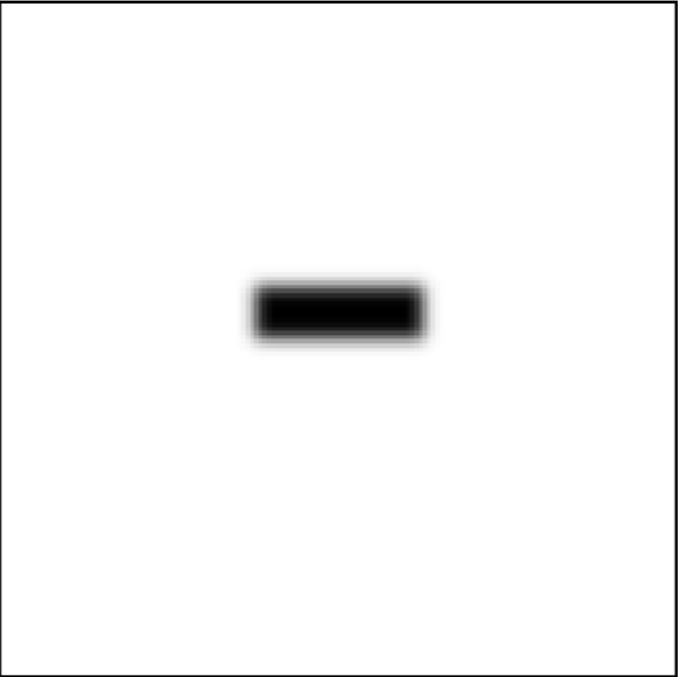
16/128



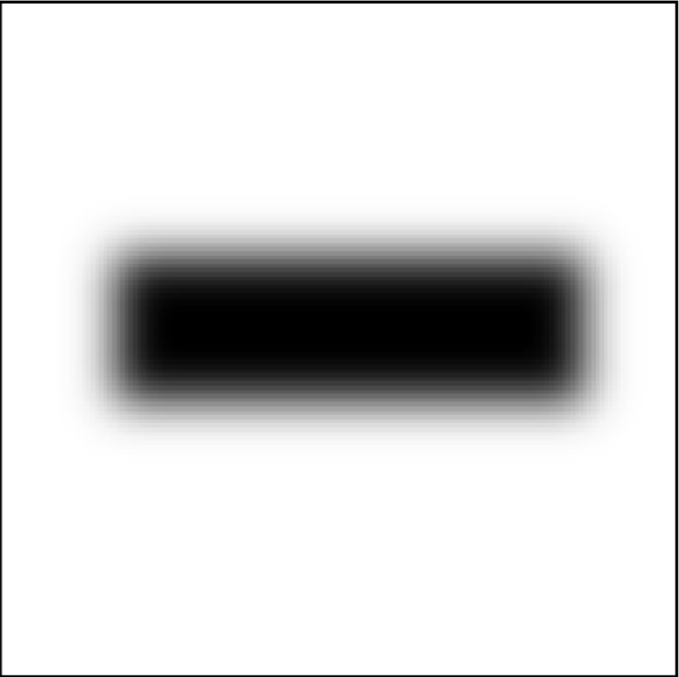




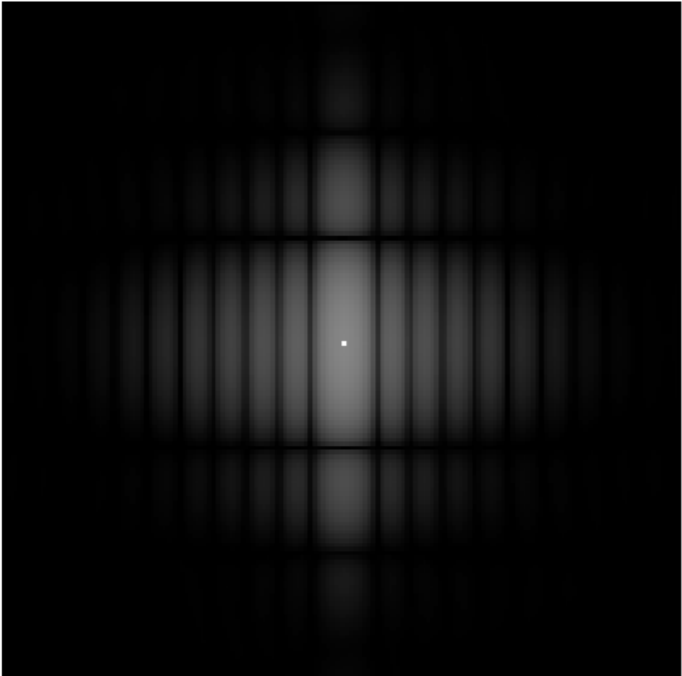
(a)



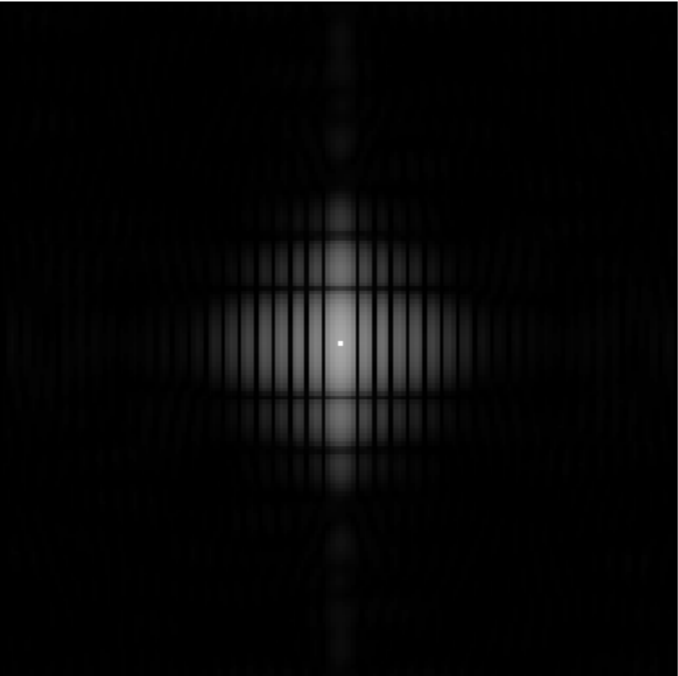
(b)



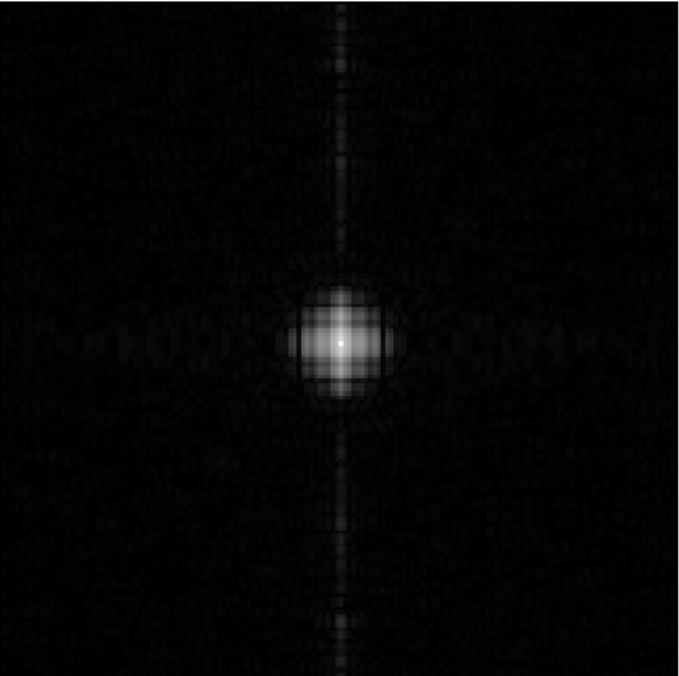
(c)



