

## CURIOSIDAD:

ELECTRODINÁMICA . TENSOR DE FARADAY  
ORDEN 2, COVARIANTE

$$\vec{E} = E_i(x_1, x_2, x_3, t) \hat{x}_i$$

$$F = F_{\alpha\beta} = \begin{bmatrix} 0 & E_1/c & E_2/c & E_3/c \\ -E_1/c & 0 & -B_3 & B_2 \\ -E_2/c & B_3 & 0 & -B_1 \\ -E_3/c & -B_2 & B_1 & 0 \end{bmatrix}$$

OBJETO DE ESTUDIO DE  
LA ELECTRODINÁMICA

⇒ NO VAMOS A ANALIZAR NADA CON ESTE FORMALISMO FÍSICO.

## REPASAMOS EC. MAXWELL

$$1) \nabla \cdot \vec{E} = -\frac{\partial \rho}{\partial t} - \vec{M}$$

$$2) \nabla \wedge \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$3) \nabla \cdot \vec{D} = \rho$$

$$4) \nabla \cdot \vec{B} = 0$$

5) UNA EC. CONSTITUTIVA DEL MATERIAL

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}(\vec{E})$$

MKS

$$\left. \begin{array}{l} E : V/m \\ H : A/m \end{array} \right\} \text{CAMPOS}$$

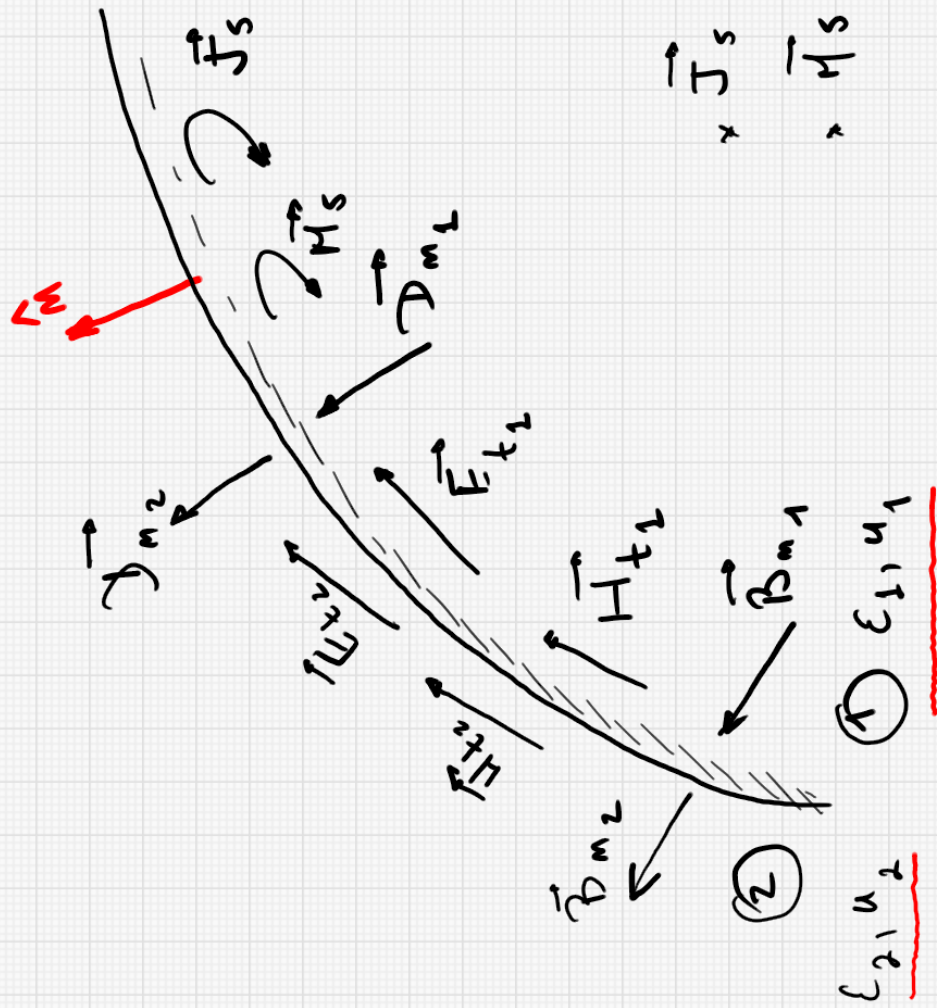
$$\left. \begin{array}{l} D : C/m^2 \\ B : Wb/m^2 \end{array} \right\} \begin{array}{l} \text{FLUJOS} \\ \text{DENSIDAD} \end{array}$$

$$\left. \begin{array}{l} M : V/m^2 \\ J : A/m^2 \\ \rho : C/m^3 \end{array} \right\} \text{SOURCES}$$

