chapter three

Structural design criteria

Contents

3.1 Modes of failure

Two basic modes of failure are assumed for the design of pressure vessels. These are: (a) elastic failure, governed by the theory of elasticity; and (b) plastic failure, governed by the theory of plasticity. Except for thick-walled pressure vessels, elastic failure is assumed. When the material is stretched beyond the elastic limit, excessive plastic deformation or rupture is expected. The relevant material properties are the yield strength and ultimate strength. In real vessels we have a multiaxial stress situation, where the failure is not governed by the individual components of stress but by some combination of all stress components.

3.2 Theories of failure

The most commonly used theories of failure are:

- Maximum principal stress theory
- Maximum shear stress theory

- Maximum distortion energy theory

According to the maximum principal stress theory, failure occurs when one of the three principal stresses reaches a stress value of elastic limit as determined from a uniaxial tension test. This theory is meaningful for brittle fracture situations.

According to the maximum shear stress theory, the maximum shear equals the shear stress at the elastic limit as determined from the uniaxial tension test. Here the maximum shear stress is one half the difference between the largest (say σ_1) and the smallest (say σ_3) principal stresses. This is also known as the Tresca criterion, which states that yielding takes place when

$$\frac{(\sigma_1 - \sigma_3)}{2} = \pm\frac{\sigma_{\mathrm{y}}}{2} \tag{3.1}$$

The distortion energy theory considers failure to have occurred when the distortion energy accumulated in the component under stress reaches the elastic limit as determined by the distortion energy in a uniaxial tension test. This is also known as the von Mises criterion, which states that yielding will take place when

$$\frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] = \pm\sigma_y \tag{3.2}$$

To understand the essential differences between the Tresca and von Mises criteria let us consider the simplified case of a biaxial stress state, where we assume that the principal stress, σ_3 is zero.

Let us first consider the case of Tresca criterion. We further assume that σ_1 and σ_2 have the same sign. Then, following Eq. (3.1), we have

$$|\sigma_1 - \sigma_3| = \sigma_{\mathrm{y}} \tag{3.3a}$$

or

$$|\sigma_2 - \sigma_3| = \sigma_{\mathrm{y}} \tag{3.3b}$$

This gives

$$\sigma_1 = \sigma_{\mathrm{y}}; \quad \sigma_1 = -\sigma_{\mathrm{y}}; \quad \sigma_2 = \sigma_{\mathrm{y}}; \quad \sigma_2 = -\sigma_{\mathrm{y}} \tag{3.4}$$

Next we that σ_1 and σ_2 are of the opposite sign. The yielding will then take place when

$$|\sigma_1 - \sigma_2| = \sigma_{\mathrm{y}} \tag{3.5}$$

This implies that

$$\sigma_1 - \sigma_2 = \sigma_y \tag{3.6a}$$

or

$$\sigma_2 - \sigma_1 = \sigma_y \tag{3.6b}$$

If Eqs. (3.4) and (3.6) are plotted with σ_1 as abscissa and σ_2 as the ordinate, then we get six straight lines (shown as dashed hexagon in Figure 3.1). The values of σ_1 and σ_2 falling on the hexagon and outside would cause yielding. We have of course assumed that the material yield strength is equal in magnitude when in tension or in compression.

Next we consider the von Mises criterion. With the assumption that $\sigma_3 = 0$, Eq. (3.2) gives

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_y^2 \tag{3.7}$$

This equation is plotted in the $\sigma_1 - \sigma_2$ plot as shown in the solid lines (forming an ellipse) in Figure 3.1. According to the von Mises criterion, the points falling on or outside of the ellipse would cause yielding.

