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RESEARCH ARTICLE

Smoothing of PV power fluctuations by geographical dispersion

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ABSTRACT

The quality and the reliability of the power generated by large grid-connected photovoltaic (PV) plants are negatively affected by the source characteristic variability. This paper deals with the smoothing of power fluctuations because of geographical dispersion of PV systems. The fluctuation frequency and the maximum fluctuation registered at a PV plant ensemble are analyzed to study these effects. We propose an empirical expression to compare the fluctuation attenuation because of both the size and the number of PV plants grouped. The convolution of single PV plants frequency distribution functions has turned out to be a successful tool to statistically describe the behavior of an ensemble of PV plants and determine their maximum output fluctuation. Our work is based on experimental 1-s data collected throughout 2009 from seven PV plants, 20 MWp in total, separated between 6 and 360 km. Copyright © 2011 John Wiley & Sons, Ltd.

KEYWORDS
large PV plants; grid-connected; power fluctuations; analytical model; convolution

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1. INTRODUCTION

The variability of the irradiance can cause significant fluctuations in the power generated by large grid-connected photovoltaic (PV) plants. As penetration of PV energy in our utility networks increases, these power fluctuations can negatively affect power quality and reliability. In particular, short term power fluctuations (below 10 min) are typically absorbed by the grid as frequency fluctuations, thus affecting power quality. A previous work [1] analyzed such power fluctuations at a single plant level, evidencing the dependence on PV plant size. Now, this paper focuses on the smoothing effect because of the aggregation of geographically dispersed PV plants. A priori, the higher the number of PV plants grouped and the further apart the PV plants are, the stronger the smoothing effect will be. This phenomenon has been previously observed at different time scales. Otani [2] worked with irradiance measurements at nine locations distributed over 16 km^2 with 1-min resolution. A cross-correlation analysis showed that for distances between the stations greater than 5 km, observed daily irradiances are essentially uncorrelated for that measurement resolution. Later on, the authors proposed a method to estimate the largest power fluctuation during a month as the product of the standard deviation fluctuation by a so-called 'largest fluctuation coefficient' [3]. They also proposed empirical equations to derive such values for PV ensembles from single PV plant data assumed to be known beforehand. These authors do not address the relationship between fluctuation and PV plant size. Neither do they address fluctuations along a day, which are of special interest for grid operators, particularly in small grids (such as islands), with high PV penetration. Other authors [4] perform a mathematical analysis which quantifies the variability reduction in power fluctuation from a fleet of PV systems, ranging from individual systems to a set of distributed systems. A relationship between the variance of the fluctuations of a single PV plant and an ensemble is suggested. They also proposed the necessity of real power data to test and validate the models. Wiemken *et al.* [5] worked with 5-min 1-y data from 100 PV sites (totaling 243 kWp) spread over Germany. They observed that, at that scale, power fluctuations of the normalized ensemble power are reduced to 10%. Murata and Otani [6] estimated the regional distribution of long term fluctuations by means of hourly simulated power output of 800 small PV systems (~3 kWp) installed nationwide in Japan.

Our work analyzes the smoothing effect on the power output of seven PV plants located in Spain, based on 1-s data recorded during 2009. Timing is controlled by means of a GPS, so that the records from all the sites can be precisely synchronized. The power of the plants ranges from 1 to 9.5 MWp, for a total of 20 MWp. All the PV plants are equipped with vertical-axis trackers (azimuth) paralleling the sun's east–west motion, and each generator tilted 45°. The PV plants are connected to a 13.2-kV grid. Power output 1-s data are obtained at the point of common coupling by means of a power meter (Allen–Bradley, Power monitor, Milwaukee, WI, USA), and are recorded by a PLC (Allen-Bradley, CompactLogix, Milwaukee, WI, USA). Six of the plants are scattered over a ~1000-km^2 area in the south of Navarra (Spain). Figure 1 details the location of the six sites considered. Distances between them range from 6 to 60 km. The seventh PV plant is located at Socuellamos (Castilla La Mancha, Spain) situated 320 km from the nearest PV plant in the south of Navarra. Table I details the power and extension of the PV plants. Additional details can be found in [1].

Observations at these seven PV plants are extrapolated to a general number of plants by means of models describing both the maximum power fluctuation along a year and the maximum power fluctuation along a particular day as a function of the number of PV plants.

2. DEFINITIONS

The power fluctuation of the *i*th PV plant, $\Delta P_{\Delta t}{}^{i}(t)$, at an instant, t, for a given sampling period, Δt, is calculated as the difference between two normalized power outputs, that is,

$$\Delta P_{\Delta t}{}^{i}(t) = \left[p^{i}(t+\Delta t) - p^{i}(t)\right] \times 100 \qquad (1)$$

$$p^{i}(t) = \frac{P^{i}(t)}{P^{*,i}} \qquad (2)$$

where $P^{i}(t)$ is the power output at instant t, and $P^{*,i}$ is the transformer power at the common coupling point.