OBJETIVOS DE ESTA CLASE: ENTENDER LO SIGUIENTE

- ①  $\{ \epsilon_r, \tan \delta \}$ , PÉRDIDAS,  $Q$
- ②  $S_s \rightarrow e^{-1}$  "SKIN" DEL MATERIAL
- ③ REPASO DE ONDAS PLANAS Y CONDICIONES DE BORDE DE CAMPOS
- ④ ALGUNAS HIPÓTESIS DE TRABAJO PARA EL RESTO DEL CURSO (RÉGIMEN ESTACIONARIO)

CONDICIONES DE CONTORNO :



- 1)  $\hat{n} \cdot (\vec{D}_2 - \vec{D}_1) = \sigma_s$
- 2)  $\hat{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0$
- 3)  $\hat{n} \wedge (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$
- 4)  $(\vec{E}_2 - \vec{E}_1) \wedge \hat{n} = \vec{M}_s$

\*  $\vec{J}_s$  : CORRIENTE SUPERFICIAL

\*  $\vec{M}_s$  : CORRIENTE MAGNETIZACION SUPERFICIAL

VAAMOS A TRABAJAR CON :

- 1) NO HAY DEPENDENCIA ARBITRARIA CON EL TIEMPO
- ⇒ ESTUDIO RÉGIMEN ARMÓNICO
- 2) STEADY-STATE (CONDICIONES ESTABLECIDAS)

PERMITE REPRESENTACIÓN FASORIAL

$$\vec{E}(x_1, x_2, x_3, t) = \text{Re} \{ \vec{E}(x_1, x_2, x_3) e^{j\omega t} \}$$

¿O QUE PASA CON LOS TRANSITORIOS?

# MATERIAL DIELECTRICO

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}(\vec{E})$$

$\rightarrow 8,85 \times 10^{-12} \text{ F/m}$

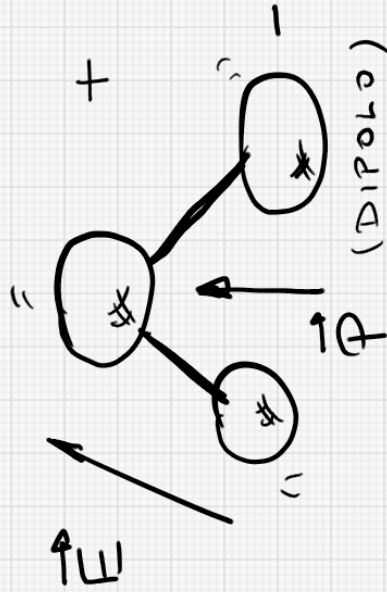
SI MEDIO LINEAL

$$\vec{P} = \epsilon_0 \chi_0 \vec{E}$$

$$\vec{D} = \epsilon \vec{E}$$

COMO ESTAMOS  
CON REPRESENTACION

FASORIAL LO CORRECTO ES USAR  $\epsilon = \epsilon' - j\epsilon''$



(LO GENERAL ES  $\vec{P}_i = \epsilon_0 \chi_{ij} E_j$  CONV. EIND.)

$$\epsilon = \epsilon' - \int \epsilon''$$

↳ TOMA SENTIDO EN EL BALANCE DE POTENCIA

$$P_{ot} = \left\langle \frac{1}{2} \frac{\partial}{\partial t} (\vec{D} \cdot \vec{E}) \right\rangle$$

$$\hookrightarrow \text{Re} \{ \cdot \} + \text{Im} \{ \cdot \}$$



$\epsilon''$  PÉRDIDA, NO SE PUEDE

USAR COMO ACTIVA

(LO DISIPA EL MATERIAL)

## VOLVEMOS A MAXWELL:

OBJETIVO: MAXWELL  $\rightarrow$  PASAR A ALGO QUE SE LEA  
EN HOJAS DE DATOS Y  
FABRICANTES DEN INFORMACIÓN

$$\begin{aligned}\nabla \times \vec{H} &= \vec{\omega D} + \vec{J} \\ &= \vec{\omega \epsilon \vec{E}} + \vec{\sigma \vec{E}} \\ &= \underbrace{(\gamma \omega \epsilon' + \omega \epsilon'' + \sigma)}_{\text{ALGEBRA}} \vec{E}\end{aligned}$$

$\epsilon = \epsilon' - j\epsilon''$   
 $\frac{\partial}{\partial t} \mapsto j\omega$

DIELECTRIC CLAMPING

$$\begin{aligned}&= \gamma \omega \epsilon' \left( 1 + \frac{\omega \epsilon'' + \sigma}{\gamma \omega \epsilon'} \right) \vec{E} \\ &\quad \underbrace{\hspace{10em}}_{-j \tan \delta}\end{aligned}$$