(d)



(e)



(f)

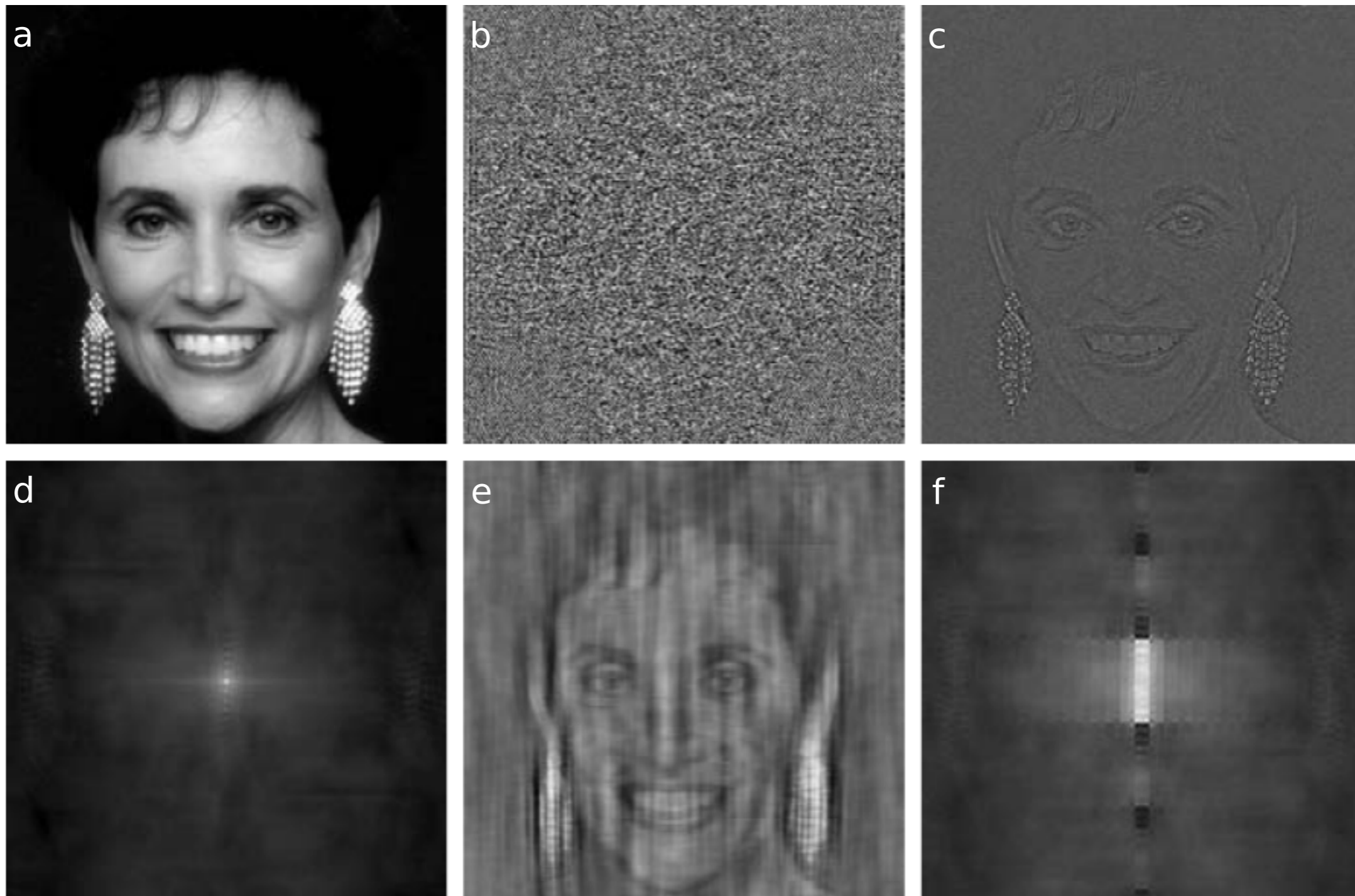
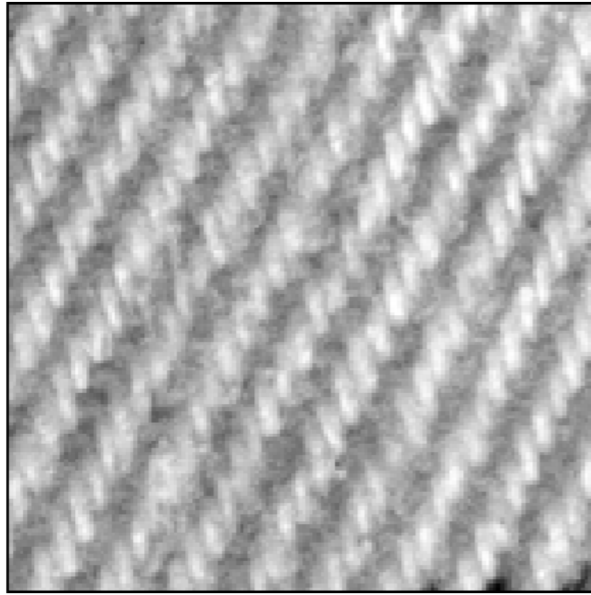
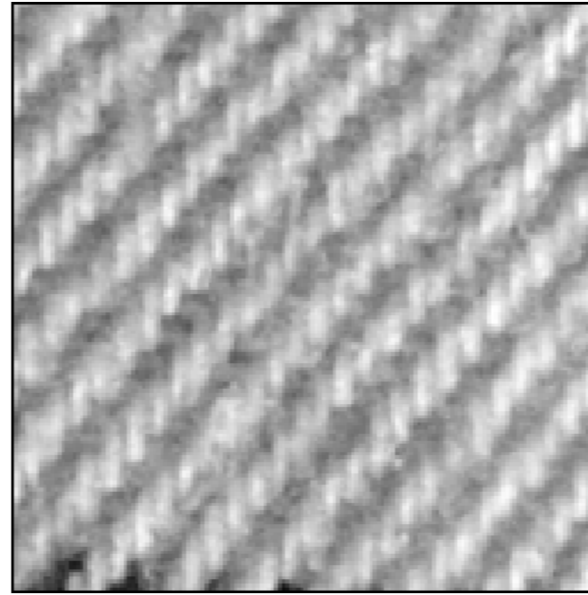


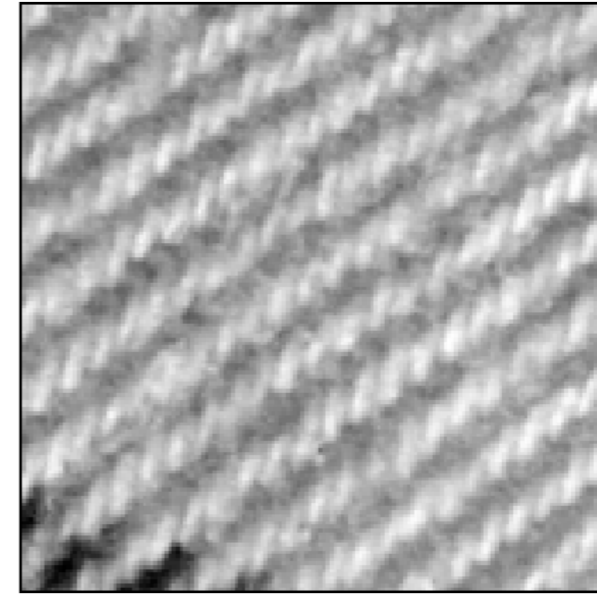
FIGURE 4.27 (a) Woman. (b) Phase angle. (c) Woman reconstructed using only the phase angle. (d) Woman reconstructed using only the spectrum. (e) Reconstruction using the phase angle corresponding to the woman and the spectrum corresponding to the rectangle in Fig. 4.24(a). (f) Reconstruction using the phase of the rectangle and the spectrum of the woman.