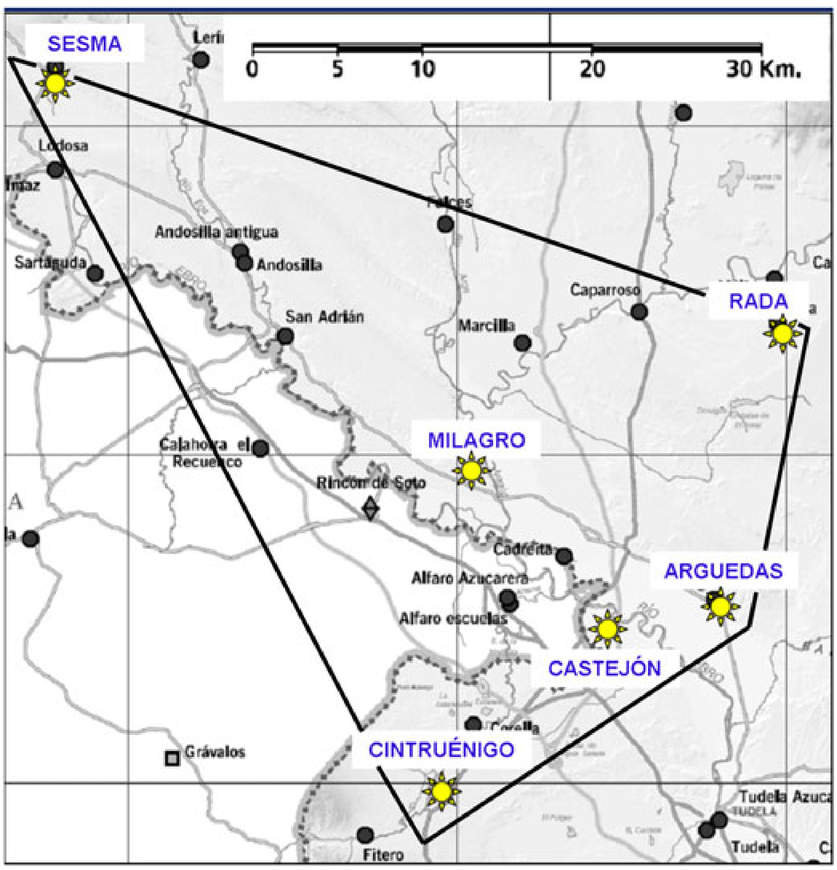

Figure 1. Location of the six photovoltaic (PV) plants under study.

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Table I. Characteristics of the photovoltaic (PV) plants.

PV plants	Peak power (kWp)	Transformer power (kW)	Area (Ha)	Location (Lat; Lon)
Arguedas	958	775	4.1	42°10′32″N 1°35′28″W
Sesma	990	800	4.2	42°27′43″N 2° 5′31″W
Cintruénigo	1438	1155	6.4	42° 3′35″N 1°47′50″W
Rada	1780	1400	8.7	42°19′3.25″N 1°34′10″W
Castejón	2640	2000	11.8	42° 9′7″N 1°39′36″W
Milagro	9500	7243	52.0	42°15′28.24″N 1°46′30″W
Socuéllamos	2600	1975	18.0	39°15′26.13″N 2°45′53.16″W
Total	19,906	15,348	—	—

Strictly speaking, the normalized power output at instant t of an aggregation of N plants is given by Equation (3):

$$p_N(t) = \frac{\sum_{i=1}^{N} P^i(t)}{\sum_{i=1}^{N} P^{*,i}} \tag{3}$$

However, when a single plant is significantly larger than the others, as it is in the case of Milagro PV plant, the smoothing effect of the geographic dispersion is masked at Equation (3) by the predominance of the largest PV plant. This represents an inconvenience when the goal is precisely to analyze geographical smoothing, which has led us to use Equation (4):

$$p_N(t) = \frac{1}{N}\sum_{i=1}^{N} \frac{P^i(t)}{P^{*,i}} \tag{4}$$

It can be argued that because power fluctuation is also smoothed by the PV plant size, Equation (4) entails some drawbacks. Mainly all the PV plants are assumed to have the same peak power because they receive the same weight at Equation (4); but their intrinsic fluctuation behavior is considered different because the rhythm is not affected by power normalization. However, as we will see later on this paper, the smoothing by geographical dispersion is significantly more important than the smoothing by size. Hence, such drawback is in fact irrelevant.

We can define the magnitude of a power fluctuation, $\Delta P_{\Delta t,N}(t)$ for a number N, of PV plants grouped at an instant t, and for a given sampling period Δt, as the difference between two normalized power outputs, Equation (5), that is,

$$\Delta P_{\Delta t,N}(t) = [p_N(t+\Delta t) - p_N(t)] \times 100 \tag{5}$$

As an example, Figure 2 shows the normalized output power p_1, recorded at Cintruénigo PV plant ($P^* = 1.155$ MW) and the ensemble of the six PV plants situated in Navarra, p_6, ($P^* = 13.373$ MW) on 2 February, from 12:30 to 13:00 h. Figure 3 shows the corresponding power fluctuation for (a) $\Delta t = 20$ s; and (b) $\Delta t = 600$ s. It can be observed that the fluctuations of the ensemble are reduced compared with the single system, and that the fluctuations increase as the time sampling increases.

3. POWER FLUCTUATIONS

The power fluctuation smoothing by geographical dispersion is seen in Figures 4 and 5, showing the power fluctuation distribution for Δt equal to 60 and 600 s correspondingly, for the Arguedas plant, the combination of three plants, namely Castejon, Milagro, and Sesma; and all the six plants located in Navarra, during a year (2009). The distribution functions are normalized. Table II gathers the values of some fluctuation intervals. Note that the magnitude and the relative frequency of the fluctuations decrease with N.

Figure 6 shows, for the whole year, the largest fluctuations observed at the PV plants versus Δt. As it was explained in [1], there is a power fluctuation smoothing effect due to size.

Now, the largest fluctuation observed during a full year for all the possible combinations, for $N = 1$…six plants, is calculated (from the six plants at Navarra). As it can be verified in Figure 7a, the smoothing effect is amplified as the number of systems grouped increases. For $\Delta t = 1$ s, the maximum fluctuation is reduced from 16.1% to 3.0%; for $\Delta t = 600$ s, it is reduced from 99.2% to 54.4%. Figure 7b details the largest fluctuation observed for each of the 15 possible combinations of $N = 4$. There is not a particular combination presenting a larger smoothing effect.

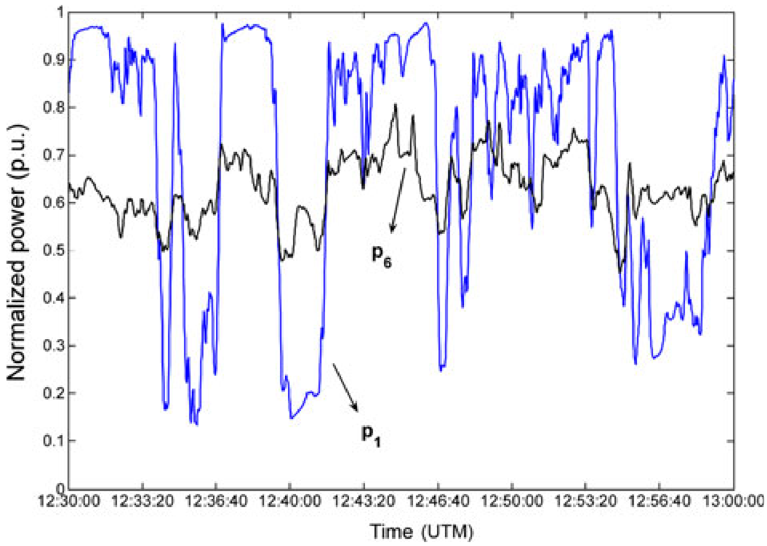

Figure 2. Normalized output power p_1 recorded at Cintruenigo PV plant ($P^* = 1.155$ MW) and the ensemble of the six PV plants p_6, ($P^* = 13.373$ MW) on 2 February, from 12:30 to 13:00 h.