## COMO RESUMEN DE LAS CUENTAS:

$$\epsilon = \epsilon' - j\epsilon''$$

$$\epsilon' = \epsilon_r \epsilon_0$$

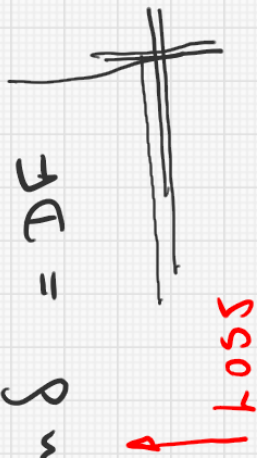
$$\tan \delta = \frac{\sigma + \omega \epsilon''}{\omega \epsilon'}$$

$$\epsilon = \epsilon_0 \epsilon_r (1 - \tan \delta)$$

$\left. \begin{array}{l} \epsilon_r, \tan \delta \\ \text{"} \end{array} \right\}$  APARECE EN  
HOJAS DE DATOS  
DF (DISSIPATION FACTOR)

\* MEDIOS HOMOGENEOS, LA ÚNICA HIPÓTESIS ES LA  
EXISTENCIA DE INTERFACES DEL CONTINUO.

$$\frac{1}{Q} = \tan \delta = DF \quad \text{DISSIPATION FACTOR}$$



DF: MEASURE OF LOSS-RATE OF ENERGY OF AN OSCILLATION IN A DISSIPATIVE SYSTEM

Q: QUALITY FACTOR (DURABILITY OF OSC.)  
(LO VAMOS A VER MAS VINCULADO A  $\epsilon, L, R$ )

## REPASO DE ONDAS PLANAS

MAXWELL FORMA FASORIAL

$$\begin{aligned} 1) \quad \nabla \times \vec{E} &= -\int \omega \mu \vec{H} \\ 2) \quad \nabla \times \vec{H} &= \int \omega \epsilon \vec{E} \\ 3) \quad \vec{B} &= \mu \vec{H} \end{aligned}$$

EC. ONDA

$$\nabla^2 \vec{E} + \omega^2 \mu \epsilon \vec{E} = 0$$

$\Rightarrow$  CUENTO: MARCONI VS. MAXWELL

## RECORDAMOS

$$\epsilon = \epsilon' - j\epsilon''$$

$$\epsilon' = \epsilon_r \epsilon_0$$

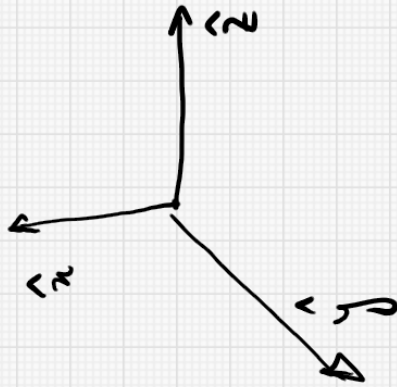
$$\epsilon = \epsilon_0 \epsilon_r (1 - j \tan \delta)$$

$$\tan \delta = \frac{\omega \epsilon'' + \sigma}{\omega \epsilon'}$$

$$k^2 = \omega^2 \epsilon \mu$$

EC ONDA NO DISIPATIVA

$$\nabla^2 E + k^2 E = 0$$



$$\vec{E}_x(z) = E_x(z) \hat{x}$$

$$\Rightarrow \frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0 \Rightarrow$$

$$E_x(z) = E^+ e^{-jkz} + E^- e^{jkz}$$

CONDICIÓN BORDE

$$\dot{E}(z,t) = [E^+ \cos(\omega t - kz) + E^- \cos(\omega t + kz)] \hat{x}$$

RECORDAMOS

$$E = E' - \int E''$$

$$E' = \epsilon_r \epsilon_0 E_0$$

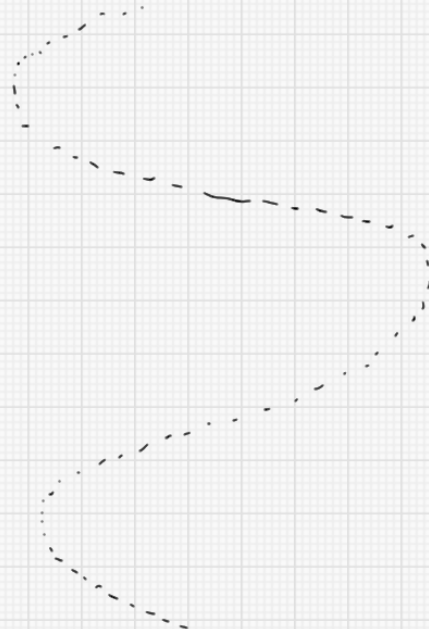
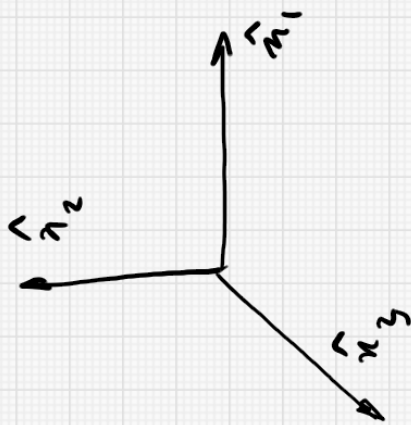
$$E = \epsilon_0 \epsilon_r (1 - \int \tan \delta)$$

$$\tan \delta = \frac{\omega \epsilon'' + \sigma}{\omega \epsilon'}$$

$$\left\{ \begin{aligned} n_p &= \frac{\omega}{k} = \frac{1}{\sqrt{\mu \epsilon}} \\ \lambda &= \frac{2\pi}{k} \end{aligned} \right.$$



$$\vec{J} = \sigma \vec{E}$$



$$\vec{E} = \text{Re} \left\{ F_0 e^{j k x_1} e^{j \omega t} \hat{x}_3 \right\} = E_0 \cos(k x_1 + \omega t) \hat{x}_3$$