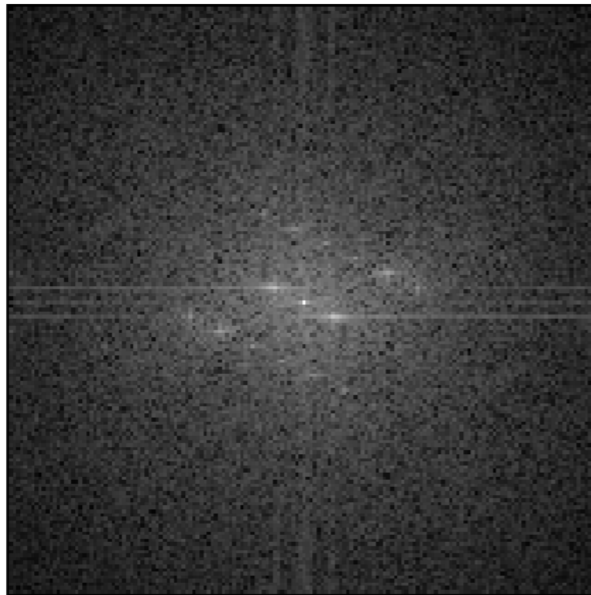
(a)



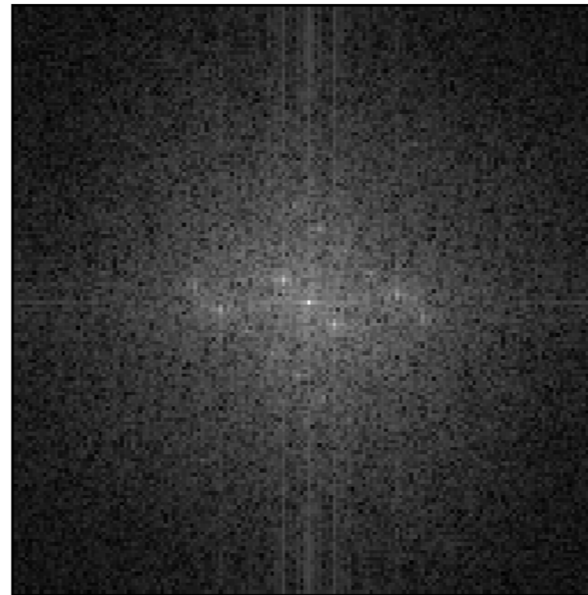
(b)



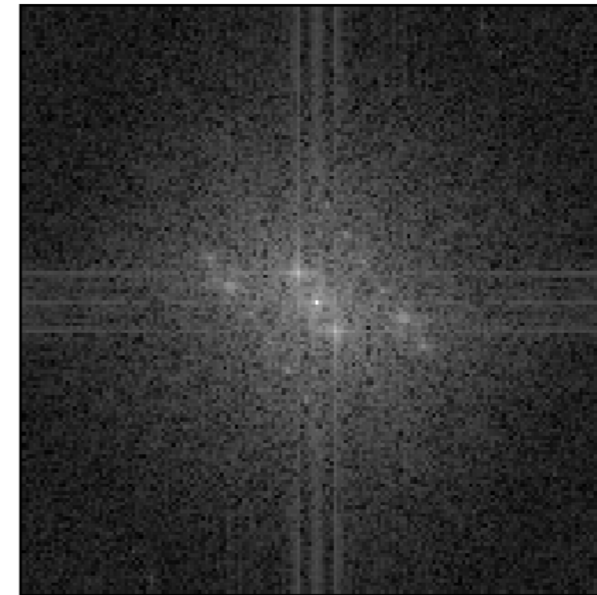
(c)



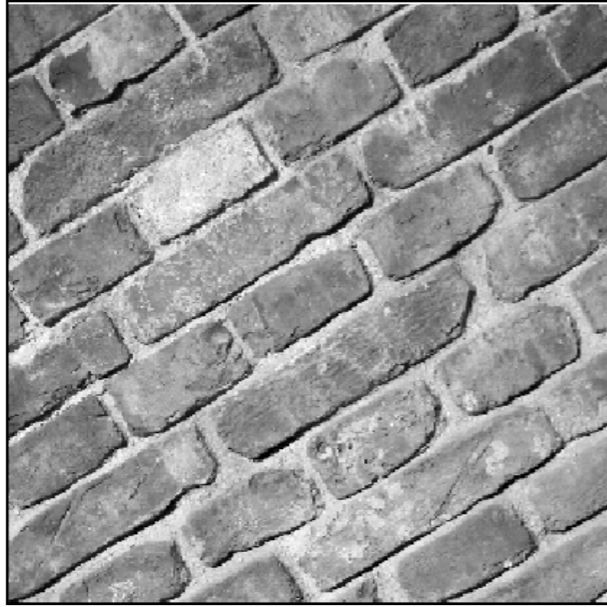
(d)



(e)



(f)



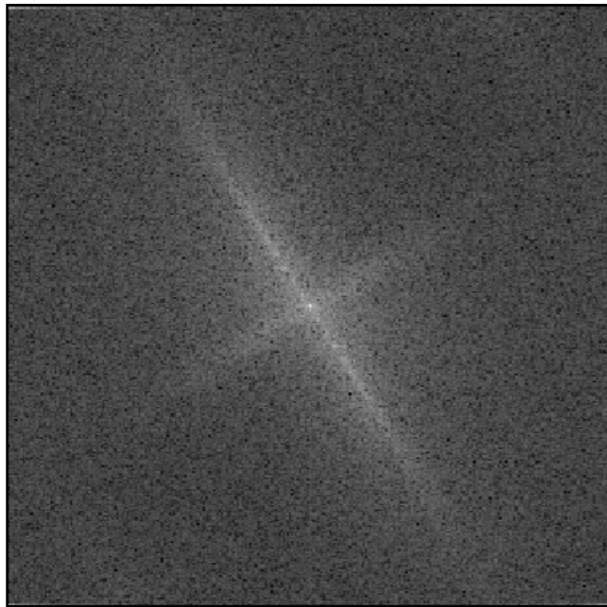
(a)