Figure 8 shows the 99th percentile[1] of the largest fluctuation observed for all the possible combinations of $N = 1\ldots$six plants 99th ($\Delta P_{\Delta t,N}$) as a function of N, with Δt as a parameter. It is worth mentioning that they fit to a geometric function, such as

$$99\text{th}\left(\Delta P_{\Delta t,N}\right) = 99\text{th}\left(\Delta P_{\Delta t,1}\right)\cdot N^{-a}, \quad a > 0 \qquad (6)$$

Table III compiles the corresponding values. The validity of Equation (6) is checked against a different PV plant located in another region: Socuellamos (Castilla La Mancha, 345 km distance from the nearest PV plant of the previous experiment). Despite adding a new PV plant, in all the cases ($\Delta t = 1$ s…600 s), the regression coefficient R^2 remains over 0.995. Hence, we can extend this relationship to determine the 99th ($\Delta P_{\Delta t,N}$) of a supposed number N of PV plants grouped.

In our previous work [1], we observed that the 90th percentiles of the fluctuations registered at a single PV plant are related to the plant area, S. In this paper, we have recalculated this relationship using the 99th percentile, 99th ($\Delta P_{\Delta t,1}$), by means of the empirical Equation (7):

$$99\text{th}\left(\Delta P_{\Delta t,1}\right) = m\cdot S^{-c}, \quad m, c > 0 \qquad (7)$$

[1]From the point of view of network operation, the relevant parameter is the maximum fluctuation, Max($\Delta P_{\Delta t,N}$). However, this value responds to very particular situations, hindering the regression analysis. This is the reason why we have analyzed the 99th percentile, 99th ($\Delta P_{\Delta t,N}$). Despite this, we have observed that Max($\Delta P_{\Delta t,N}$) has never exceeded 99th ($\Delta P_{\Delta t,N}$) more than 9% during the period under analysis. This difference can be assumed as a safety factor.

where S is given in Ha, and m and c depend on Δt. Some values are compiled in Table IV. The parameter c represents the attenuation of power fluctuations because of the area of PV plants. It is worth mentioning that for large Δt (600 s in our case), attenuation becomes practically irrelevant ($c = 0.02$), which means that power fluctuations are not influenced by S under these conditions. This makes sense from the point of view that 600 s is long enough for shadows to completely cover a PV plant (within this experiment size ranges). In fact, this is the reason why the regression coefficient R^2 value falls for large Δt. On the other hand, the parameter m has been found strongly dependent on the sample period Δt (Figure 9, $R^2 = 0.98$), according to the expression in Equation (8):

$$m = 99\text{th}\left(\Delta P_{600,1}\right)\cdot\left(1 - e^{-0.24\cdot\Delta t}\right) \qquad (8)$$

Obviously, when PV plants of the same size are grouped, Equations (6) and (8) can be combined in Equation (9):

$$99\text{th}\left(\Delta P_{\Delta t,N}\right) = 99\text{th}\left(\Delta P_{600,1}\right)\cdot\left(1 - e^{-0.24\cdot\Delta t}\right)\cdot S^{-c}\cdot N^{-a}\, a, c > 0 \qquad (9)$$

This equation describes the fluctuation smoothing because of both size and number of PV plants. Table V summarizes the values of the attenuation coefficients c and a, for different Δt. It is noteworthy that power fluctuations are much more attenuated by number than by size for the same rated PV power installed. Making the appropriate transformations, these experimental results confirm the theoretical findings [4] for small Δt and at single PV plant level called *Crowded Region*, that fluctuations decrease proportionally to $\sqrt{S}$ ($c = 0.49$ in

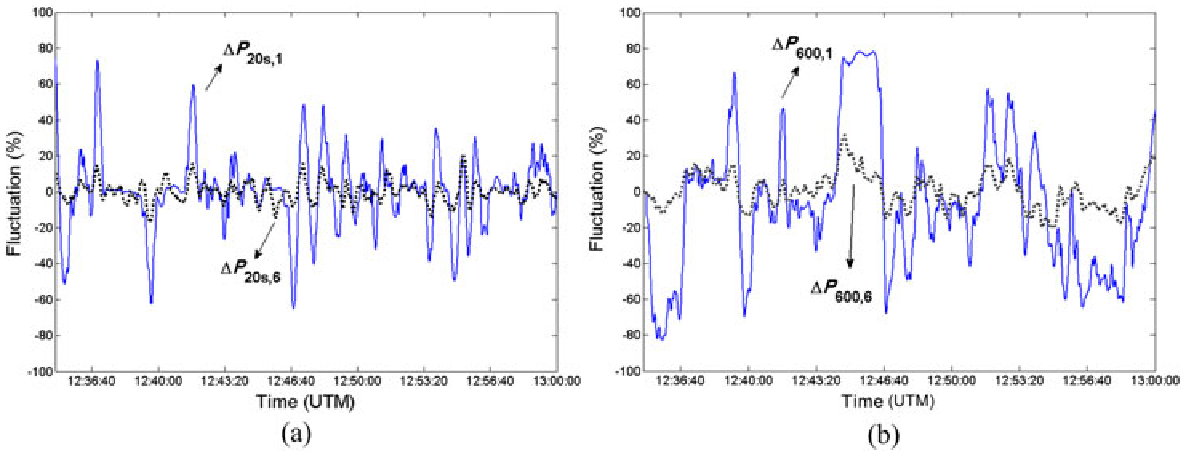

Figure 3. Power fluctuation evolutions at Cintruenigo PV plant $\Delta P_{\Delta t,1}$ and at the ensemble of the six PV plants site $\Delta P_{\Delta t,6}$ during a 30-min period, for (a) $\Delta t = 20$ s; and (b) $\Delta t = 600$ s.