## MAXWELL EN CONDUCTORES

$$\begin{cases} \nabla \cdot \vec{E} = -\rho \\ \nabla \times \vec{H} = \rho \omega \epsilon \vec{E} + \sigma \vec{E} \end{cases}$$

→ NOS QUEDA  $\nabla^2 E + \omega^2 \gamma \epsilon \left(1 - \frac{\sigma}{\omega \epsilon}\right) E = 0$

$-\gamma^2$

$$\gamma = \alpha + j\beta = \rho \omega \sqrt{\mu \epsilon} \sqrt{1 - \frac{\sigma}{\omega \epsilon}} \quad \vec{E} = ?$$



$$\nabla^2 E - \gamma^2 E = 0$$

con

$$\gamma = \alpha + j\beta = j\omega \sqrt{\mu\epsilon} \sqrt{1 - j\frac{\sigma}{\omega\epsilon}}$$

(CUALQUIER EJE COMPATIBLE)

$$\Rightarrow \vec{E} = (E^+ e^{-\gamma z} + E^- e^{\gamma z}) \hat{x}$$

$$\text{PERO } e^{-\gamma z} = e^{-\alpha z} \cdot e^{-j\beta z}$$

!! ATENUACIÓN

$$\Rightarrow \vec{E} = [E^+ e^{\alpha z} \cos(\omega t - \beta z) + E^- e^{-\alpha z} \cos(\omega t + \beta z)] \hat{x}$$

$\rightarrow \infty \Rightarrow E^+ = 0$

$$\Rightarrow e^{-\alpha z} \Big|_{z=\delta_s} = e^{-1} \quad \uparrow \text{SKIN}$$

$$\gamma = \alpha + j\beta = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}$$

$\sigma \gg \omega\epsilon$  (BUEN CONDUCTOR)

$$\delta_s = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

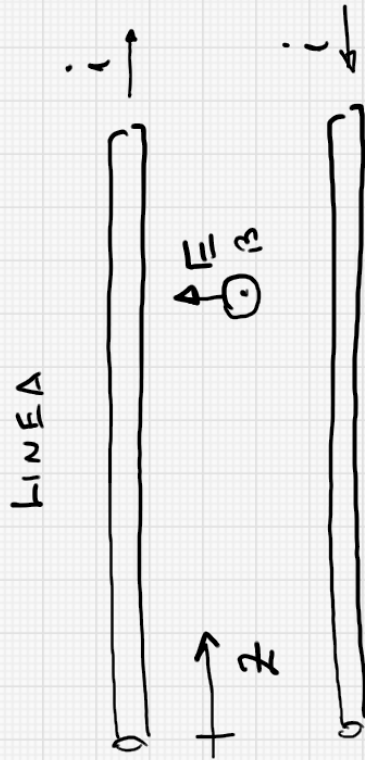
$\delta_s \downarrow, \omega \uparrow$

$\delta_s \downarrow, \sigma \uparrow$

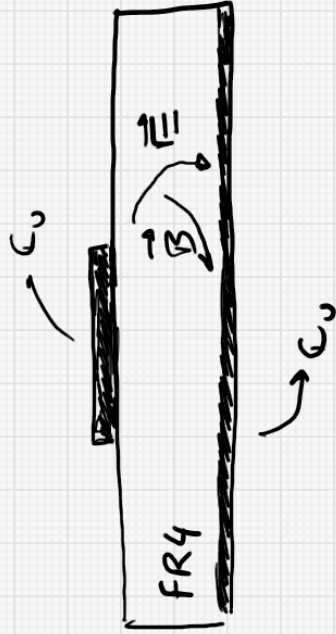
DATOS	
Al	$\delta_s = 0,84 \text{ mm}$
Cu	$\delta_s = 0,664 \text{ mm}$
Au	$\delta_s = 0,84 \text{ mm}$
	@ 500 MHz



