VERSIÓN 1:

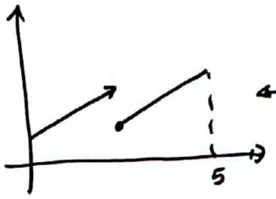
V o F:

Af 1: $\int_1^2 x^2 dx = \frac{x^3}{3}\Big|_1^2 = \frac{2^3}{3} - \frac{1^3}{3} = \frac{8-1}{3} = \frac{7}{3} > 2$

Como $S^*(f,P) \geq \int_1^2 f(x)dx \ \forall P$ y $\int_1^2 f > 2$, no puede existir P tal qe $S^*(f,P) \leq 2$. FALSO

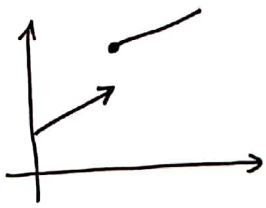
Af 2: f NO es continua en $a=2$ y Af 2 es la negación de la def. de cont. para $\varepsilon = \frac{1}{2}$. VERDADERO

Af 3: FALSO:



f es integrable pero no contina.

Af 4: FALSO:

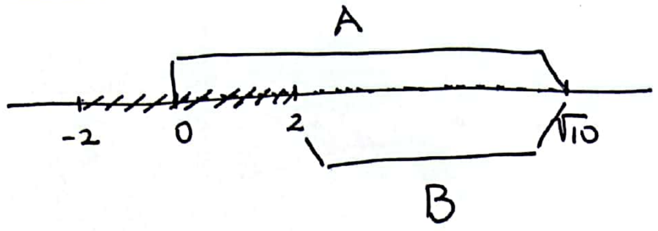


f es est. $\nearrow$ pero no contina (i.e. no es derivable).

Af 5: VERDADERO:

$\exists \lim_{h \to 0} \frac{f(a+h)-f(a)}{h} \Rightarrow f$ es derivable en a $\Rightarrow f$ es contina en a.

Mo 1:



$\sup(A) = \sqrt{10} \notin A$ pues $\sqrt{10} \notin \mathbb{Q}$

$\Downarrow$

A no tiene máximo

$\sup(B) = \sqrt{10} \notin B$

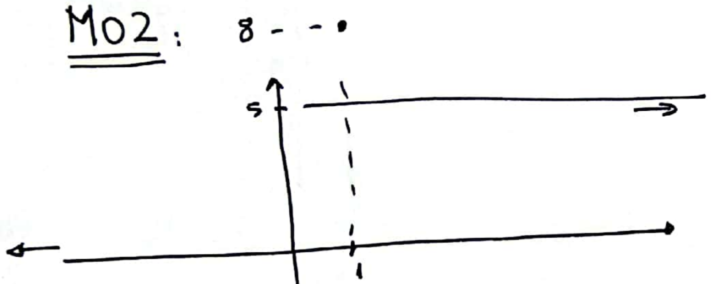
$\inf(A) = 0 \in A \Rightarrow$ es un mínimo

$\inf(B) = 2 \notin B \Rightarrow$ no es un mínimo.

Luego:

- $\sup(A) = \sup(B)$
- $\inf(A) < \inf(B)$ $\Rightarrow$ E
- A no tiene máximo

Mo2:



Candidato a máximo:

$$M = \sup\left\{ \underbrace{\lim_{x \to -\infty} f(x)}_{= 0},\ \underbrace{\lim_{x \to +\infty} f(x)}_{= 5},\ \overbrace{\underbrace{\sup_{x \in [-N,N]} f(x)}_{(*)}}^{\geq 8} \right\} \longrightarrow \text{el sup. tiene que salir de } (*)$$

Como f es cont. en $[-N,N] \underset{\text{Weierstrass}}{\Longrightarrow} \exists\ \tilde{M} = \max_{x \in [-N,N]} f(x)$

Tomando N suf. grande, tiene que ser $M = \tilde{M}$.

Candidato a mínimo:

$$m = \inf\left\{ \underbrace{\lim_{x \to -\infty} f(x)}_{= 0},\ \underbrace{\lim_{x \to +\infty} f(x)}_{5},\ \underbrace{\inf_{x \in [-N,N]} f(x)}_{(*)} \right\}$$

No tenemos información para determinar que $(*) \leq 0 \Rightarrow$ puede ser que $(*) > 0$ y $\inf\{0, 5, (*)\} = 0 \notin Im(f) \Rightarrow$

f no tiene mínimo absoluto.

Si fuera ⊛ ≤ 0 ⟹ f tiene mínimo absoluto y vale ⊛

Conclusión: con la información dada:

(*) f tiene máximo absoluto

(*) f puede o no tener mínimo absoluto.