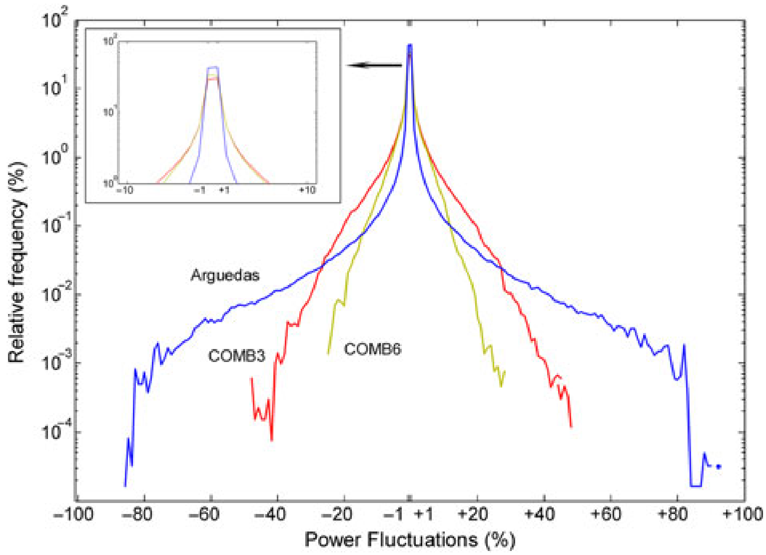

Figure 4. Distributions of the power fluctuations $\Delta P_{\Delta t,N}$, registered during a full year (2009) at Arguedas PV plant and the combination of $N = 3$ (COMB3) and $N = 6$ (COMB6) different PV plants, all at Navarra, for $\Delta t = 60$ s.

our case). The influence of the dispersion in the smoothing effect at this time scale is $1/N$ ($a = 0.77$) Alternatively, for systems sufficiently far apart and for large Δt, denominated as *Spacious Region* in [4], the fluctuations decrease by a $1/\sqrt{N}$ law ($a = 0.46$). This was also pointed out in [3].

In an attempt to illustrate how N and S influence the power fluctuation smoothing, we propose an exercise analog to Hoff and Perez [4], which contrasts the power output variability of centralized generation versus distributed generation. Imagine a situation where 100 MW of PV power must be installed, and the network operator has to decide the power clustering degree (size P^* and number N of PV plants), so that $100\,\text{MW} = N{\cdot}P^*$. The Canary Islands are a good example: the PV power installed raises up to 96 MW, which corresponds to a penetration level of 3.3% [8]. If we make the assumption that all the PV plants have the same constructive parameters such as the ground coverage ratio, PV generator, and so on, there is a direct relationship k, between the rated power P^*, of the PV central and its area S, so that $100\,\text{MW} = kNS$. In our case, k is essentially 6.51 Ha/MW. Table VI shows the 99th ($\Delta P_{\Delta t,N}$) values for different Δt and N,P^* combinations according to the empirical expression, Equation (9), and assuming that 99th ($\Delta P_{600,1}$) = 98% shows. These results are displayed in Figure 10. It can be observed that the fluctuation of 100 1-MW plants is around 10% the fluctuation of a

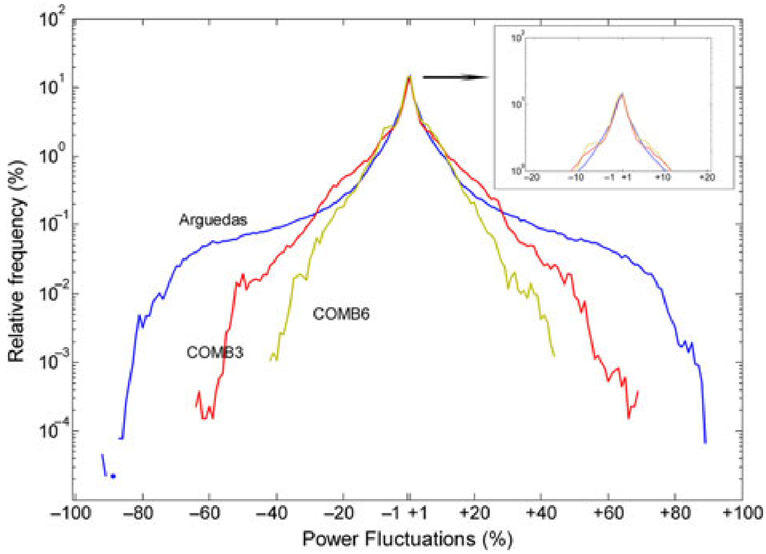

Figure 5. Distributions of the power fluctuations, $\Delta P_{\Delta t,N}$, registered during a full year (2009) at Arguedas PV plant and the combination of $N=3$ (COMB3) and $N=6$ (COMB6) different PV plants, all at Navarra, for $\Delta t = 60$ s.

Table II. Influence of Δt and the number of plants grouped, N, on the frequency distribution of the fluctuations. Values are percentage of the total time.