# LÍNEAS DE TRANSMISIÓN



LÍNEA MICROSTRIP

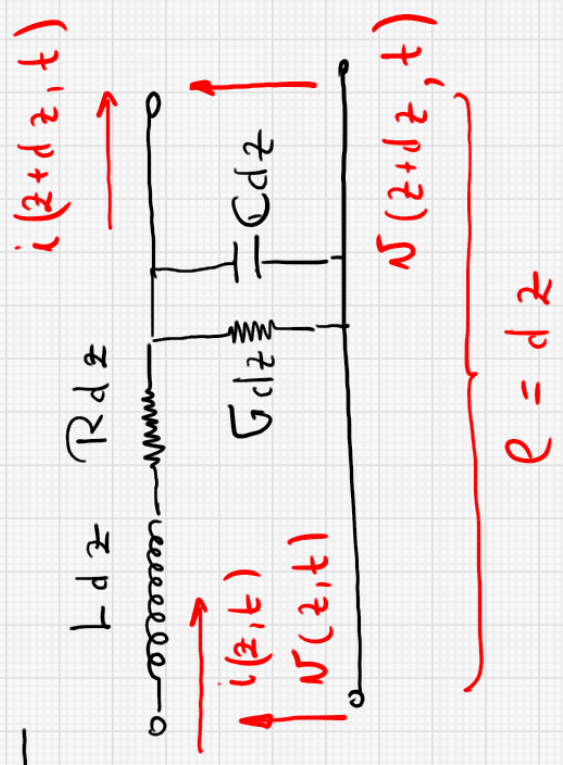
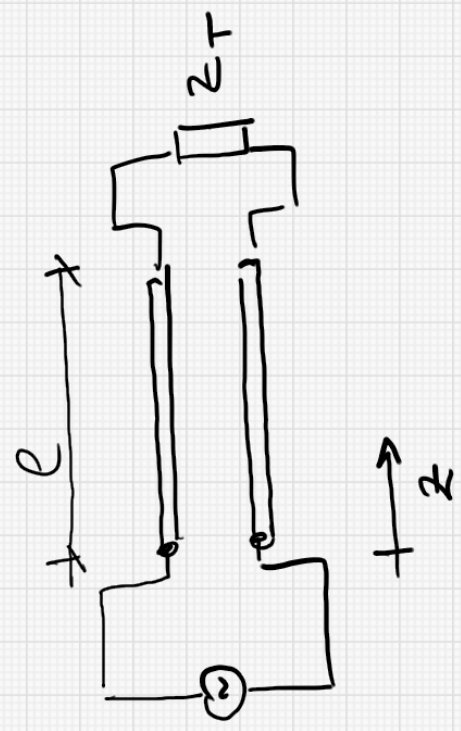


⇒ SE CARACTERIZAN POR

1) PARÁMETROS DISTRIBUIDOS

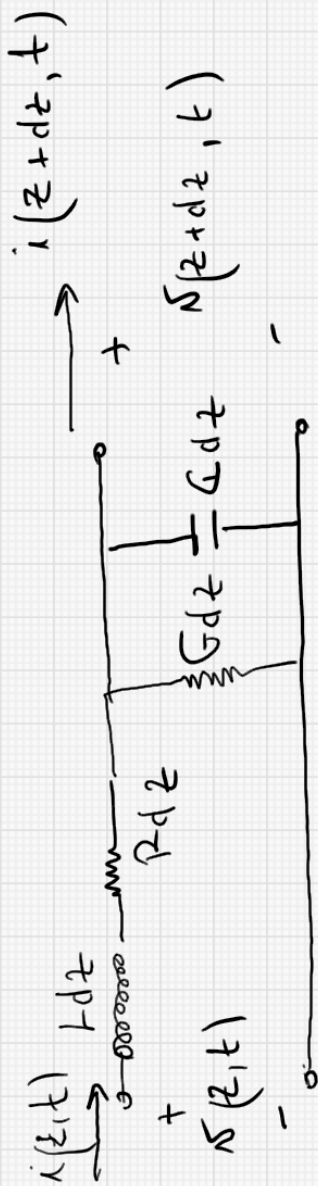
2) MODOS DE PROPAGACIÓN TEM

# MODELO DE LINEA DE "DOS HILOS"



MODELO

- R [ $\Omega/m$ ] PERDIDA EN LA LINEA
  - G [ $F/m$ ] ENERGIA DEL CAMPO ELÉCTRICO
  - L [ $H/m$ ] " " MAGNÉTICO
  - C [ $V/m$ ] PERDIDA EN EL MEDIO POR PROPAGACIÓN DEL CAMPO
- ↳  $\omega f$



$$\Rightarrow \text{SE OBTIENE : } \left\{ \begin{array}{l} -\frac{\partial v}{\partial z} = R i + L \frac{\partial i}{\partial t} \\ -\frac{\partial i}{\partial z} = G v + C \frac{\partial v}{\partial t} \end{array} \right.$$