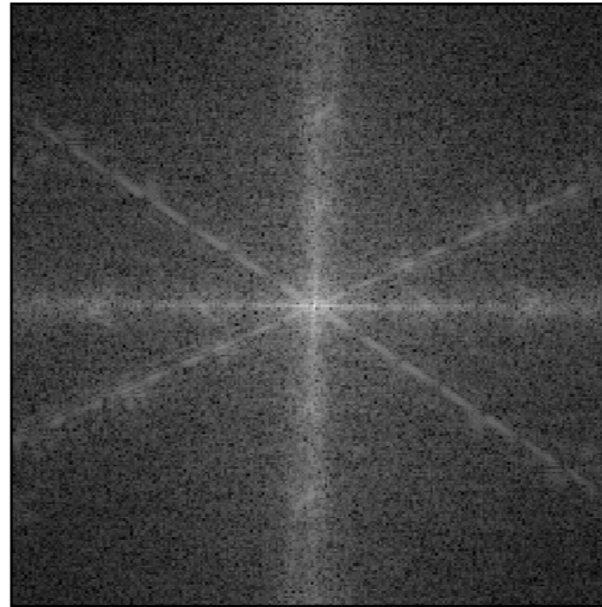
(b)



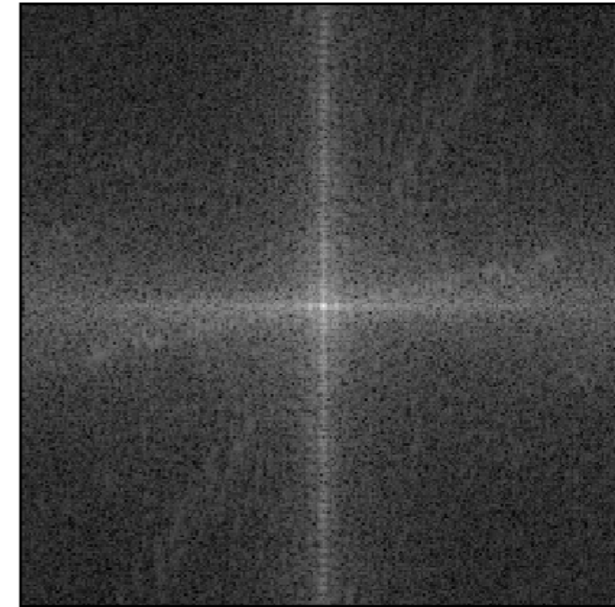
(c)



(d)



(e)



(f)



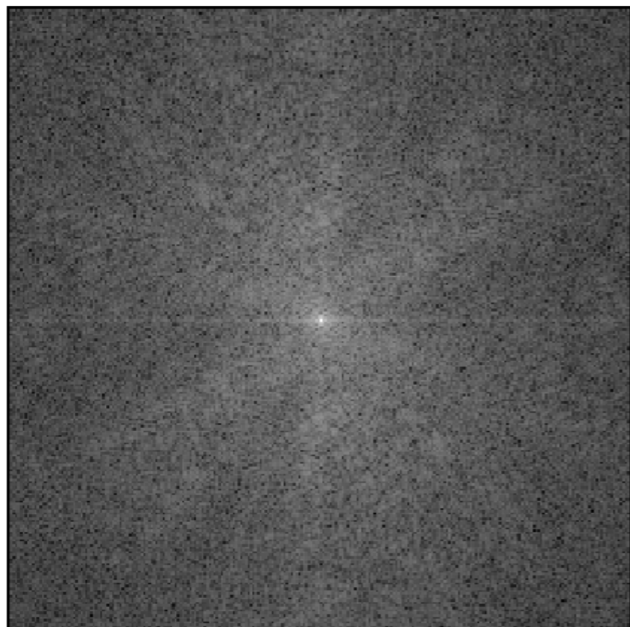
(a)



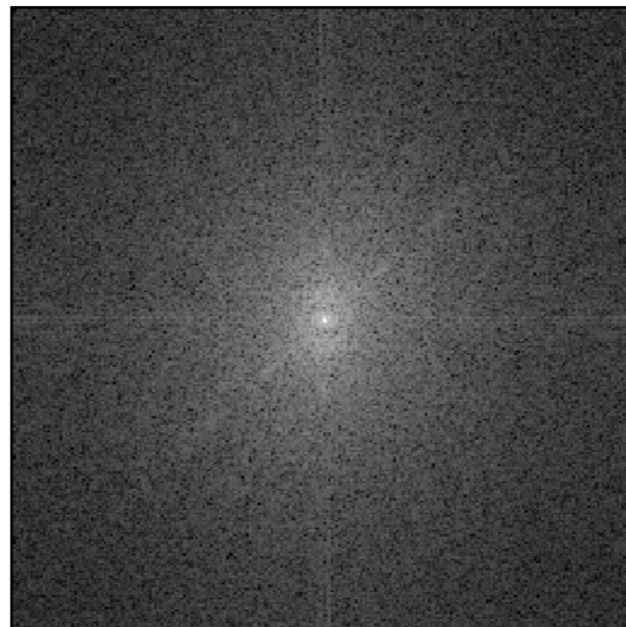
(b)



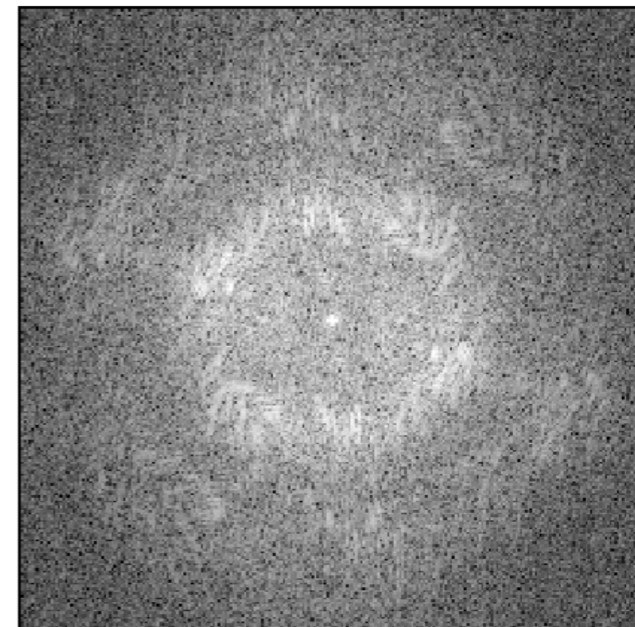
(c)



(d)