$\Delta P_{\Delta t,N}$ (%)	Δt (s)					
	60 s			600 s		
	Arguedas	COMB3	COMB6	Arguedas	COMB3	COMB6
0 ≤ ΔP ≤ 3	92.8	78.5	85.3	80.0	48.7	54.3
3 ≤ ΔP ≤ 10	4.4	15.9	13.2	11.5	30.5	33.7
10 ≤ ΔP ≤ 50	2.6	5.6	1.5	7.5	21.1	12.0
50 ≤ ΔP ≤ 100	0.2	0.0	0.0	0.9	0.1	0.0
0 ≤ ΔP ≤ 100	100.0	100.0	100.0	100.0	100.0	100.0

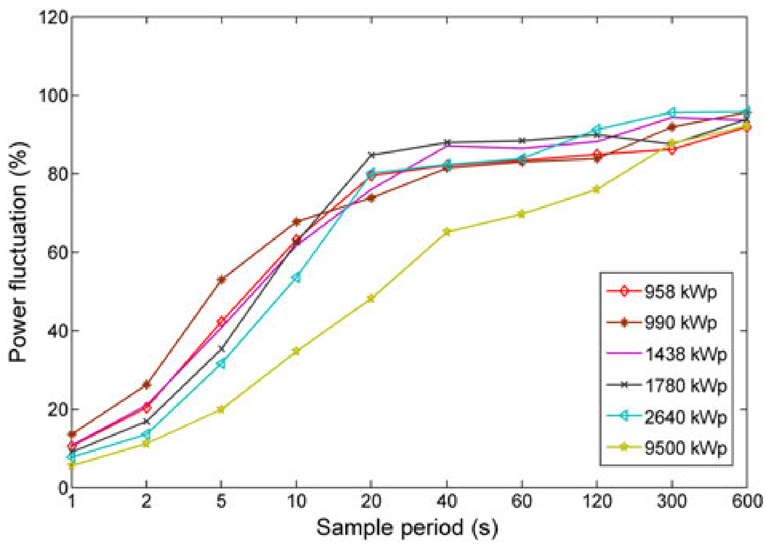

Figure 6. Maximum power fluctuations registered during a year at each PV plant at Navarra versus Δt.

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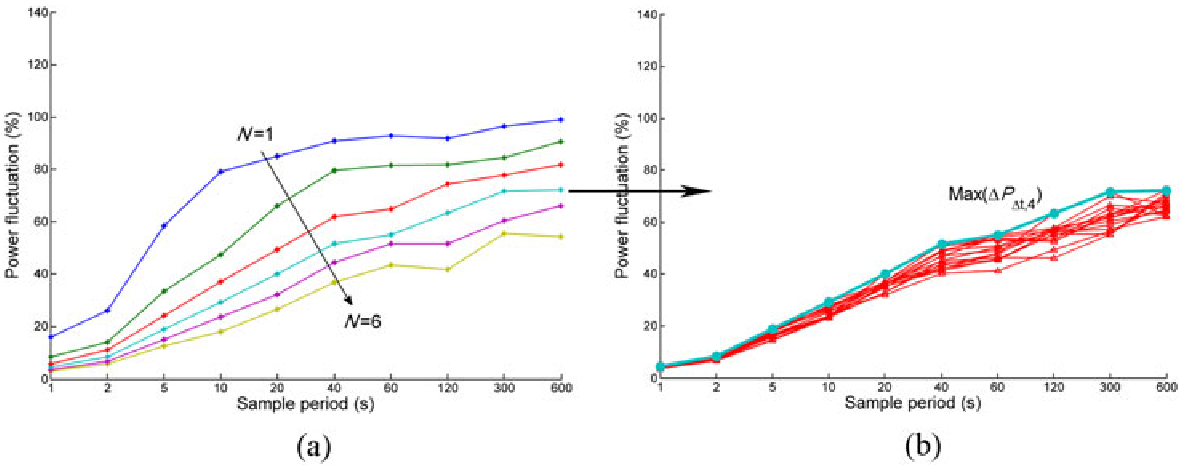

Figure 7. (a) Maximum power fluctuations $\Delta P_{\Delta t,N}$, registered during a year for all possible combinations for $N = 1$…six plants (all at Navarra). (b) Maximum power fluctuation observed for all the 15 possible combinations of $N = 4$ during a year.

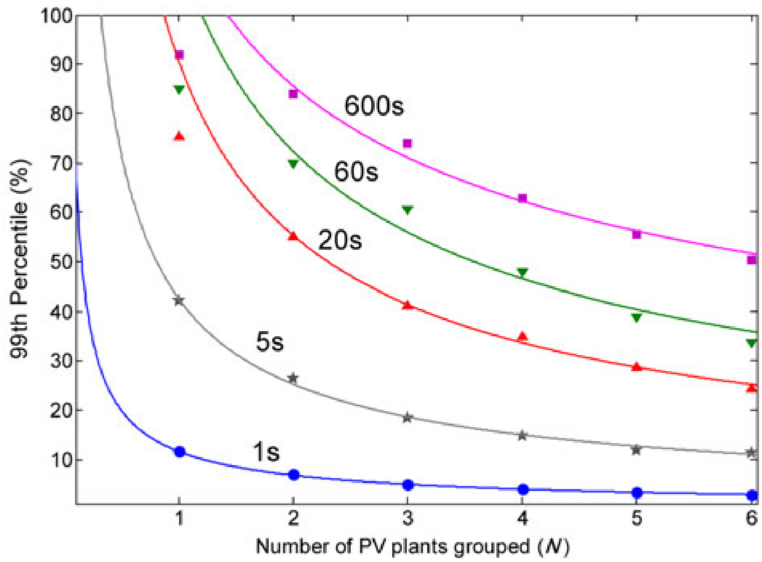

Figure 8. Ninety-ninth percentile of largest fluctuation observed for all the possible combination of N plants, 99th ($\Delta P_{\Delta t,N}$) for $\Delta t = 1$, 5, 20, 60, and 600 s.

Table III. Estimated parameter *a* and coefficient regression for the empirical equation proposed in Equation (6).

Δt(s)	*a*	R^2
1	0.77	0.99
5	0.75	0.99
20	0.71	0.99
60	0.63	0.98
600	0.46	0.98

single 1-MW plant, which agrees with the previously mentioned observations in Germany [5] and the analytic studies developed in [4]. Likewise, the smoothing factor is considerably reduced in the case of 50 2-MW plants: 99th ($\Delta P_{600,50}$) is under 20% in 600 s. Typically, the electric generator ramping rates range from 25% to 40%

Table IV. Estimated parameter *m* and *c* and coefficient regression for the empirical equation proposed in Equation (7).