$\Rightarrow$  COMO  $v(z, t) = \operatorname{Re} \{ v(z) e^{j\omega t} \}$   
PASAMOS A FASORES

$$\Rightarrow \left. \begin{array}{l} \frac{dV(z)}{dz} = -(R + j\omega L) I(z) \\ \frac{dI(z)}{dz} = -(G + j\omega C) V(z) \end{array} \right\}$$

$$\frac{dV(z)}{dz} = -(R + f\omega L)I(z)$$

$$\frac{dI(z)}{dz} = -(G + C f\omega) V(z)$$

DESACOPLO

$$\left. \begin{aligned} & \frac{d^2 V}{dz^2} - \underbrace{-(R + f\omega L)(G + C f\omega)}_{\gamma^2} V = 0 \\ & \frac{d^2 I}{dz^2} - (R + f\omega L)(G + C f\omega) I = 0 \end{aligned} \right\}$$

SOLUCIONES:

$$\left. \begin{aligned} V(z) &= V_1 e^{-\gamma z} + V_2 e^{\gamma z} \\ I(z) &= I_1 e^{-\gamma z} + I_2 e^{\gamma z} \end{aligned} \right\}$$

$$s^2 = (R + j\omega L)(G + j\omega C)$$

$$s = \alpha + j\beta$$

$$\begin{cases} v(z) = v_1 e^{-\gamma z} + v_2 e^{\gamma z} \\ I(z) = I_1 e^{-\gamma z} + I_2 e^{\gamma z} \end{cases}$$

$$\begin{cases} v(z,t) = v_1 e^{-\alpha z} \operatorname{Re}\{e^{j(\omega t - \beta z)}\} + v_2 e^{\alpha z} \operatorname{Re}\{e^{j(\omega t + \beta z)}\} \\ i(z,t) = I_1 e^{-\alpha z} \operatorname{Re}\{e^{j(\omega t - \beta z)}\} + I_2 e^{\alpha z} \operatorname{Re}\{e^{j(\omega t + \beta z)}\} \end{cases}$$

$$\alpha_p = \frac{\omega}{\beta}, \quad \gamma = \frac{2\pi}{\beta}$$

$$[\alpha] = \text{dB/m}, \quad [\beta] = \text{rad/m}$$

⇒ OBJETIVO: MANIPULAR LAS

ECUACIONES PARA TENER  
POCOS PARAMETROS Y USARLAS  
DE MANERA "INGENIERIL"

DE LA EQ. TRANSMISIÓN:  $\frac{dV(z)}{dz} = -(R + j\omega L) I$

⇒ SI SOLO ASUMIMOS ONDA PROGRESIVA TENEMOS QUE:

$$- \gamma V_1 = -(R + j\omega L) I_1$$

⇒ SE PUEDE DEDUCIR  $\frac{V_1}{I_1} = -\frac{V_2}{I_2} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = Z_0$

$$\left. \begin{aligned} V(z) &= V_1 e^{-\gamma z} + V_2 e^{\gamma z} \\ I(z) &= \frac{V_1}{Z_0} e^{-\gamma z} - \frac{V_2}{Z_0} e^{\gamma z} \end{aligned} \right\}$$

con  $\gamma^2 = (R + j\omega L)(G + j\omega C)$   
 $= \alpha + j\beta$

$$\Rightarrow Z_0 = R_0 + jX_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Y = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \frac{2\pi}{\lambda} \quad , \quad \lambda = v_p \cdot f$$

$$\Rightarrow (\alpha, \beta, Z_0) @ \omega \Leftrightarrow \underbrace{R, L, G, C, \omega}_{\text{DATOS DE LA ESTRUCTURA FÍSICA}}$$

DATOS DE LA ESTRUCTURA FÍSICA

$\Rightarrow$  SE CONSTRUYE PARA TRABAJAR A  $\omega / \frac{\omega L}{R} \gg 1$  Y  $\frac{\omega C}{G} \gg 1$

(LINEAS SIN PERDIDAS IDEALES  $R = G = 0$ )  $\Rightarrow \alpha = 0$

EN ESTAS CONDICIONES

$$Z_0 = R_0 = \sqrt{L/C}$$

$$\alpha \approx 0$$

$$\beta = \omega \sqrt{LC}$$

$$\gamma_P = \frac{\omega}{\beta}$$

$$\frac{\omega L}{R} \gg 1, \quad \frac{\omega C}{G} \gg 1$$

"LÍNEAS INDEPENDIENTES DE  $B_{\omega}$

PARA UN  $\omega_0$  (CENTRAL) Y SIN

PERDIDAS"



$$I = \frac{1}{Z_0} (V_1 e^{-\gamma z} - V_2 e^{\gamma z})$$

$$\Rightarrow Z(z) = \frac{V(z)}{I(z)} = Z_0 \frac{V_1 e^{-\gamma z} + V_2 e^{\gamma z}}{V_1 e^{-\gamma z} - V_2 e^{\gamma z}}$$

\* ESPECIFICAMENTE  $Z(z=l) = Z_T$

\* COEFICIENTE DE REFLEXIÓN :  $\rho_T = \frac{V_T e^{j\phi_T}}{V_1 e^{-\gamma l}} = \frac{V_2 e^{\gamma l}}{V_1 e^{-\gamma l}} = \frac{V_2 e^{2\gamma l}}{V_1}$

$$\rho_T = \frac{V_2 e^{2\gamma l}}{V_1}$$

$$z_T = z_0 \frac{V_1 e^{-\gamma l} + V_2 e^{+\gamma l}}{V_1 e^{-\gamma l} - V_2 e^{+\gamma l}}$$

