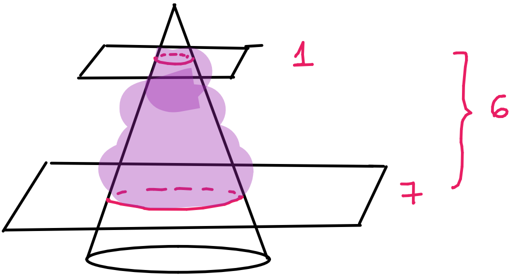
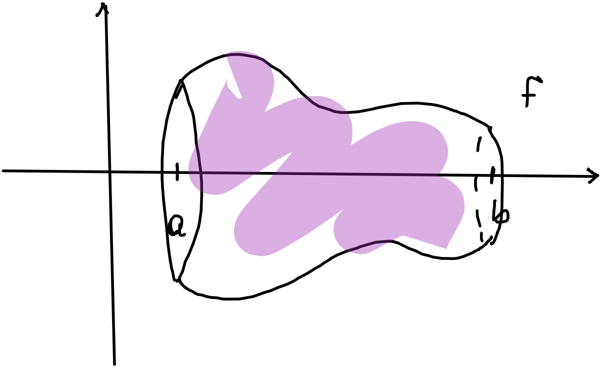
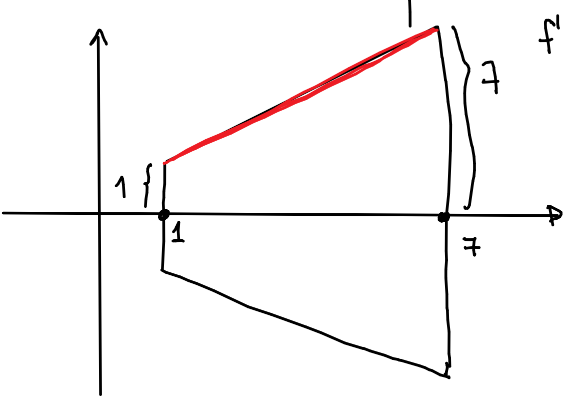
Ejercicio 7, dic 19.





→ $f(x)=x$



$f'(x) = 1$

$$\int_1^7 2\pi f(x)\sqrt{1+f'(x)^2}\,dx =$$

$$= \int_1^7 2\pi x\sqrt{2}\,dx = 2\pi\sqrt{2}\,\frac{x^2}{2}\Big|_1^7 =$$

$$= 2\pi\sqrt{2}\underbrace{\left(\frac{49}{2}-\frac{1}{2}\right)}_{\frac{48}{2}=24} = \overset{48}{\cancel{24}}\sqrt{2}\pi$$

Resp. 6.

Ej. 4, feb. 2023

entre $-\frac{\pi}{2}$ y $\frac{\pi}{2}$

entre -2 y 2

$$f : [0, +\infty) \longrightarrow \mathbb{R}$$

$$f(x) = \begin{cases} \frac{4}{\pi} \operatorname{Arctan}\left(\frac{2x}{1-x^2}\right) & x \neq 1 \\ \alpha & x = 1 \end{cases}$$

Para ver ①, tenemos que saber si

$$\exists \lim_{x \to 1} f(x).$$

Cuando $x \to 1$

$$2x \to 2$$

$$1 - x^2 \longrightarrow 0$$

$$\lim_{x \to 1^-} 1 - x^2 = 0^+$$

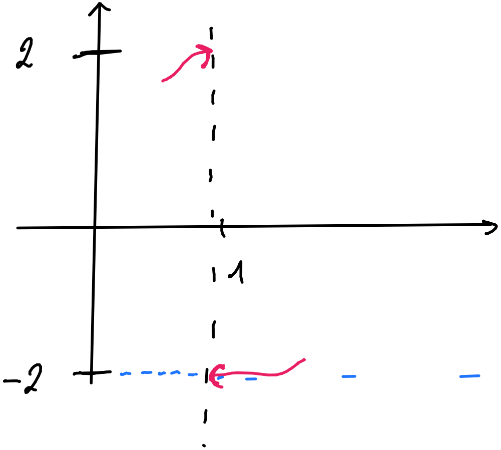
$$\lim_{x \to 1^+} 1 - x^2 = 0^-$$

$$\lim_{x \to 1^{\mp}} \frac{2x}{1-x^2} = \pm \infty$$

$$\lim_{x \to 1^{\mp}} \operatorname{Arctan}\left(\frac{2x}{1-x^2}\right) = \pm \frac{\pi}{2}$$

$\lim_{x \to 1^{\mp}} f(x) = \pm 2$ ← Este límite no existe.

Existen los límites laterales y son distintos.



① Existe $\alpha \in \mathbb{R}$ tal que f es continua para ese valor de α. FALSA.

$$f(x) = \begin{cases} \frac{4}{\pi} \operatorname{Arctan}\left(\frac{2x}{1-x^2}\right) & x \neq 1 \\ \alpha & x = 1 \end{cases}$$

$$\left.\begin{aligned} f(0) &= 0 \\ \lim_{x\to 1^-} f(x) &= 2 \end{aligned}\right]$$

$$f'(x) = \frac{4}{\pi} \frac{1}{1+\left(\frac{2x}{1-x^2}\right)^2} \cdot \frac{2(1-x^2)+2x\cdot 2x}{(1-x^2)^2} =$$

$$= \frac{4}{\pi} \frac{2x^2-2}{\left[1+\left(\frac{2x}{1-x^2}\right)^2\right](1-x^2)^2}$$