Δt(s)	*m*	*c*	R^2
1	25.55	0.49	0.99
5	64.09	0.29	0.96
20	88.2	0.11	0.89
60	91.6	0.05	0.86
600	94.07	0.02	0.61

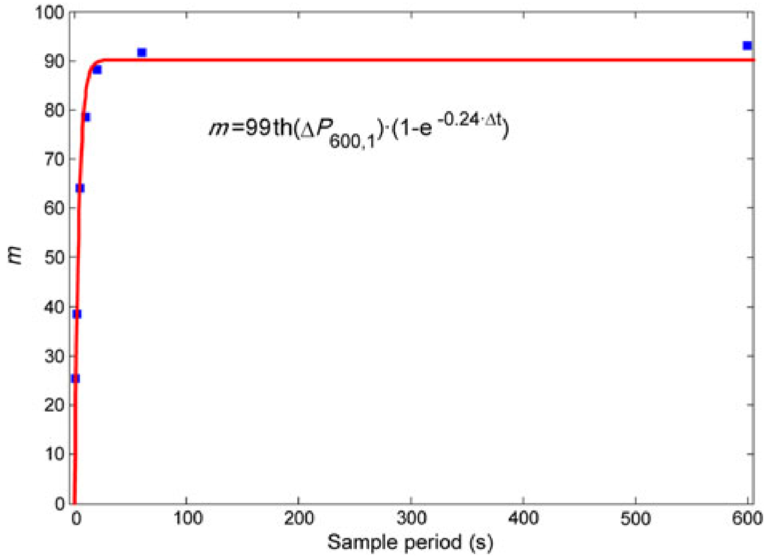

Figure 9. Estimated relationship between the empirical coefficient *m* (Equation (8)) and the sample period Δt.

Table V. Estimated parameter *a* and *c* for the empirical Equation (9).

Δt(s)	*a*	*c*
1	0.77	0.49
5	0.75	0.29
20	0.71	0.11
60	0.63	0.05
600	0.46	0.02

in 600 s [9], so that the necessity of a higher clustering degree is arguable.

The influence of aggregation between distant plants has also been studied. Figure 11 shows the power fluctuation distribution for Δt equal to 60 and 600 s, respectively, for the combination of the PV plants in Arguedas and Castejón (distanced 6 km) and the plants in Arguedas and Socuéllamos, (distanced 345 km). Both distributions essentially coincide, which support the idea that 6 km is enough to ensure the decorrelation between short-term power fluctuations. In fact, the corresponding cross-correlation coefficients ρ, of these particular combinations for different time scales (daily, monthly, and yearly) are very low (Table VII). For $\Delta t = 600$ s and distance 6 km, $\rho_d = 0.07$. This result agrees with the findings made in [2].

4. FREQUENCY FLUCTUATION MODEL

Power fluctuation analysis in daily terms is of particular interest for grid operators. A previous work [1] has led us to model daily fluctuation frequency by establishing a certain threshold u, (for example 3%), such that all fluctuations below it are considered irrelevant. Then, frequency distribution of fluctuations along a particular day is properly described by Equation (10):

$$f(x)\begin{cases} x \le u; & \left(1-\dfrac{T_{\Delta P>u}}{T_D}\right)\dfrac{1}{u} \\ x > u; & \dfrac{T_{\Delta P>u}}{T_D} b \cdot e^{-b(x-u)} \end{cases} \qquad (10)$$

Table VI. 99th ($\Delta P_{\Delta t,N}$) values for different Δt and N, S combinations; obtained via Equation (9) and Table V.

N	P^* (MW)	S (Ha)	99th ($\Delta P_{\Delta t,N}$)				
			Δt = 1 s	Δt = 5 s	Δt = 20 s	Δt = 60 s	Δt = 600 s
1	100.00	651.000	0.9	10.5	47.7	70.9	86.1
10	10.00	65.100	0.5	3.6	12.0	18.6	31.3
100	1.00	6.510	0.2	1.3	3.0	4.9	11.3
1000	0.10	0.650	0.1	0.4	0.8	1.3	4.1
10,000	0.01	0.065	0.1	0.2	0.2	0.3	1.5

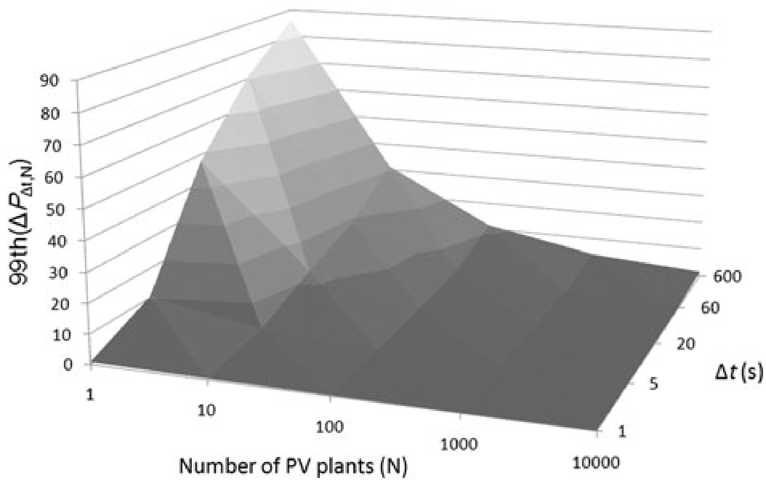

Figure 10. Estimated 99th ($\Delta P_{\Delta t,N}$) values for different t and N, P^* PV plants combinations; obtained via Equation (9) and Table V. Around $N = 50$ in advance, the smoothing factor is considerably reduced: the 99th ($\Delta P_{600,50}$) is under 20% in 600 s.