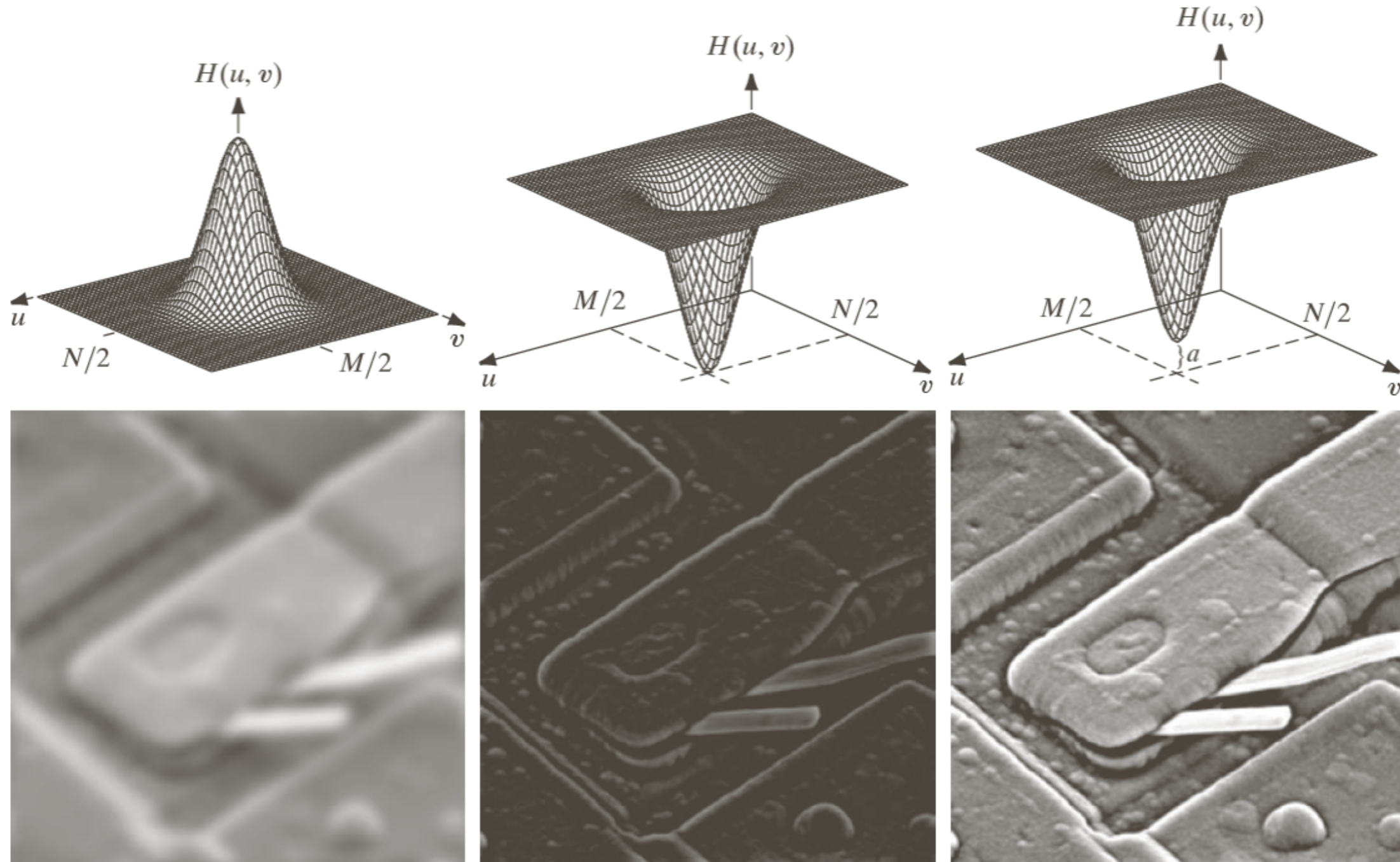
(e)



(f)

- Propiedad importante de la transformada de Fourier
- A una operación de convolución en el espacio le corresponde una multiplicación puntual en frecuencia

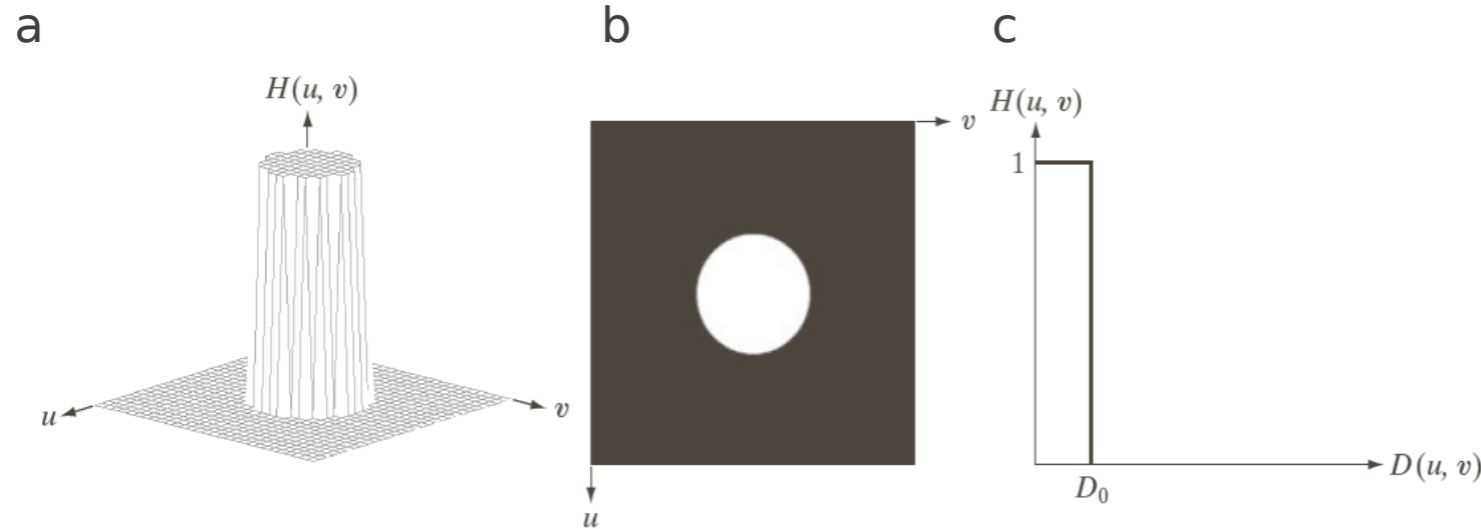
$$\begin{array}{ccccc} \text{Image space: } & g(u, v) & * & h(u, v) & = & g'(u, v) \\ & \downarrow & & \downarrow & & \uparrow \\ & \text{DFT} & & \text{DFT} & & \text{DFT}^{-1} \\ & \downarrow & & \downarrow & & \uparrow \\ \text{Frequency space: } & G(m, n) & \cdot & H(m, n) & \longrightarrow & G'(m, n) \end{array}$$