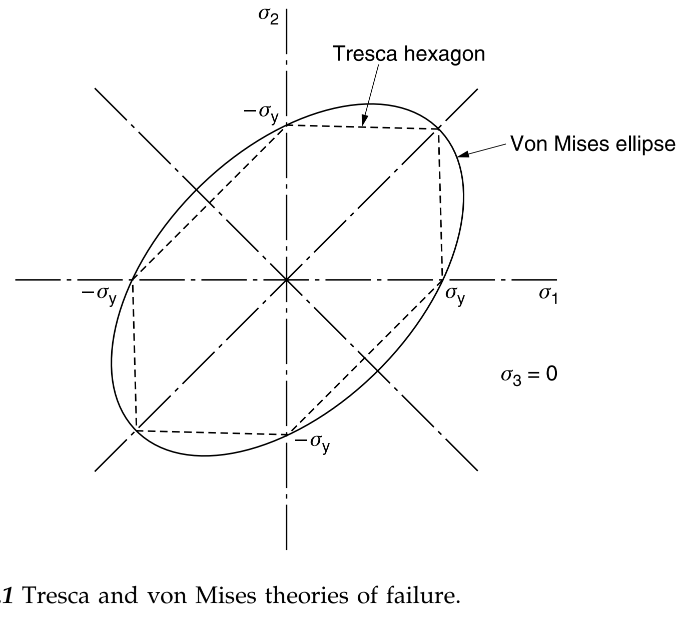

Figure 3.1 Tresca and von Mises theories of failure.

3.3 Theories of failure used in ASME Boiler and Pressure Vessel Code

Two basic theories of failure are used in the American Society of Mechanical Engineers (ASME) Boiler and Pressure Vessel Code, Section I, Section IV, Section III Division 1 (Subsections NC, ND, and NE), and Section VIII Division 1 use the maximum principal stress theory. Section III Division 1 (Subsection NB and the optional part of NC) and Section VIII Division 2 use the maximum shear stress theory or the Tresca criterion. The maximum principal stress theory (sometimes called Rankine theory) is appropriate for materials such as cast iron at room temperature, and for mild steels at temperatures below the nil ductility transition (NDT) temperature (discussed in Section 3.7). Although this theory is used in some design codes (as mentioned previously) the reason is that of simplicity, in that it reduces the amount of analysis, although often necessitating large factors of safety.

It is generally agreed that the von Mises criterion is better suited for common pressure vessels, the ASME Code chose to use the Tresca criterion as a framework for the design by analysis procedure for two reasons: (a) it is more conservative, and (b) it is considered easier to apply. However, now that computers are used for the calculations, the von Mises expression is a continuous function and is easily adapted for calculations, whereas the Tresca expression is discontinuous (as can be seen from Figure 3.1).

In order to avoid dividing both the calculated and the yield stress by two, the ASME Code defines new terms called stress intensity, and stress difference. The stress differences (S_{ij}) are simply the algebraic differences of the principal stresses, σ_1, σ_2, and σ_3, so that

$$S_{1,2} = \sigma_1 - \sigma_2, S_{2,3} = \sigma_2 - \sigma_3, S_{3,1} = \sigma_3 - \sigma_1 \tag{3.8}$$

The stress intensity, S, is the maximum absolute value of the stress difference

$$S = \max\left(\left|S_{1,2}\right|, \quad \left|S_{2,3}\right|, \quad \left|S_{3,1}\right|\right) \tag{3.9}$$

In terms of the stress intensity, S, Tresca criterion then reduces to

$$S = \sigma_{\mathrm{y}} \tag{3.10}$$

Throughout the design by analysis procedure in the ASME Code stress intensities are used.

3.4 Allowable stress limits in the ASME Boiler and Pressure Vessel Code

The overall objective in determining the allowable stress limits is to ensure that a pressure vessel does not fail within its established design life. The modes that are most likely to cause a failure, as identified by the ASME Code, are as follows:[2]

- Excessive elastic deformation including elastic instability
- Excessive plastic deformation
- Brittle fracture
- Stress rupture or creep deformation (inelastic)
- Plastic instability and incremental collapse
- High strain and low cycle fatigue
- Stress corrosion
- Corrosion fatigue

The first failure mode, namely that of excessive elastic deformation, is generally related to functional requirements. The aspect of elastic instability deals with the propensity of buckling in thin shells. The aspect of excessive plastic deformation could lead to complete collapse as outlined in the previous chapter. This failure mode requires that the analysis be addressed from the standpoint of bursting and gross distortion from a single load application. The failure mode associated with brittle fracture is related to the fracture toughness and is addressed later in this chapter. The failure mode associated with stress rupture or creep is appropriate for pressure vessels operating at high temperatures and as such will not be discussed here. The failure associate with plastic instability and incremental collapse was identified in the previous chapter as ratcheting causing progressive growth due to cyclic load application and should be addressed at the analysis stage. The high strain and low cycle fatigue is an important consideration for cyclic thermal loads. The crack initiation from fatigue damage should be addressed in the analysis. The failure modes associated with stress corrosion and corrosion fatigue are related to the environmental considerations as well as mode of operation.