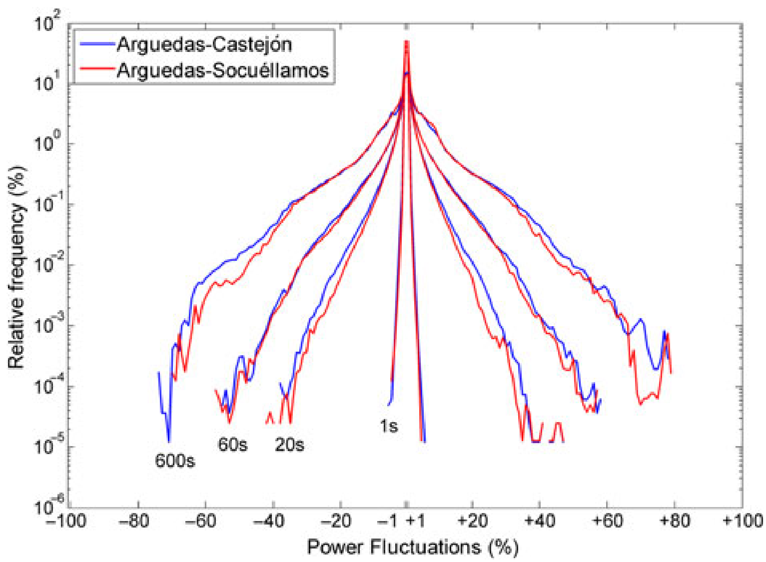

Figure 11. Distribution of the power fluctuations $\Delta P_{\Delta t,2}$, registered during a full year (2009) at the combination of Arguedas–Castejón PV plants (distance = 6 km) and Arguedas–Socuéllamos (distance = 345 km) for Δt = 1, 20, 60, and 600 s.

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Table VII. Cross-correlation coefficients, ρ, for daily, monthly, and yearly output power fluctuations for $\Delta t = 60$ s and 600 s. The daily term corresponds to 23 July, the day when the largest fluctuation was observed for the combination Arguedas–Socuéllamos. The monthly term corresponds to May 2009, one of the months with the highest fluctuation frequency.

PV plant combination	Arguedas–Castejón (6 km)		Arguedas–Socuéllamos (345 km)	
Δt (s)	60 *s*	600 *s*	60 *s*	600 *s*
ρ_d (23 July)	0.03	0.17	0.027	0.09
ρ_m(May 2009)	0.01	0.07	0.01	0.002
ρ_y(2009)	0.004	0.12	0.005	0.01

where, $T_{\Delta P>u}/T_D$ represents the fraction of daytime which exhibits relevant fluctuations, and b is an empirical coefficient depending on the day and the PV plant size. On the other hand, the frequency distribution function of the sum of two independent random variables is given by the convolution of the two single functions [7]. This property can be used here to derive the frequency distribution function of power fluctuations for any combination of PV plants, previously assuming that the distance between them is large enough to assure decorrelation between their individual fluctuations. As previously described, this is the case for fluctuations below 10 min and distances above 6 km.

Let us consider two PV plants with respective transformer power $P^{*,1}$ and $P^{*,2}$. The fluctuation of their aggregate is given by the sum of their weighted fluctuations, Equation (11):

$$\Delta P_{\Delta t,2} = fw^1 \cdot \Delta P^1_{\Delta t} + fw^2 \cdot \Delta P^2_{\Delta t} \tag{11}$$

where

$$fw^1 = \frac{P^{*,1}}{P^{*,1} + P^{*,2}};\ fw^2 = \frac{P^{*,2}}{P^{*,1} + P^{*,2}} \tag{12}$$

are the weighted factors. Because the power fluctuation range of the PV plants aggregation must remain between the interval [−100, 100], this condition makes it compulsory to redefine the power fluctuation range of each PV plant via these weighted factors, now $(-100 \cdot fw^i, +100 \cdot fw^i; i = 1 \ldots N)$, and to the fluctuation distribution function, now as follows

$$f^i(x) = \begin{cases} x \le u \cdot fw^i; & \left(1 - \frac{T_{\Delta P>u}}{T_D}\right) \frac{1}{fw^i \cdot u} \\ x > u \cdot fw^i; & \frac{T_{\Delta P>u}}{T_D} \frac{b}{fw^i} \cdot e^{-\frac{b}{fw^i}(x - u \cdot fw^i)} \end{cases} \tag{13}$$

Hence, the frequency distribution function $f_2(z)$ of $\Delta P_{\Delta t,2}$ is given by the convolution of the individual distributions, Equation (14):

$$f_2(z) = \int_{-100 f_1}^{100 f_1} f^1(x) f^2(z - x) dx \tag{14}$$

which can be directly solved by widely available software tools. The result can be transformed into a frequency distribution function with similar shape of Equation (10), by Equation (15):

$$\left(\frac{T_{\Delta P>u}}{T_D}\right)_2 = 1 - \int_{-u}^{u} f_2(z) \mathrm{d}z;\ b_2 = \frac{2 f_2(u)}{(T_{\Delta P>u}/T_D)_2} \tag{15}$$

This way, fluctuations of two PV plants are statistically described as fluctuations of a single equivalent PV plant. The procedure can be iterated as many times as desired to derive the frequency distribution function for any ensemble of PV plants. One advantage of this model is that now, we can estimate the value for the daily maximum output fluctuation for any combination of N PV plants; following the suggestion made in [3], this value coincides with the fluctuation which has a 0.25% of probability to occur. In our terms, Equation (16) is given as follows:

$$\begin{aligned} F\left(\Delta P_{\Delta t,N} > \mathrm{Max}(\Delta P_{\Delta t}, N)\right) = 0.25\% = \left.\frac{T_D}{T_{\Delta P>u}}\right|_N e^{-b_N\left(\mathrm{Max}\left(\Delta P_{\Delta t,N}\right) - u\right)} \Longrightarrow \\ \mathrm{Max}\left(\Delta P_{\Delta t,N}\right) = u - \frac{1}{b_N} \ln\left(0.25\% \left.\frac{T_D}{T_{\Delta P>u}}\right|_N\right) \end{aligned} \tag{16}$$

The validity of Equation (16) has been checked for a particular day, 29 April 2009, a day with large fluctuations. We have compared the value of Max $(\Delta P_{600,N})$ observed for the ensemble of $N = 1 \ldots 7$ PV plants and the corresponding values calculated via convolution, Equation (16). For the convolution method, we have supposed that all the plants grouped are like the Cintruénigo one, 1.1 MW ($b_1 = 0.079$, $T_{\Delta P>u}/T_D|_1 = 0.68$ on that particular day). Regarding the observed values of Max$(\Delta P_{600,N})$, they correspond to the mean of all possible combinations of N plants (for example, 35 possible combinations for $N = 4$). Results are compiled in Table VI and shown in Figure 12a. Despite the approximations made, the proposed method is relatively precise. This convolution exercise has been extended and used to obtain the values of Max($\Delta P_{600,N}$) for that particular day and a supposed number up to 96 systems grouped (Table VIII and Figure 12b). The value for the attenuation coefficient a, on that particular day is −0.38. Hence, assuming that the power fluctuation distribution of a single PV plant is known, the daily largest output power fluctuation for a number N of PV plants sufficiently separated can be found through the convolution technique.

