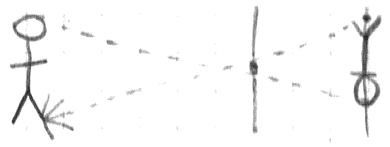
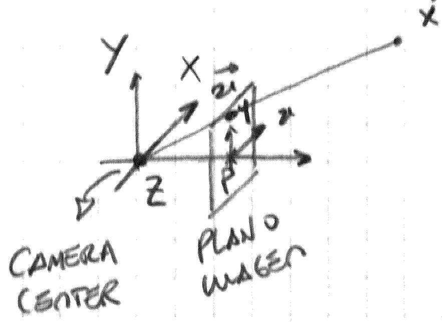
CLASE 2: CAMARA MODEL AND PROJECTIVE GEOMETRY

(0) **CAMERA**: TALK and show a real camera — **DO DEMO**

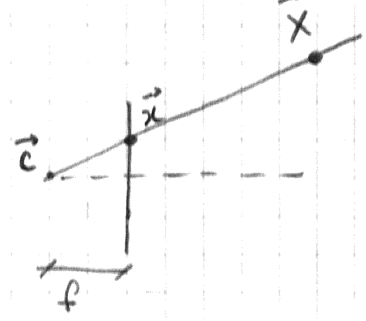
(I) **Modelo Pinhole**:





- Is an ideal model with an aperture reduced to a point.
- Doesn't account for: blur, nor for the lens distortion

- Central projection in camera coordinates:



$$\vec{Cx} = \lambda \vec{CX} \to \begin{pmatrix} x \\ y \\ f \end{pmatrix} = \lambda \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\to \lambda = \frac{f}{Z} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = f \begin{pmatrix} X/Z \\ Y/Z \end{pmatrix}$$

In pixel coordinates:

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \alpha x + c_x \\ \alpha y + c_y \end{pmatrix} = \begin{pmatrix} (\alpha f)\, X/Z + c_x \\ (\alpha f)\, Y/Z + c_y \end{pmatrix} \to \alpha f: \text{focal length in px}$$

$\begin{pmatrix} c_x \\ c_y \end{pmatrix}$: position of the principal point in px.

(II) **Projective Geometry in 2D**: $\mathbb{P}^2$

We identify two points in $\mathbb{R}^3$ on the same ray from the origin through the equivalence relation:

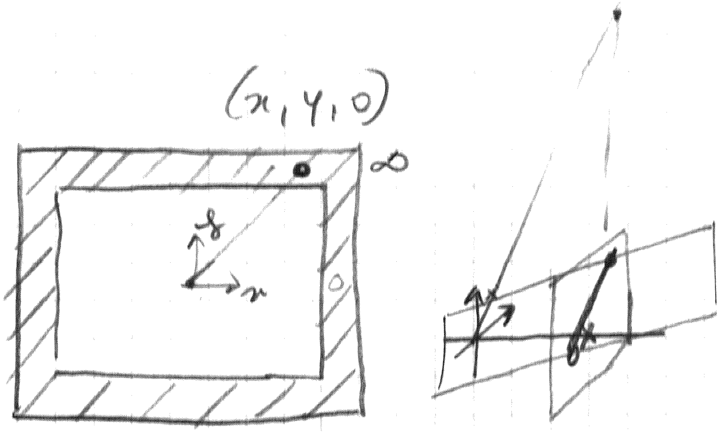
$R: \; x R y \iff \exists \lambda \neq 0 : x = \lambda y$.

$\mathbb{P}^2 \overset{\text{Def}}{=} (\mathbb{R}^3 \setminus 0)/R$, this means $(x \; y \; z) \overset{\mathbb{P}^2}{=} (x/z \;\; y/z \;\; 1)$

(III) **Properties of $\mathbb{P}^2$**:

- the point $(x \; y \; \varepsilon) = (\frac{x}{\varepsilon} \; \frac{y}{\varepsilon} \; 1)$ is a point "far away" in the direction of (y/x). $(x, y \; 0)$ represents that direction at ∞.

- A plane in $\mathbb{R}^3$ through the origin is represented by the equation:



$$aX + bY + cZ = 0$$

It represents a line in $\mathbb{P}^2$ expresed by $(a, b, c)^T$, whit the equation:

$$(a \; b \; c)(x \; y \; z)^T = 0$$

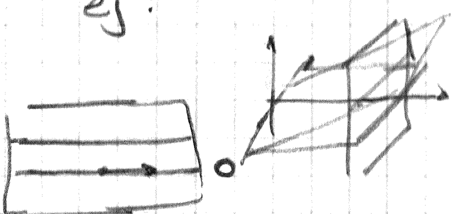
- Examples:

- line through x_1 and x_2 : $l = x_1 \times x_2$ since $(x_1 \times x_2)^T \cdot x_i = 0$

$$|x_1 \; x_2 \; x_i| = 0$$

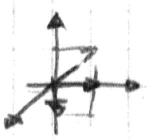
- Intersections of two lines: $x = l_1 \times l_2$

ej:



$y = 0 \to (0 \; 1 \; 0)$

$y - c = 0 \to (0 \; 1 \; -c)$



$x \in l_1 \cap l_2 \Rightarrow l_1 \times l_2 =$

$$\begin{pmatrix} -c \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$ se cortan en ∞ horizontal

$\hookrightarrow \begin{pmatrix} +1 \\ 0 \\ 0 \end{pmatrix}$

(IV) CALIBRATION MATRIX:

- Let us get back to the calibration equation:

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \alpha f X/Z + c_x \\ \frac{\alpha f Y}{Z} + c_y \end{pmatrix} \xRightarrow{\text{in } \mathbb{P}^2} Z\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha f & 0 & c_x \\ 0 & \alpha f & c_y \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

(we typically call $\alpha f =$ "f")

the 3D coordinate system can be expresed in a different orthonormal coordinate frame:

$$x = \begin{pmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{pmatrix} [R \; T] \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}_{\in \mathbb{P}^3}$$