The allowable stress limits in the ASME Code are established on two modes of failure and are characterized as:

- Avoidance of gross distortion or bursting
- Avoidance of ratcheting

In order for sustained loads to produce collapse in a structure, it is necessary that the loads produce full plasticity over the cross-section bearing the load, leading to what is commonly termed as the ''plastic hinge.'' The stresses they produce are designated primary stresses. The set of primary mean stresses (or primary membrane stresses), P_m, represent the sustained load acting on the structure divided by the cross-sectional area resisting the load. In fact P_m is the stress intensity derived from the stress distribution and as such is the difference between the largest and the smallest of the principal stresses. P_m determines the susceptibility of the structure to fail by plastic collapse. In order to avoid gross distortion it is necessary to avoid a significant portion of the wall of the vessel from becoming fully plastic. For an elastic–perfectly plastic stress strain law (Figure 3.2) such a vessel would be fully plastic when the membrane stress reaches the yield stress. A safety factor of 1.5 is provided to avoid this situation (see Figure 3.3 for the design limit for P_m/S_y). The allowable design stress (primary membrane) is therefore limited to a stress limit typically two-thirds of the yield (referred to as material allowable S_m).

Large bending moments acting over the full cross-section can also produce structural collapse. The set of bending stresses generated by sustained bending moments are termed primary bending stresses, P_b, and at any particular point in the structure, being the stress intensities, they represent the differences between the largest and the smallest values of the principal stresses. The mode of collapse is bending, as opposed to extension, and the collapse will take place only when there is complete plastic yielding of the net cross-section. The pattern of plasticity in this plastic hinge so formed, consists of part of the cross-section becoming plastic in tension and the remainder of the section becoming plastic in compression.

When there are both direct (membrane) as well as bending stresses, the avoidance of gross distortion or bursting in a vessel is treated in the same way as direct and bending stresses in a rectangular beam. If such a beam is loaded in bending, collapse does not occur until the load has been increased by a factor known as the ''shape factor'' of the cross-section when a plastic hinge is formed. The shape factor of a rectangular section in bending is 1.5.

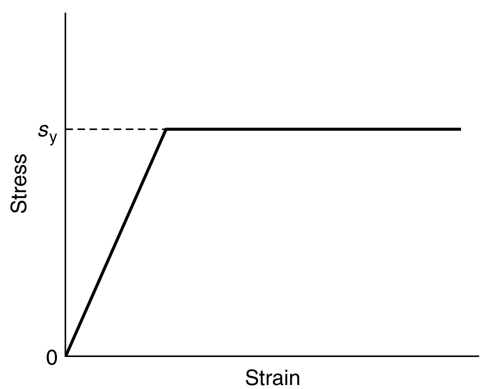

Figure 3.2 Strain–strain characteristics for an elastic–perfectly plastic material.

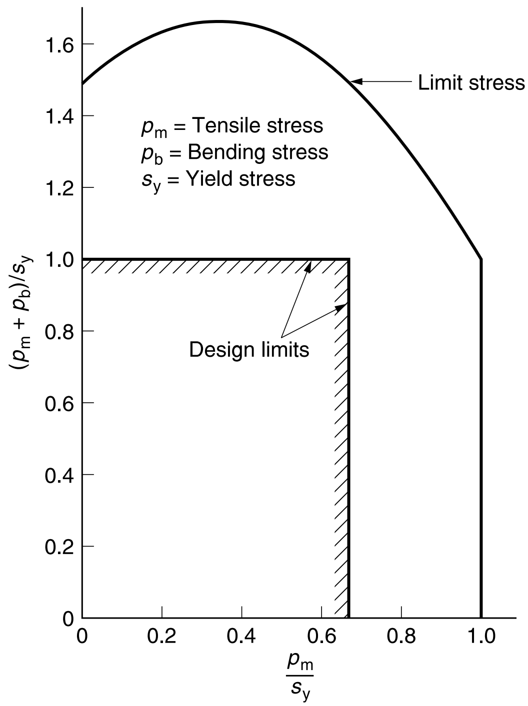

Figure 3.3 Membrane plus bending versus membrane stress for a rectangular beam.