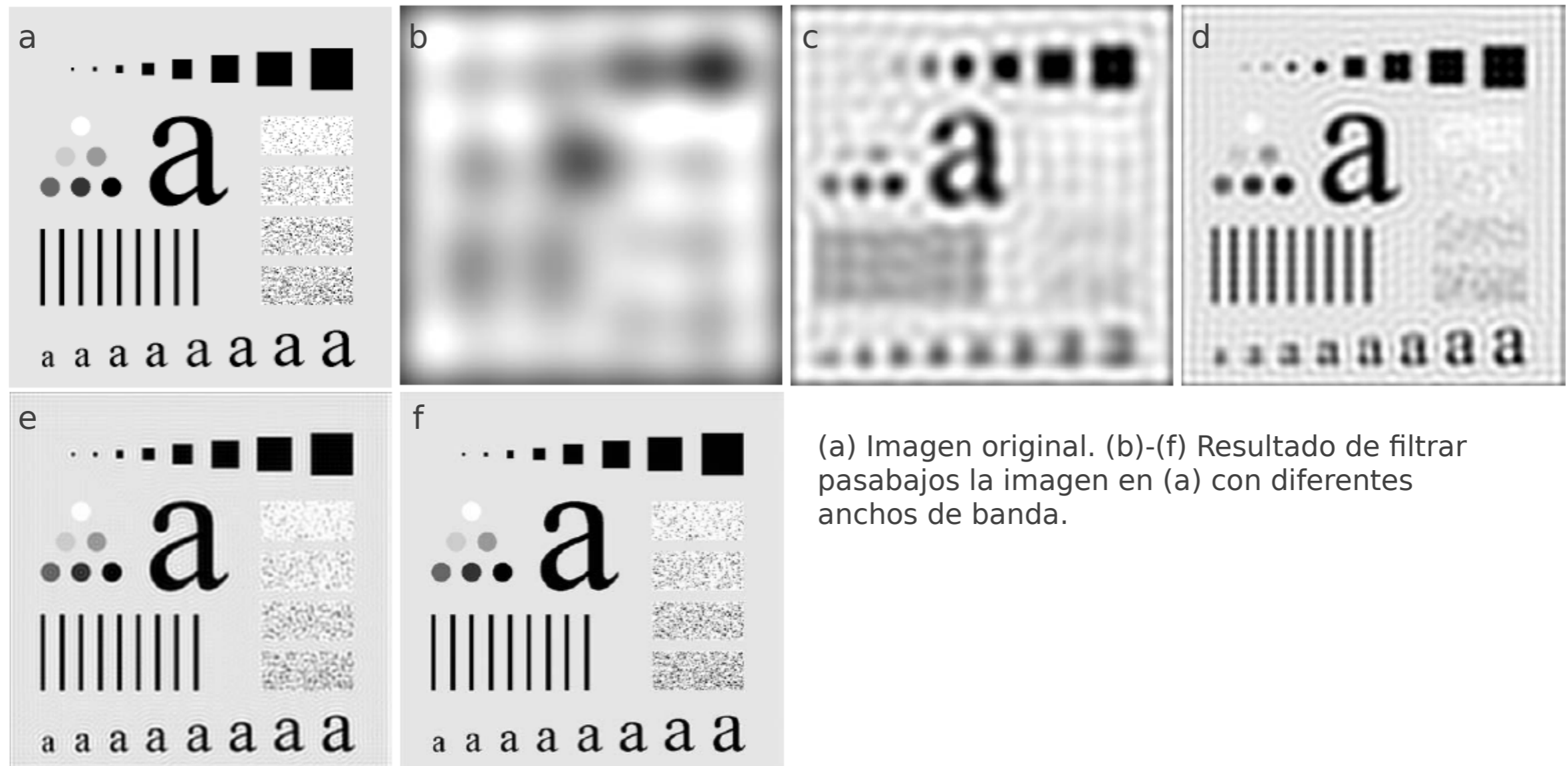
a	b	c
d	e	f

FIGURE 4.31 Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used $a = 0.85$ in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).

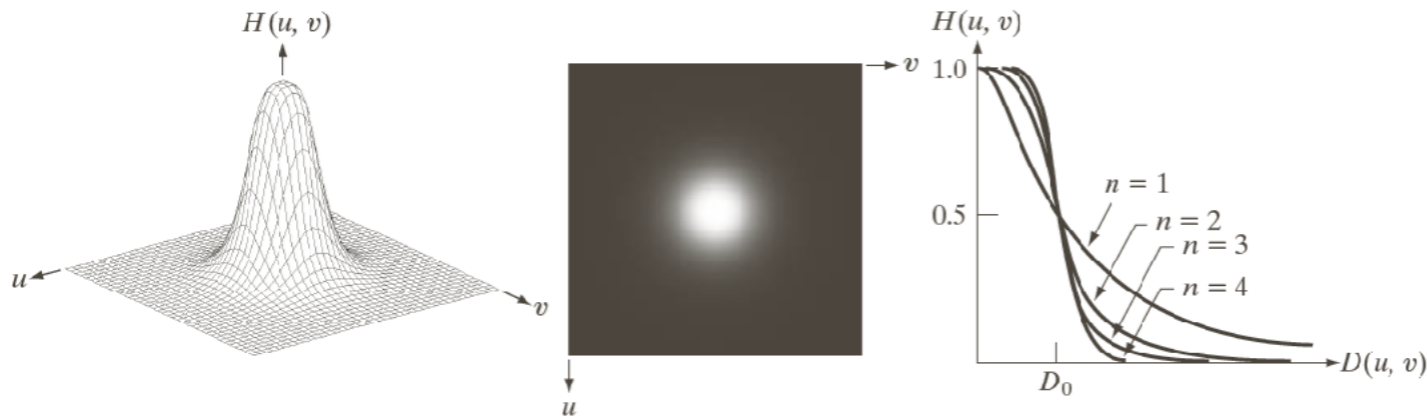
Filtrado pasabajos "ideal"



(a) Perspectiva de la función de transferencia de un filtro "pasabajos" ideal. (b) El filtro mostrado como una imagen. (c) Sección radial del filtro