5. CONCLUSIONS

Short-term power fluctuations generated by an ensemble of geographical dispersed large PV plants are considerably reduced compared with a single one; not only regarding the largest output fluctuation, but also their relative frequency. The results obtained in this paper support the idea that PV plants power fluctuations are attenuated more by the number of plants grouped than by their size for the same rated PV power installed. An empirical expression is proposed to calculate the value of the 99th percentile fluctuation value for a number N, of PV plants with a size S, grouped: an example of 100 plants of 1 MW dispersed reveals an important smoothing effect over power fluctuations. A separation gap between PV plants of 6 km has been shown to be sufficient to ensure uncorrelated output power fluctuations; it would be interesting to analyze these effects below this distance. Finally, applying convolution techniques to the analytic model proposed in [1], which describes the fluctuation frequency, it is possible to estimate the largest fluctuation for a number N of PV plants grouped, from single PV plants model parameters. Hence, the relationship between these parameters and meteorological standard data will make easier the integration of large dispersed PV plants into the power network.

Table VIII. Mean of the observed values of Max($\Delta P_{600,N}$) on 29 April 2009 for the ensemble of N = 1…7 PV plants and the corresponding values calculated via convolution, Equation (16). This technique has been arbitrarily extended up to 96 systems grouped.

N	Max ($\Delta P_{600,N}$) (%)	
	Observed	Convolution
1	69.38	73.53
2	62.70	55.08
3	53.40	49.15
4	46.72	44.77
5	42.80	41.06
6	40.06	38.42
7	33.40	35.44
8	—	33.96
16	—	26.14
32	—	19.58
64	—	14.05
96	—	11.28

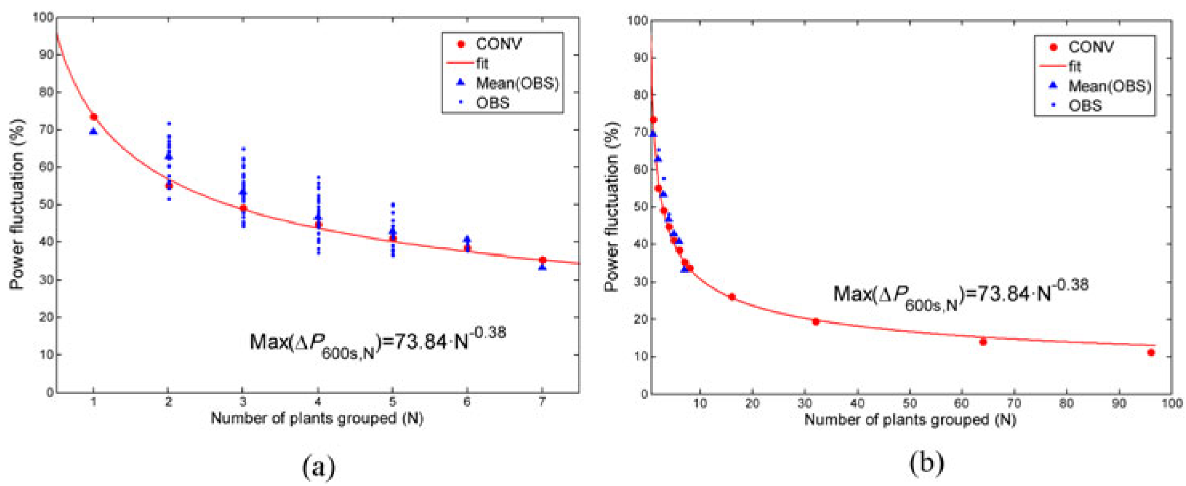

Figure 12. (a) Max($\Delta P_{600,N}$) observed on 29 April 2009 for the ensemble of N = 1…7 PV plants (blue dots) and their corresponding mean (blue triangles). The red dots are the values of Max($\Delta P_{600,N}$) calculated via convolution, Equation (16). The same figure shows the value of the attenuation coefficient for that particular day $a = 0.38$. (b) These convolution exercises have been extended to obtain the values of Max($\Delta P_{600,N}$) for up to 96 grouped systems.

NOMENCLATURE

$P^i(t)$	PV plant output power at an instant t (ith PV plant).
Δt	Sampling period between observations.
$P^{*,\,i}$	PV plant rated power (ith PV plant).
$p^i(t)$	Normalized output power at instant t (ith PV plant).
$\Delta P_{\Delta t}{}^{i}(t)$	Power fluctuation at an instant t for a given sampling period Δt of the ith PV plant.
N	Number of PV plants aggregated.
$p_N(t)$	Normalized output power at instant t of an aggregation of N plants.
$\Delta P_{\Delta t,N}(t)$	Power fluctuation at an instant t for a given sampling period Δt and for the aggregation of N PV plants.
99th ($\Delta P_{\Delta t,N}$)	99th of the largest fluctuation observed for all the possible combinations of N PV plants.
S	PV plant surface (Ha).
a	Attenuation of power fluctuations as a function of N.
c	Attenuation of power fluctuations as a function of S.
m	Value of 99th ($\Delta P_{\Delta t,1}$)as a function of Δt.
k	Relationship between the rated power P^* of the PV central and its area S, [MW/Ha].
u	Threshold which delimiters relevant fluctuations.
ρ	Cross-correlation coefficients between measured power series.
$f^i(x)$	Daily frequency distribution of fluctuations x (ith PV plant).
$f_N(x)$	Daily frequency distribution of fluctuations x for the aggregation of N PV plants.
$\frac{T_{\Delta P>u}}{T_D}$	Fraction of daytime (T_D) which exhibits relevant fluctuations.
b	Daily frequency distribution coefficient, depending on the day and the PV plant size.
$fw^i = \frac{P^{*,i}}{\sum_{i=1}^{N} P^{*,i}}$	Weighted factor (ith PV plant).
$F(x>y)$	Cumulative distribution function of a power fluctuation x to be larger than y.

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