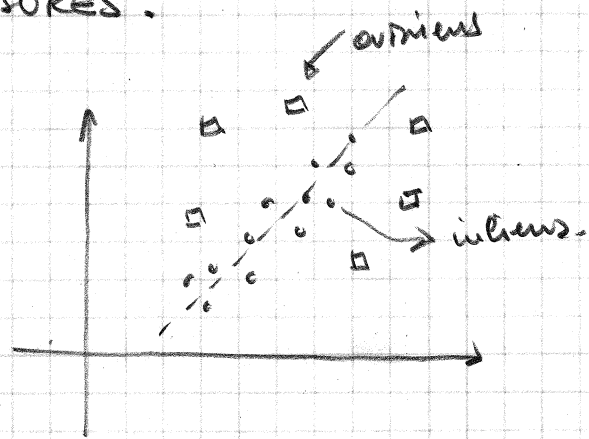


# CASE 6: RANSAC and ERROR MEASURES.



\* RANSAC:

Problem Definition

model  $x \rightarrow y$

$$f(x) = y$$

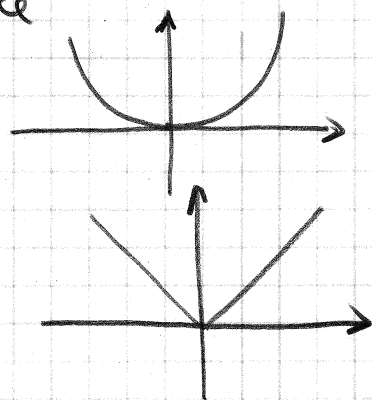
Empirical data i.e.  $(\tilde{x}_i, \tilde{y}_i)$  ← this input data may be (a) a point that fits the model corrupted with gaussian noise

$$\begin{aligned} \tilde{x}_i &= x_i + n_i \\ \tilde{y}_i &= y_i + n_i \end{aligned} / f(x_i) = y_i ; \text{ or } (b) \text{ it may be an outlier}$$

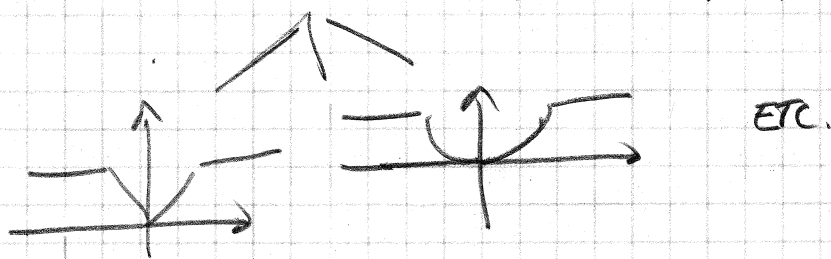
Error measurement.  $e(\tilde{x}_i, \tilde{y}_i)$ , for example

$$\| \cdot \|_2 \rightarrow e(x_i, y_i) = \| f(x_i) - y_i \|_2^2$$

$$\| \cdot \|_1 \rightarrow e(x_i, y_i) = |f(x_i) - y_i|$$



? more sophisticated penalty functions



RANSAC algorithm:

① Select a random set of  $\frac{1}{n}$  points  $\left( n = \text{min number that allows us to estimate } f \right)$

$$(\tilde{x}_1, \tilde{y}_1) \dots (\tilde{x}_n, \tilde{y}_n) \rightarrow (\tilde{x}_{p_1}, \tilde{y}_{p_1}), \dots, (\tilde{x}_{p_n}, \tilde{y}_{p_n}) \quad p_i \in \{1, \dots, n\}$$

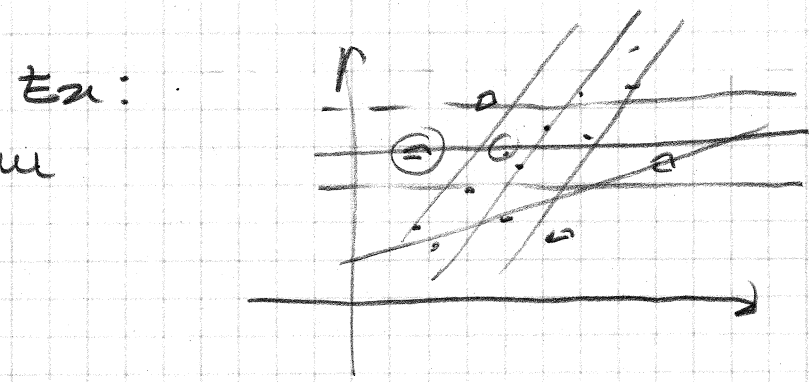
② Using this point fit the model:

$$f^* = \underset{f \in \mathcal{F}}{\text{argmin}} e(f(\tilde{x}_{p_n}), \tilde{y}_{p_n})$$

$\mathcal{F} \rightarrow \text{space of models.}$

measure how many of the remaining points fits the model

Repeat this steps N times and keep the model with more inliers.



and graph:  $(x_1, z_1) - (x_8, z_8) \rightarrow H \rightarrow \frac{x_H \times z'_H}{k} < th?$

Error MEASURE: (For example for H estimation)

algebraic error:  $\frac{x_H \times z'_H}{k} (=0)$  (for example if we multiply by an const)  
 easy ← is numerically what we aim to minimize.

problem: geometrically meaningless

advantage: numerically easy to implement and minimize

gold standard:  $E(H, \hat{x}) = d(x, \hat{x})^2 + d^2(x', H)^2$

problem: difficult to minimize, and we need to estimate  $\hat{x}$

advantage: geometrically meaningful

Sampson error: Taylor expansion around  $\hat{x}$  of gold standard error (see CASE 5 - add instead)