When the primary stress in a rectangular section consists of a combination of bending and axial tension, the value of the limit load depends on the ratio between the tensile and bending loads. Figure 3.3 shows the value of the maximum calculated stress at the outer fiber of a rectangular section required to produce a plastic hinge plotted against the average tensile stress across the section, with both values expressed as multiples of the yield stress S_y. When the average tensile stress P_m is zero, the failure stress for bending is 1.5 S_y. The ASME Code limits the combination of the membrane and bending to the yield stress S_y. It can be seen from Figure 3.3 that there are variable margins depending on the particular combination of stresses, but it was decided to keep the design limits simple.

The repeated plastic straining or ratcheting is sometimes termed incremental collapse. If a structure is repeatedly loaded to progressively higher levels, one can imagine that at some highly stressed region a stage will be reached when the plastic strain will accumulate during each cycle of load, a situation that must be avoided. However, some initial plastic deformation is judged permissible during the first few cycles of load provided the structure shakes down to elastic behavior for subsequent loading cycles. Consider, for example, the outer fiber of a beam strained in tension to a value ε_1, somewhat beyond the yield strain as shown in Figure

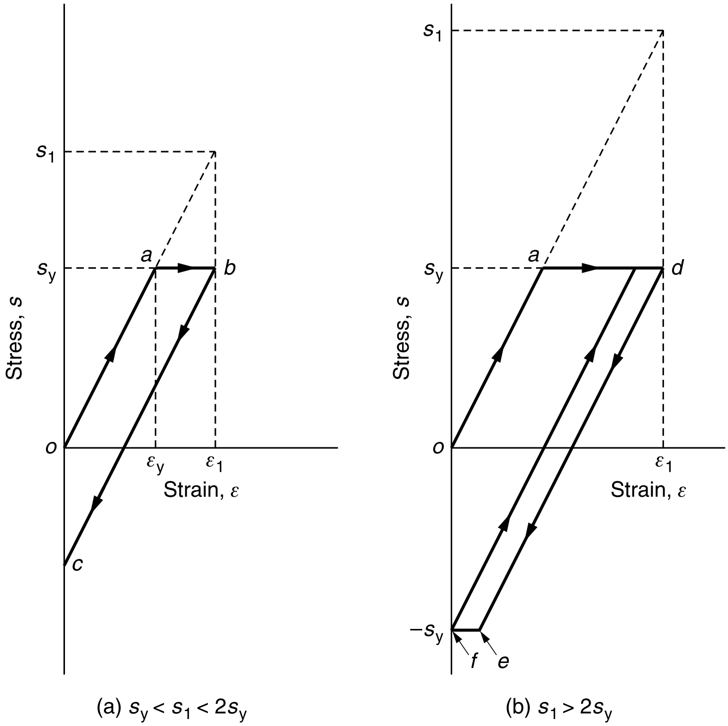

Figure 3.4 Ratcheting behavior.

3.4(a) by the path OAB. The calculated elastic stress would be $S = S_1 = E\varepsilon_1$. When the beam is returned to its undeformed position O, the outer fiber has a residual compressive stress of magnitude $S_1 - S_y$. On any subsequent loading, the residual compression must be removed before the stress goes into tension and thus the elastic stress range has been increased by the quantity $S_1 - S_y$. If $S_1 = 2S_y$, the elastic range becomes $2S_y$, but if $S_1 > 2S_y$, the fiber yields in compression, as shown by the line EF in Figure 3.4(b) and all subsequent cycles produce plastic strain. Therefore the limit of $2S_y$ could be regarded as a threshold beyond which some plasticity action would progress.