(*) No hay cambio de signo como para poder determinar que existe x_0 / $f(x_0)=0$.

Opción correcta: A

Mo 3:

$$f'(x) = \left(\log(x^2+1)\right)' e^{x^3-1} + \log(x^2+1)\left(e^{x^3-1}\right)'$$

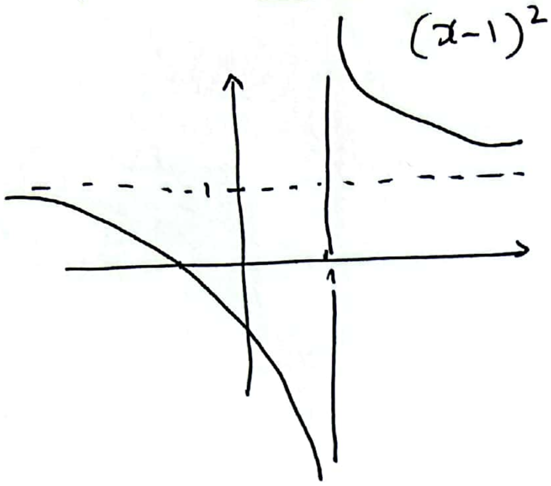
$$= \frac{2x}{x^2+1} e^{x^3-1} + \log(x^2+1)\, e^{x^3-1} \cdot 3x^2$$

$$= e^{x^3-1}\left[\frac{2x}{x^2+1} + 3x^2 \log(x^2+1)\right]$$

$$f'(1) = e^0\left[\frac{2}{1+1} + 3\log(2)\right] = 1 + 3\log(2) \longrightarrow A$$

M04:

$$f'(x) = \frac{\overbrace{(x+1)'}^{1}(x-1) - (x+1)\overbrace{(x-1)'}^{1}}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2} = \frac{-2}{(x-1)^2} < 0 \ \forall x$$



- $\lim\limits_{x \to \pm\infty} \frac{x+1}{x-1} = 1$
- $\lim\limits_{x \to 1^+} \frac{\overset{\to 2}{x+1}}{\underset{\downarrow 0^+}{x-1}} = +\infty$
- $\lim\limits_{x \to 1^-} \frac{\overset{2}{x+1}}{\underset{\downarrow 0^-}{x-1}} = -\infty$

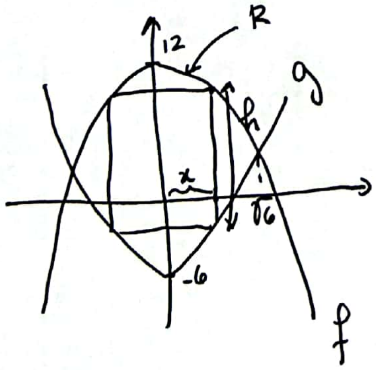
Conclusión:

(*) f es ↘ en $(-\infty, 1)$, y en $(1, +\infty)$

(*) f no tiene máximo ni mínimo absoluto.

Opción correcta: B

M05:



Sea $0 \le x \le \sqrt{6}$

Area (R) $= 2x \times h$

$h = f(x) - g(x) = 12 - x^2 - (-6 + 2x^2) = -3x^2 + 18$

$\Rightarrow A(x) = 2x \times (-3x^2 + 18) = -6x^3 + 36x$

$A'(x) = -18x^2 + 36 = 0 \Leftrightarrow x^2 = \frac{36}{18} \Leftrightarrow x^2 = 2 \Leftrightarrow x = \pm\sqrt{2}$

sgn A': $-$ $(-\sqrt{2})$ $+$ (0) $(\sqrt{2})$ $-$

M

$M = A(\sqrt{2}) = 2\sqrt{2}\,(\underbrace{-3.2+18}_{12}) = \sqrt{2}.24 \longrightarrow$ C

M06: Usamos el teorema fundamental:

$$f'(x) = \left(\sqrt{x^4 - x^2 + 1} + 1\right) 2x \Longrightarrow f'(1) = (\underbrace{\sqrt{1-1+1} + 1}_{2})\, 2 = 4 \rightarrow \text{E}$$

M07:

Sea $I = \int \operatorname{sen}(x) e^{3x} dx$

$$I = \left[\begin{array}{l} f' = \operatorname{sen} x \rightarrow f = -\cos x \\ g = e^{3x} \rightarrow g' = 3e^{3x} \end{array}\right] = -\cos x\, e^{3x} + \int \cos x \cdot 3 e^{3x} dx$$

$$= \left[\begin{array}{l} f' = \cos x \rightarrow f = \operatorname{sen} x \\ g = e^{3x} \rightarrow g' = 3e^{3x} \end{array}\right] = -\cos x\, e^{3x} + 3\left(\operatorname{sen} x\, e^{3x} - 3 \underbrace{\int \operatorname{sen} x\, e^{3x} dx}_{I}\right)$$

$$\Longrightarrow I = (-\cos x + 3\operatorname{sen} x) e^{3x} - 9I$$

$$\Longrightarrow 10 I = (-\cos x + 3 \operatorname{sen} x) e^{3x} \Longrightarrow \boxed{I = \frac{(-\cos x + 3\operatorname{sen} x) e^{3x}}{10}}$$

$$\int_0^{\frac{\pi}{2}} \operatorname{sen} x\, e^{3x} dx = \left.\frac{(-\cos x + 3 \operatorname{sen} x) e^{3x}}{10}\right|_0^{\frac{\pi}{2}}$$

$$= \frac{(-\overbrace{\cos\left(\frac{\pi}{2}\right)}^{=0} + 3\overbrace{\operatorname{sen}\left(\frac{\pi}{2}\right)}^{=1}) e^{3\frac{\pi}{2}}}{10} - \frac{(\overbrace{-\cos(0)}^{=-1} + 3\overbrace{\operatorname{sen}(0)}^{=0}) e^0}{10}$$

$$= \frac{3e^{\frac{3}{2}\pi} + 1}{10} \longrightarrow \text{A}$$

MO 8:

$$\text{Area} = \int_{-3}^{0} (g(x)-f(x))\,dx = \int_{-3}^{0} g(x)\,dx - \int_{-3}^{0} f(x)\,dx = \text{I} - \text{II}$$

$$\text{I} = \int_{-3}^{0} \frac{10}{x+10}\,dx = 10\log(x+10)\Big|_{-3}^{0} = 10\log(10) - 10\log(\underbrace{-3+10}_{7})$$

$$\text{II} = \int_{-3}^{0} \frac{2x+16}{16-x^2}\,dx$$

$$\frac{2x+16}{16-x^2} = \frac{A}{4-x} + \frac{B}{4+x} = \frac{A(4+x)+B(4-x)}{16-x^2} = \frac{\overbrace{(A-B)}^{2\,=}x + \overbrace{4(A+B)}^{16\,=}}{16-x^2}$$

$$\begin{cases} A-B=2 \rightarrow B=A-2 \\ A+B=4 \rightarrow A+A-2=4 \rightarrow 2A=6 \rightarrow \boxed{A=3} \end{cases}$$

$$y\ B=3-2=1$$

$$\Rightarrow \text{II} = \int_{-3}^{0} \frac{3}{4-x} + \frac{1}{4+x}\,dx = -3\log(4-x) + \log(4+x)\Big|_{-3}^{0}$$

$$= \underbrace{-3\log(4)+\log(4)}_{-2\log(4)} + 3\log(\underbrace{4-(-3)}_{7}) - \underbrace{\log(\underbrace{4-3}_{1})}_{0\,=}$$

$$= -2\log(4) + 3\log(7)$$

Luego, $\text{I} - \text{II} = 10\log(10) - 10\log(7) - (-2\log(4) + 3\log(7))$

$$= 10\log(10) - 13\log(7) + 2\log(4) \longrightarrow \text{A}$$

Mo 9:

$$\lim_{x\to 0} \frac{\overbrace{e^x - x - 2 + \cos x - \frac{x^3}{6}}^{f(x)}}{x^4} = \lim_{x\to 0} \frac{P_4(f)(x) + R_4(f)(x)}{x^4}$$

$$= \lim_{x\to 0} \frac{P_4(f)(x)}{x^4} + \underset{0}{\underset{\shortparallel}{\frac{R_4(f)(x)}{x^4}}}$$

$$P_4(f)(x) = \left(\cancel{1} + \cancel{x} + \frac{\cancel{x^2}}{\cancel{2!}} + \frac{\cancel{x^3}}{\underset{6}{3!}} + \frac{x^4}{4!}\right) - \cancel{x} - \cancel{2} + \left(\cancel{1} - \frac{\cancel{x^2}}{\cancel{2!}} + \frac{x^4}{4!}\right) - \frac{\cancel{x^3}}{\cancel{6}}$$

$$= 2\frac{x^4}{4!}$$

$$\Rightarrow \lim_{x\to 0} \frac{P_4(f)(x)}{x^4} = \lim_{x\to 0} \frac{2\frac{\cancel{x^4}}{4!}}{\cancel{x^4}} = \frac{2}{4!} = \frac{\cancel{2}}{4\times 3\times \cancel{2}} = \frac{1}{12}$$

$\downarrow$

E