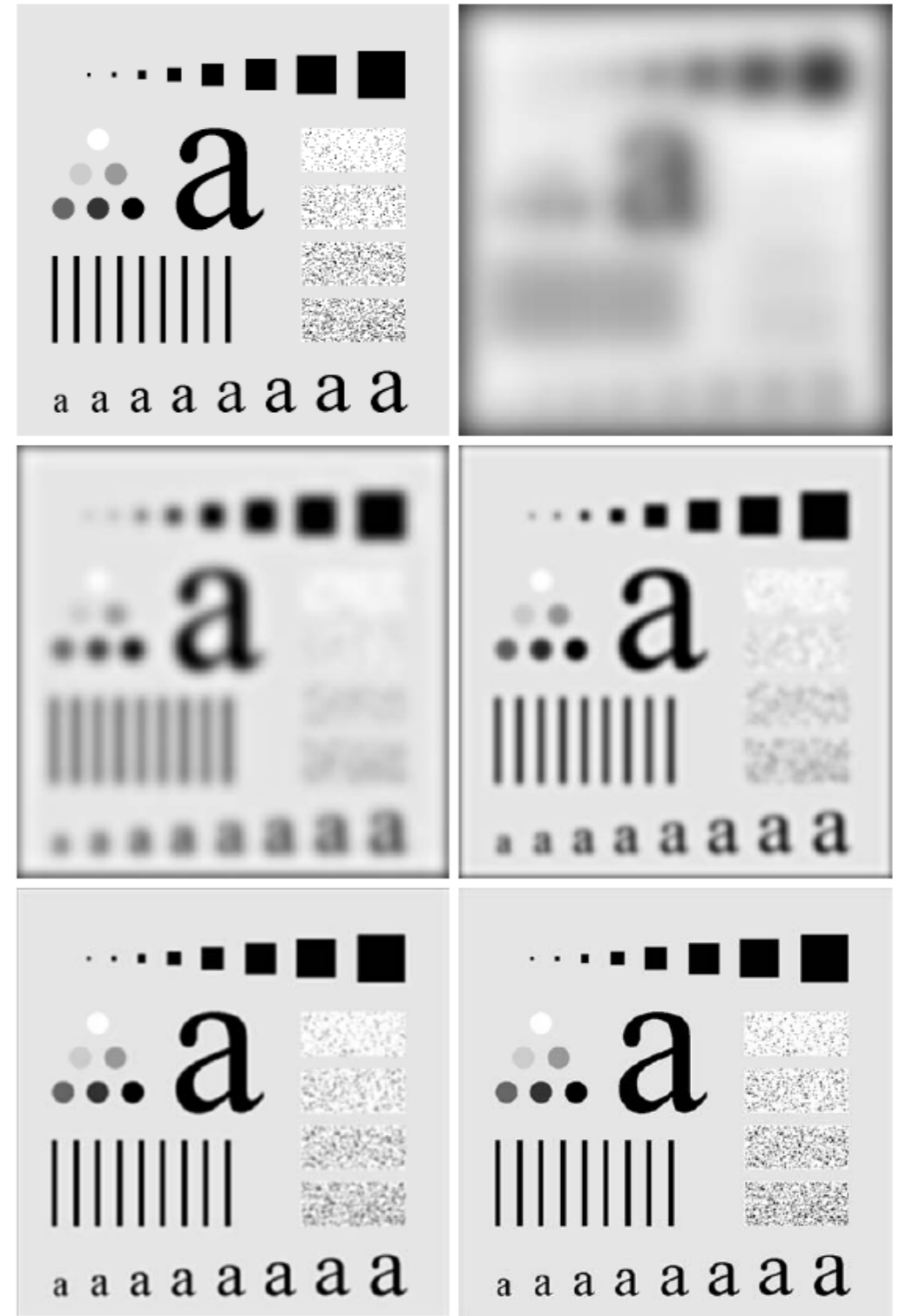


(a) Imagen original. (b)-(f) Resultado de filtrar pasabajos la imagen en (a) con diferentes anchos de banda.



a b c

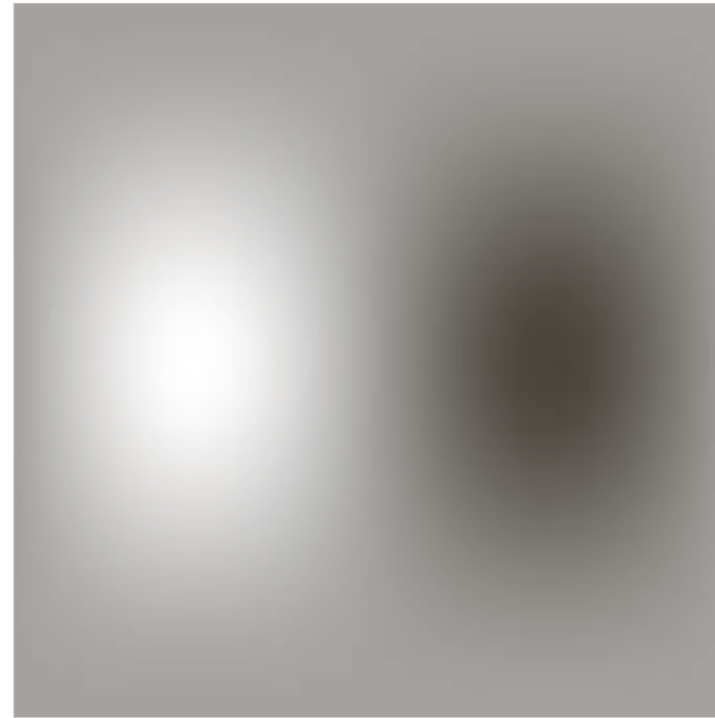
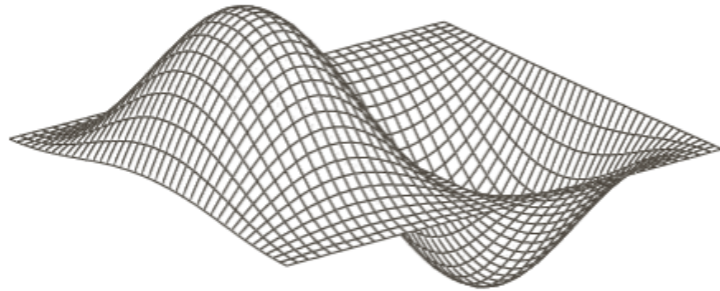
FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.



a b
c d
e f

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

-1	0	1
-2	0	2
-1	0	1



a b
c d

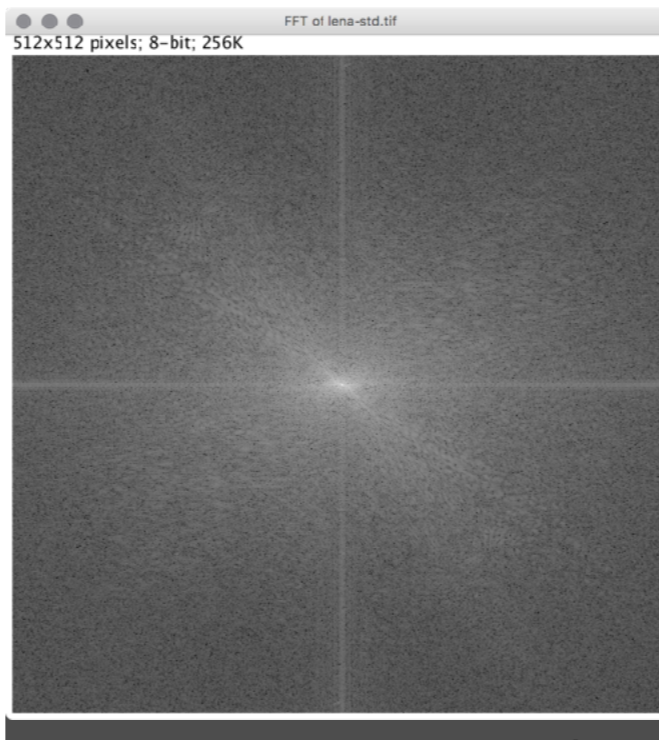
FIGURE 4.39
(a) A spatial mask and perspective plot of its corresponding frequency domain filter. (b) Filter shown as an image. (c) Result of filtering Fig. 4.38(a) in the frequency domain with the filter in (b). (d) Result of filtering the same image with the spatial filter in (a). The results are identical.