3.5 Service limits

The loading conditions that are generally considered for the design of pressure vessels include pressure, dead weight, piping reaction, seismic, thermal expansion and loadings due to wind and snow. The ASME Boiler and Pressure Vessel Code delineates the various loads in terms of the following conditions:

1. Design
2. Testing
3. Level A
4. Level B
5. Level C
6. Level D

Test conditions refer to the hydrostatic tests that are performed on the pressure vessel during its operating life. Level A service limits correspond to those of normal operating conditions. Level B service limits are sometimes referred to as ''upset'' conditions, and are those for which the component must withstand without sustaining damage requiring repair. Typically this includes the operating basis earthquake (OBE) and thermal transients for which the power level changes are on the order of 10 to 20 percent. Level C service limits constitute the emergency conditions in which large deformations in the area of discontinuity are created. Level D service limits are so called faulted conditions, for which gross deformation with a loss of dimensional stability is permitted. The component may require repair or removal. Examples are safe shutdown earthquake (SSE), pipe break or a combination of such events.

Specifically for the ASME Code, the primary membrane stress intensity, P_m, and the combined membrane plus bending stress intensity, $P_m + P_b$, (also the local membrane plus bending stress intensity, $P_L + P_b$ in some cases) for the various loading conditions are shown below.

1. Design condition:

$$
\begin{aligned}
P_m &\leq S_m \\
P_m + P_b &\leq 1.5S_m
\end{aligned} \tag{3.11}
$$

2. Testing condition:

$$
\begin{aligned}
P_m &\leq 0.9S_y \\
P_m + P_b &\leq 1.35S_y, \text{ for } P_m \leq 0.67S_y \\
P_m + P_b &\leq (2.15S_y - 1.2P_m) \text{ for } 0.67S_y \leq P_m \leq 0.9S_y
\end{aligned} \tag{3.12}
$$

3. Level C condition (emergency):

$$
\begin{aligned}
P_m &\leq S_y \\
P_L + P_b &\leq 1.5S_y, \text{ for } P_m \leq 0.67S_y \\
P_L + P_b &\leq (2.5S_y - 1.5P_L) \text{ for } P_L > 0.67S_y
\end{aligned} \tag{3.13}
$$

4. Level D condition (faulted):

$$\begin{aligned} P_{\mathrm{m}} &\leq \text{ lesser of } 0.7S_{\mathrm{u}} \text{ and } 2.4S_{\mathrm{m}} \\ P_{\mathrm{m}} + P_{\mathrm{b}} &\leq \text{ lesser of } 1.05S_{\mathrm{u}} \text{ and } 3.6S_{\mathrm{m}} \end{aligned} \tag{3.14}$$

In the following chapter, the above limits have been critically appraised by introducing the shape factor of the cross-section. The above limits are strictly applicable for rectangular cross-sections. The new limits have been proposed and discussed.

3.6 Design for cyclic loading

Due to loads that are applied in a cyclic fashion the material can fail by fatigue when sufficient cycles of loading are applied. The number of cycles that will cause fatigue failure depends on the magnitude of strain that is incurred during each cycle of loading. Fatigue data are generally obtained at room temperature and plotted in the form of nominal stress amplitude (one half of stress range) versus number of cycles to failure. The stress range is obtained by multiplying the strain range from the fatigue test by the modulus of elasticity. The endurance limit is defined as the cyclic stress amplitude, which will not cause fatigue failure regardless of the number of applied cycles of stress. However, for pressure vessels sometimes the endurance limit and one-million cycle fatigue limit are used interchangeably. Pressure vessel codes commonly use a factor of safety of 2 on the fatigue stress and a safety factor of 20 on fatigue life (number of cycles to failure). The design for cyclic loading is performed to check whether a pressure vessel designed statically will not fail due to multiple stress cycling. The process entails:

1. Identifying design details which introduce stress concentrations and therefore potential sites for fatigue failure
2. Identifying cyclic (or repeated) stresses experienced during service
3. Using appropriate *S–N* curves and deducing design life.