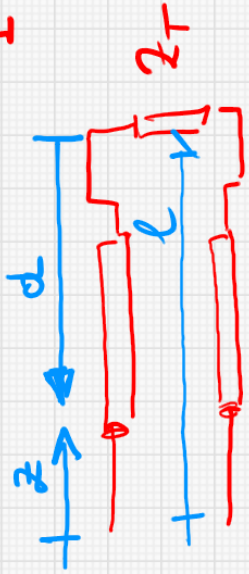
- COMO GENERALIZAMOS:

$$z(z) = z_0 \frac{V_1 e^{-\gamma z} + V_2 e^{+\gamma z}}{V_1 e^{-\gamma z} - V_2 e^{+\gamma z}}$$

$$z(z) = \frac{e^{\gamma(l-z)} + \rho_T e^{-\gamma(l-z)}}{e^{\gamma(l-z)} - \rho_T e^{-\gamma(l-z)}}$$

$$\Rightarrow \left[ \begin{aligned} \frac{z_T}{z_0} &= \frac{1 + \rho_T}{1 - \rho_T} \\ \rho_T &= \frac{z_T/z_0 - 1}{z_T/z_0 + 1} \end{aligned} \right]$$

$$\rho_T e^{-2\gamma l} = \frac{V_2}{V_1}$$



ME QUEDA :

$$\frac{Z(d)}{Z_0} = \frac{1 + \rho_T e^{-2\gamma d}}{1 - \rho_T e^{-2\gamma d}}$$

$$\rho_T = \frac{Z_T - Z_0}{Z_T + Z_0}$$

(EN TERMINAL)

CON BAJAS PERDIDAS :

$$\alpha \approx 0$$
$$\beta = \omega \sqrt{LC}$$
$$\frac{\omega L}{R} \gg 1 \quad \gamma \quad \omega C \gg 1$$

$$\frac{Z(d)}{Z_0} = \frac{Z_T/Z_0 + \tanh(\gamma d)}{1 + Z_T/Z_0 \tanh(\gamma d)}$$

$\Rightarrow$

### \* TRANSFORMADORES

$$\frac{z(d)}{z_0} = \frac{z_T/z_0 + \tanh(\beta d)}{1 + z_T/z_0 \tanh(\beta d)}$$

"  $\tanh(\beta d)$  "

CASO 1:

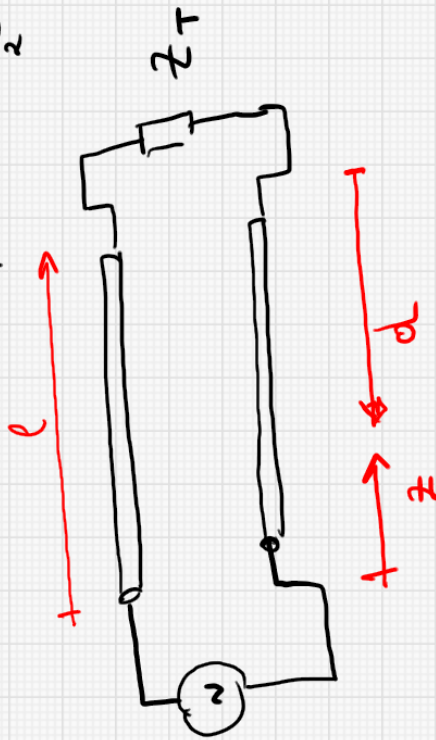
$$\left. \begin{aligned} \beta d &= m\pi \\ \tanh(m\pi) &= 0 \end{aligned} \right\} \Rightarrow z(d) = z_T \quad @ \quad d = \frac{m\pi}{\beta} = \frac{m\pi}{2\pi/\lambda} = \frac{\lambda m}{2}$$

CASO 2:

$$\left. \begin{aligned} \beta d &= \frac{m\pi}{2} \\ \tanh\left(\frac{m\pi}{2}\right) &= \infty \end{aligned} \right\} \Rightarrow z_0 = \sqrt{z_T z(d)} \quad @ \quad d = \frac{m\pi}{2\beta} = m \frac{\lambda}{4}$$

## RESUMEN DE RELACIONES DE IMPEDANCIA :

$$Z(z) = Z_0 \frac{V_1 e^{-\gamma z} + V_2 e^{\gamma z}}{V_1 e^{-\gamma z} - V_2 e^{\gamma z}}$$



