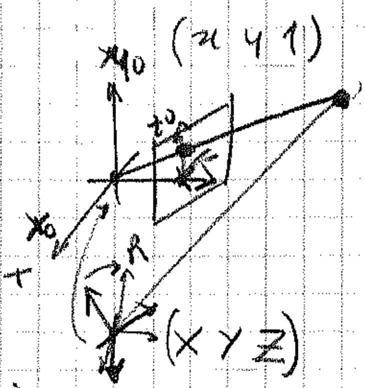
CLASE 3: CALIBRATION MATRIX: 9/3

- the last class we probed that the position in the camera plane (in homogeneous coordinates) and the position in the 3D world are related through:

$$Z\begin{pmatrix}u\\v\\1\end{pmatrix} = \underbrace{\begin{pmatrix}f & 0 & c_x\\ 0 & f & c_y\\ 0 & 0 & 1\end{pmatrix}}_{K}\underbrace{[\underbrace{R}_{3\times3}\ \underbrace{T}_{3\times1}]}_{x_0\ y_0\ z_0}\begin{pmatrix}X\\Y\\Z\\1\end{pmatrix}$$



K is called the internal calibration matrix (Depend on the camera)

$P \overset{\text{Def}}{=} K[R\ T]$ is called the projection matrix

To understand how this matrix can be estimated in practice, it is necessary to introduce the concept of HOMOGRAPHIES.

- HOMOGRAPHIES.

- Definition: An homography is a invertible mapping of P^2 on itself for which if x_1, x_2 and x_3 lie in a line $\Longleftrightarrow$ $h(x_1), h(x_2), h(x_3)$ do.

  - Theorem: $h: \mathbb{P}^2 \to \mathbb{P}^2$ is an homography if and only if there $\exists\, H \in \mathbb{R}^{3\times3}$ (non-singular) such that $\forall x \in \mathbb{P}^2$ $h(x) = Hx$.

(Proof $\Leftarrow$) $x_1, x_2, x_3 \in l_i \to l^T x_i = 0 \quad i=1,2,3$

H is no-singular $\Rightarrow \exists H^{-1}$. $\quad l^T x_i = \underbrace{l^T H^{-1}}_{l'^T} H x_i = 0$

$l'^T H x_i = 0$

$\Rightarrow Hx_i = x_i' \in l' \quad i=1,2,3.$

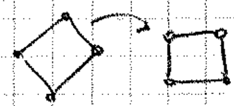
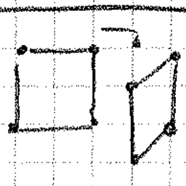
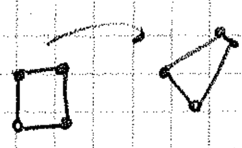
- interesting fact: lines are transformed as $\left(l' = H^{-T} l\right)$

• Property :

$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \overbrace{\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}}^{H} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$ H invertible.

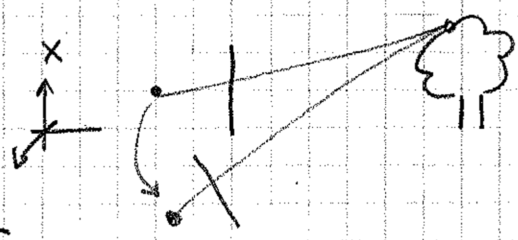
$\Rightarrow$ λH represents the same transformation as $\lambda H x = \lambda x' \overset{\mathbb{P}^2}{=} x'$

• GROUPS OF HOMOGRAPHIES •

GROUP	MATRIX	DISTORTION	INVARIANT PROPERTIES.
Euclidean 3 dof	$\begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		length, Area. (and all the following)
Similarity 4 dof	$\begin{bmatrix} s r_{11} & s r_{12} & t_x \\ s r_{21} & s r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Ratio of length, angles,
Affine 6 dof	$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Parallelism, Ratio of areas, The line at ∞ (l_∞),
Projective 8 dof	$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$		Concurrency, Colinearity, cross-ratio

• HOMOGRAPHY BETWEEN IMAGES •

$$x_1 = P_1 X = K[R\ T_1]\, X$$



(i) If we just rotate the camera and we use de center of coords as the one on the first camera position. then
$$x_1 = K\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \quad x_2 = K\left[R \,\middle|\, \begin{matrix}0\\0\\0\end{matrix}\right]\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = KR\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$\Rightarrow$ $\boxed{x_2 = K R K^{-1} x_1}$

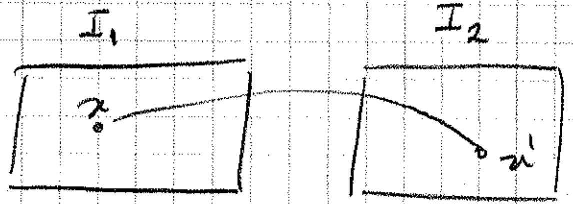
K is inv
R is inv $\Big\}$ $\Rightarrow$ $K R K^{-1} = H$ (3x3 inv. MATRIX)

(ii) When the scene we are looking at is flat. $(Z=0)$

$$x' = K\left[R_1\, R_2\, R_3\, T\right]\begin{pmatrix} X \\ Y \\ 0 \\ 1 \end{pmatrix} = \left[R_1\ R_2\ T\right]\begin{pmatrix} X \\ Y \\ 1 \end{pmatrix} := H\begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

• HOMOGRAPHY ESTIMATION • (e.g. for Panorama Construction)

Let us assume that two images are related through an hom...



and that we have established correspon... between the images $x \rightarrow x'$ and we want to estimate H / $H(x_i) = x'_i$ $\forall i: 1..N$.

If two points are related through the homography, we have:

$\lambda x' = Hx$ for some $\lambda \in \mathbb{R}$ $\Rightarrow$ x' and Hx should be colinear which can be mathematically stated as:

$x' \times Hx = 0$. This expression gives two independ... equations for each correspondence (x, x').

$\Rightarrow$ We need at least 4 correspondences to estima... an homography.

$$x'_i \times H x_i = \begin{pmatrix} x'_i \\ y'_i \\ 1 \end{pmatrix} \times \begin{pmatrix} h_{11} x_i + h_{12} y_i + h_{13} \\ h_{21} x_i + h_{22} y_i + h_{23} \\ h_{31} x_i + h_{32} y_i + h_{33} \end{pmatrix} = 0$$

$$\begin{vmatrix} i & j & k \\ \times & \times & \times \\ \times & \times & \times \end{vmatrix} : \left[y'_i(h_{31} x_i + h_{32} y_i + h_{33}) - (h_{21} x_i + h_{22} y_i + h_{23})\right]\hat{i}$$

$$\left[(h_{11} x_i + h_{12} y_i + h_{13}) - x'_i(h_{31} x_i + h_{32} y_i + h_{33})\right]\hat{j}$$

$$\begin{pmatrix} x_i & y_i & 1 & 0 & 0 & 0 & -x'_i x_i & -x'_i y_i & -x'_i \\ 0 & 0 & 0 & x_i & y_i & 1 & -y'_i x_i & -y'_i y_i & -y'_i \end{pmatrix} \begin{pmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$