The concept of cumulative damage factor is a simple yet reliable method to determine the factor of safety against fatigue failure. If N_i denotes the allowable number of cycles corresponding to a stress range S_i, then the usage factor U_i at the material point due to n_{i} applied number of cycles of stress range S_i is

$$U_i = \frac{n_i}{N_i} \tag{3.15}$$

If the material is subjected to m different cycles of frequency n_i and corresponding to stress ranges S_i ($I = 1, 2, \ldots m$), then the cumulative damage factor, U, is given by

$$U = \sum_{i=1}^{m} U_i = \sum_{i-1}^{m} \frac{n_i}{N_i} \tag{3.16}$$

Safety from fatigue failure requires

$$U \leq 1 \tag{3.17}$$

The ASME design fatigue curves are based on strain controlled data in which the best fit curves are constructed by a factor of 2 on stress or a factor of 20 on cycles to account for environment, size effect, and data scatter.

3.7 Protection against fracture

Pressure vessel materials are primarily steels, and the main point of concern is the effect of temperature on the fracture toughness of steel. Steels are generally ductile, but their resistance to brittle fracture diminishes as the temperature is lowered. The lower limit of the operating temperature is therefore determined by the transition point at which there is a change from ductile to brittle fracture. The value of the stress at fracture under those situations can be considerably lower than the yield strength. The fracture properties including the transition temperature depend on the composition, heat treatment, prior cold work, and the size of the flaws that may be present. As the carbon content is increased from 0.1 to 0.8 percent, the NDT (nil ductility transition) temperature increases from –45°C to +50°C. Small amounts of manganese or niobium can produce large decrease in transition temperature. The four design criteria for mild steels can be summarized as follows:

1. NDT design criterion: The maximum principal stress should not exceed 34.5 MPa, to assure fracture arrest at temperatures below NDT temperature.
2. NDT +17°C design criterion: The temperature of operation must be maintained above an NDT of +17°C, to assure that brittle fracture will not take place at stress levels up to one half the yield strength.
3. NDT +33°C design criterion: The temperature of operation must be maintained above an NDT of +33 °C, to assure that brittle fracture will not take place at stress levels up to the yield strength.
4. NDT +67°C design criterion: The temperature of operation must be maintained above an NDT of +67°C, to assure that brittle fracture will not take place at any stress level.

The margin of safety from brittle fracture is therefore dependent on the stress level as well as the expected minimum temperature of operation. Some design codes use a single margin of safety criterion based on energy absorption in a Charpy test conducted at the minimum expected temperature of operation.[1]

References

1. Burgreen, D., *Design Methods for Power Plant Structures*, C. P. Press, 1975.
2. Anon., Criteria of the ASME Boiler and Pressure Vessel Code for Design by Analysis in Sections III and VIII, Division 2, American Society of Mechanical Engineers, New York.

Problems

1. The in-plane normal stresses in a flat plate are 10 MPa and 60 MPa and the shear stress is 30 MPa. Find the stress intensity and the von Mises equivalent stress. What is the factor of safety corresponding to (a) Tresca criterion, and (b) von Mises criterion if the material yield strength is 150 MPa?
2. The in-plane stresses in a flat plate are –50 MPa and –150 MPa on two perpendicular planes and a shear stress of 40 MPa on those planes. Compute the maximum shear stress, the stress intensity and the von Mises equivalent stress. What is the factor of safety corresponding to (a) Tresca criterion, and (b) von Mises criterion if the material yield strength is 200 MPa?
3. The hoop stress in a cylindrical shell with closed ends is pR/t and the longitudinal stress is $pR/(2t)$, where p is the internal pressure, R the mean radius and t the thickness. If the shell is of diameter 0.5 m and a thickness of 12.5 mm, and is subjected to an internal pressure of 7 MPa, determine the maximum shear stress, the stress intensity and the von Mises equivalent stress. What is the factor of safety corresponding to (a) Tresca criterion, and (b) von Mises criterion if the material yield strength is 160 MPa?
4. A carbon steel pressure vessel is subjected to 1000 pressure cycles at an alternating stress of 300 MPa. At this alternating stress the number of cycles to failure is 7000 from the design fatigue curve. Subsequently the vessel is subjected to 400 temperature cycles at an alternating stress of 700 MPa for which the number of cycles to failure is 600 from the fatigue curve. Is the vessel adequate for the given cyclic loading?