File > Open Samples > Lena



Process > FFT > FFT

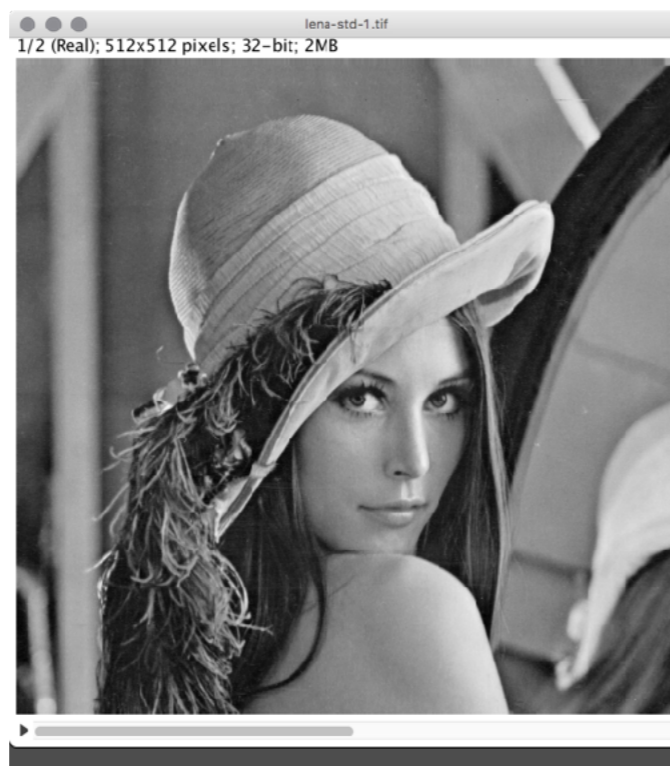


Modulo

Process > FFT > Inverse FFT



Process > FFT > FFT Options... Complex Fourier Transform



Parte Real

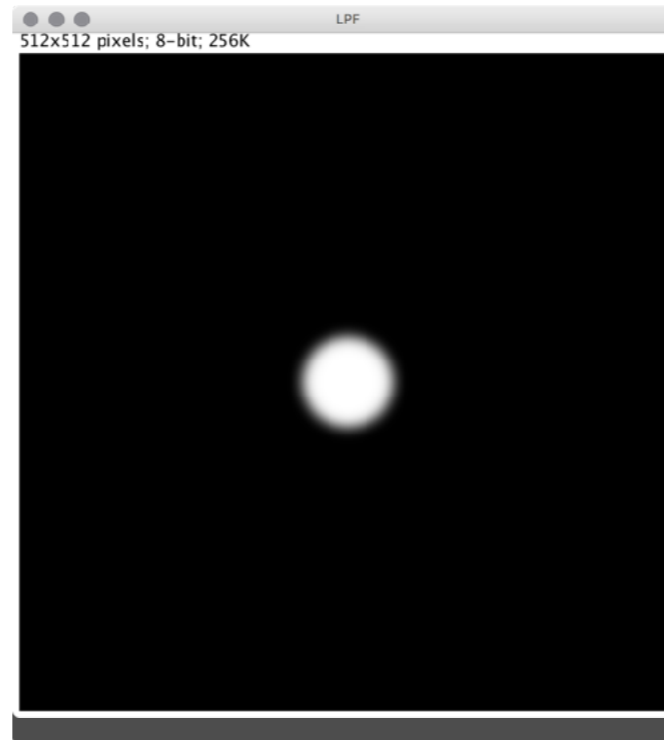


Parte Imaginaria

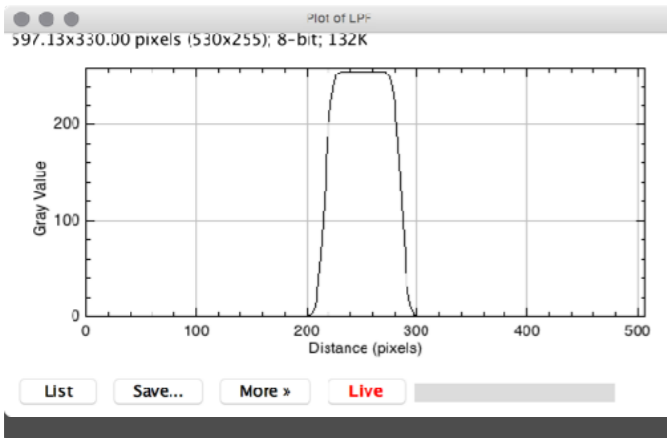
File > Open Samples > Lena



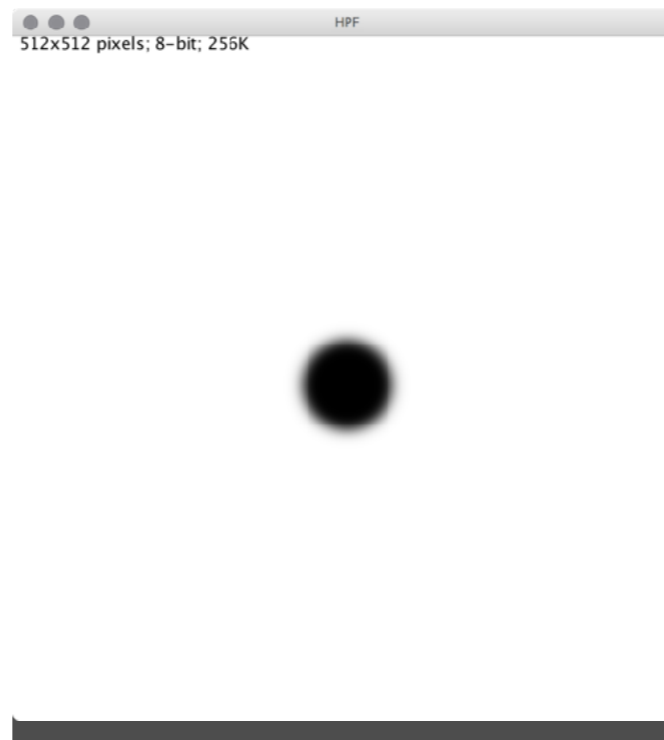
Filtro Pasabajos



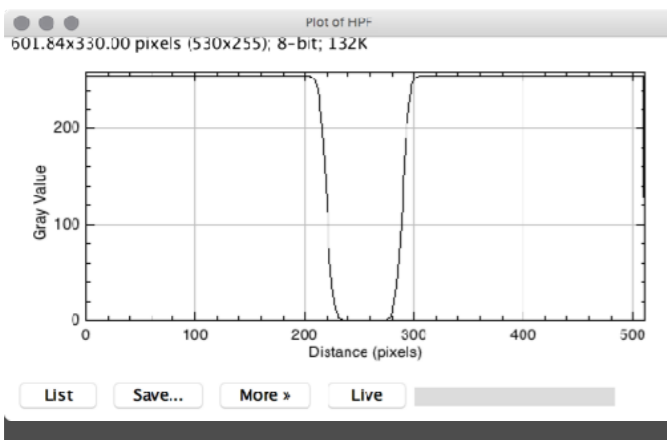
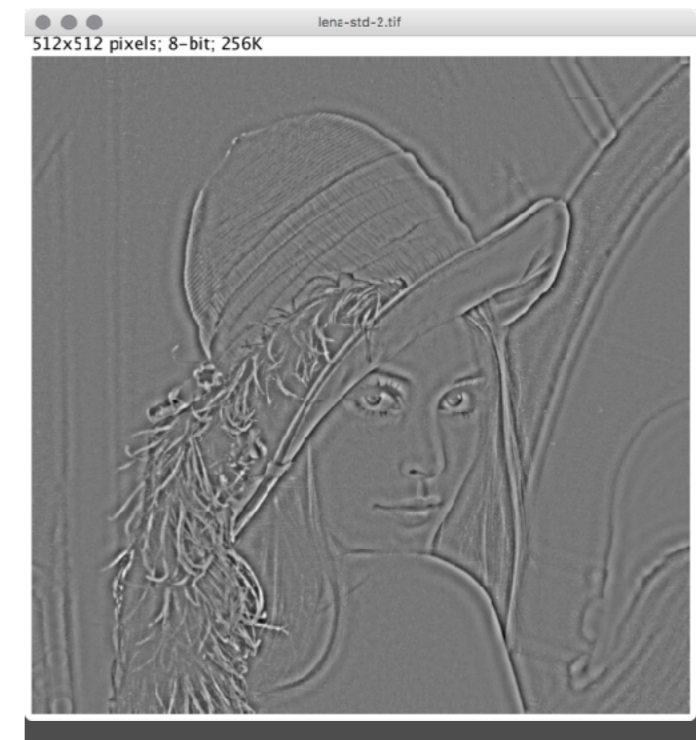
Process > FFT > Custom Filter...



Filtro Pasaaltos



Process > FFT > Custom Filter...



- File > New > Image...
 - LPF / 8-bit / Black / 512 / 512 / 1
- Círculo centrado en (256,256) con radio 80 (apretar Control).
- Edit > Fill
- Process > Filter > Gaussian Blur... (5 píxeles)
- Edit > Invert