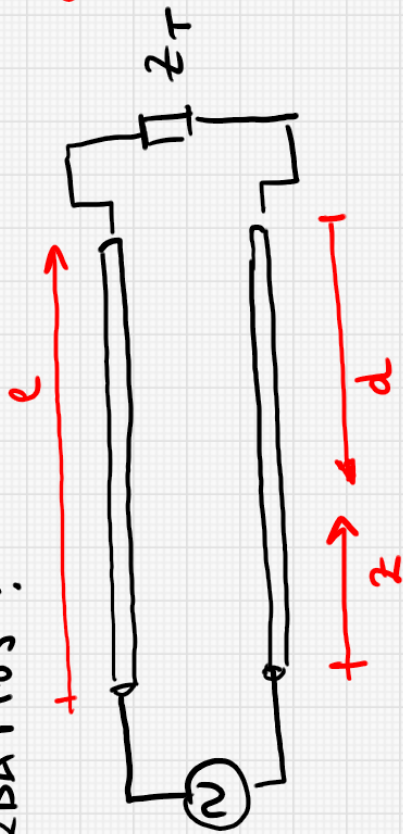
$$\left. \begin{aligned} \gamma &= \alpha + j\beta \\ Z(z=l) &= Z_T \\ \alpha &\approx 0 \\ \beta &= \omega \sqrt{LC} = \frac{2\pi}{\lambda} \end{aligned} \right\}$$

$$\Rightarrow \frac{Z(d)}{Z_0} = \frac{Z_T/Z_0 + \tanh(\gamma d)}{1 + Z_T/Z_0 \tanh(\gamma d)}$$

$$\rho_T = \frac{Z_T/Z_0 - 1}{Z_T/Z_0 + 1}$$

COEFICIENTE DE REFLEXIÓN Y POTENCIA :

RECORDAMOS :



$$d = l - z$$

$$V(z) = V_1 e^{-\gamma z} + V_2 e^{\gamma z} \Rightarrow$$

$$P(z) = \frac{V_2 e^{\gamma z}}{V_1 - \gamma z} = \frac{V_2}{V_1}$$

$$2\gamma(e-d)$$

$$2\gamma e^{-2\gamma d} = \rho_T e^{-2\gamma d}$$

$$P(d) = \frac{V_2}{V_1}$$

$$\rho_T$$

$$\gamma = \alpha + j\beta$$

$$-2\gamma d$$

$$= \rho_T e^{-2\gamma d}$$

$$\frac{Z(d)}{z_0} = \frac{1 + p_T e^{-2\gamma d}}{1 - p_T e^{-2\gamma d}}$$

$$p_T = \frac{z/z_0 - 1}{z/z_0 + 1}$$

⇒ SE PUEDE ESCRIBIR :

$$\begin{cases} V(z) = v_1 e^{-\gamma z} (1 + v_2/v_1 e^{2\gamma z}) \\ I(z) = \frac{v_1}{z_0} e^{-\gamma z} (1 - v_2/v_1 e^{2\gamma z}) \end{cases}$$

⇒ POTENCIAL

$$P = V(z) \pm I(z) \Rightarrow P_r = \left| \frac{v_1}{z_0} \right|^2 R_0 e^{-2\alpha z} \left\{ 1 - |P(z)|^2 - \frac{2X_0}{R_0} \text{Im}[P(z)] \right\}$$

$$z_0 = R_0 + jX_0$$

$$P_i = \left| \frac{v_1}{z_0} \right|^2 X_0 e^{-2\alpha z} \left\{ 1 - |P(z)|^2 + \frac{2R_0}{X_0} \text{Im}[P(z)] \right\}$$

REAL

IMAGINARIA

A VECES  $T'(z)$

$$\frac{1}{2} \frac{|V_1|^2}{R_0} (1 - |P(z)|^2)$$

$$P_{REF} = P_{INC} (1 - |P(z)|^2)$$

o EN "d"

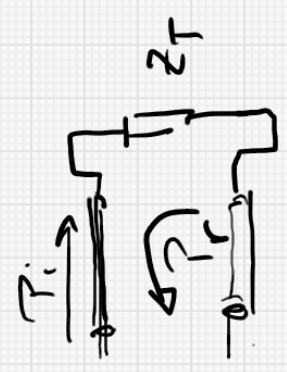
$$P_{REF} = P_{INC} (1 - |P(d)|^2)$$

$$P_{REF} = P_{INC} (1 - |P_T|^2)$$

EN LA CARGA

SE PUEDE MOSTRAR QUE ES LA POTENCIA

INCIDENTE EN UN PUNTO "z" CUALQUIERA





RECORDAR

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 + jX_0 = \sqrt{\frac{L}{C}} = R_0$$

$\omega L \gg R$  @  $\omega$  DE TRABAJO  
 $\omega C \gg G$

\* JAMOS A VER AHORA EL PARAMETRO SWR (ROE)

$$ROE = \frac{|V(d)|_{\max}}{|V(d)|_{\min}}$$

Y SE PUEDE PROBAR QUE

$$ROE = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

\* SAFE DE USAR:

$$\left\{ \begin{array}{l} |V(d)|_{\max} = |V_i| (1 + |\Gamma|) \\ |V(d)|_{\min} = |V_i| (1 - |\Gamma|) \end{array} \right.$$