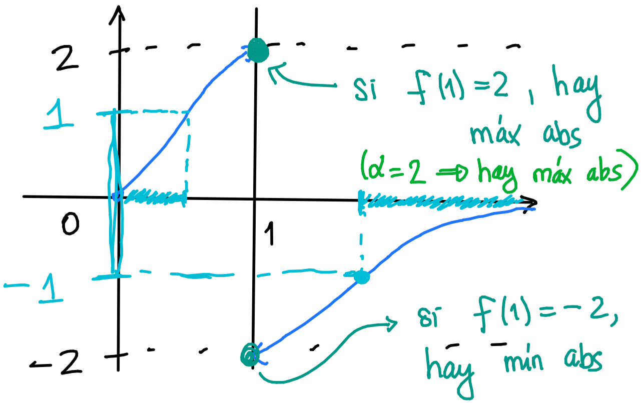
$2x^2-2=0 \Longleftrightarrow x=1$ (para $x\geq 0$)

$f'(x) \neq 0$ en $[0,1) \cup (1,+\infty)$

En $[0,1)$, la función f es creciente.

En $(1,+\infty)$, como $f' \neq 0$, va a ser creciente o decreciente. Como $f(x) \geq -2$ en $(1,+\infty)$ y $\lim_{x\to 1^+} f(x) = -2$, f crece en $(1,+\infty)$.

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{4}{\pi}\,\operatorname{Arctg}\left(\underset{\substack{\downarrow\\0}}{\frac{2x}{1-x^2}}\right) = 0$$



si $f(1)=2$, hay máx abs

($\alpha=2 \Rightarrow$ hay máx abs)

si $f(1)=-2$, hay mín abs

② Existe $\alpha\in\mathbb{R}$ t.q. f tiene tanto máximo como mín abs en $[0,+\infty)$ FALSA

③ $[-1,1]$ está contenido en el recorrido de f para todo α. VERDADERA

$$[0,2) = f\big([0,1)\big) \Rightarrow [0,2) \subset f\big([0,+\infty)\big)$$

$$\Rightarrow [0,1] \subset f\big([0,+\infty)\big)$$

$$(-2,0) = f\big((1,+\infty)\big) \Rightarrow [-1,0) \subset f\big([0,+\infty)\big)$$

Entonces $[-1,0) \cup [0,1] = [-1,1] \subset f\big([0,+\infty)\big)$

Ej 4, 2do sem 2017, parcial 2

$$f(x) = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

$$f'(x) = \frac{1}{2}\left(e^{x} + e^{-x}\right) > 0 \text{ siempre.}$$

$\Rightarrow f$ es estrictamente creciente en $\mathbb{R}$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1}{2}\Big(\underset{\substack{\downarrow\\ 0}}{e^{x}} - \underbrace{e^{-x}}_{\substack{\downarrow\\ +\infty}}\Big) = -\infty$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} f(x) = +\infty$$

$f : \mathbb{R} \to \mathbb{R}$ es biyectiva.

$f^{-1}: \mathbb{R} \to \mathbb{R}$

$$f \circ f^{-1}(x) = x \xrightarrow{\text{Derivo}}$$

$$(f \circ f^{-1})'(x) = 1$$

$$f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1$$

$$\boxed{(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}}$$

$f'(x) = \cosh(x)$

$$(f^{-1})'(x) = \frac{1}{\cosh(\operatorname{arcsenh}(x))} =$$

$$= \frac{1}{\sqrt{1 + \underline{\operatorname{senh}^2(\operatorname{arcsenh}(x))}}} =$$

$=$

$\downarrow$

$\cosh^2(x) - \operatorname{senh}^2(x) = 1$

$$= \frac{1}{\sqrt{1+x^2}}$$

$$\cosh(x) = \sqrt{1+\operatorname{senh}^2(x)}$$ | G.

Ej 2, 2020, 17/8

$$f(x) = \int_x^{e^x} \underbrace{\left(\operatorname{sen}^3(\pi t) + 2\cos(\pi t)\right)(1+t-t^2)}_{=g(t)}\, dt$$

$$= \int_0^{e^x} g(t)\,dt + \int_x^0 g(t)\,dt =$$

$$= \underbrace{\int_0^{e^x} g(t)\,dt}_{\Downarrow} - \underbrace{\int_0^x g(t)\,dt}$$

Por TFC, la derivada de esto es $g(x)$.

$$F(x) = \int_0^x g(t)\,dt$$

$$\int_0^{e^x} g(t)\,dt = F(e^x)$$

$$\left(\int_0^{e^x} g(t)\,dt\right)' = F'(e^x)\cdot e^x =$$

$$= g(e^x)\cdot e^x$$

$$f'(x) = g(e^x)e^x - g(x)$$

$$f'(0) = g(1)\cdot 1 - g(0)$$

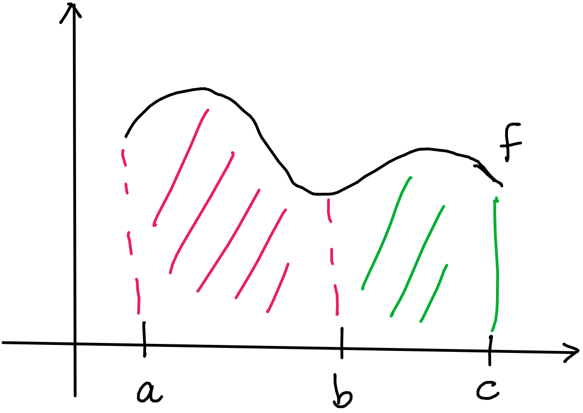
$$g(t) = \left(\operatorname{sen}^3(\pi t) + 2\cos(\pi t)\right)(1+t-t^2)$$

$$g(0) = 2$$

$$g(1) = -2$$

$$f'(0) = g(1) - g(0) = -2-2 = -4$$

$$\boxed{d}$$



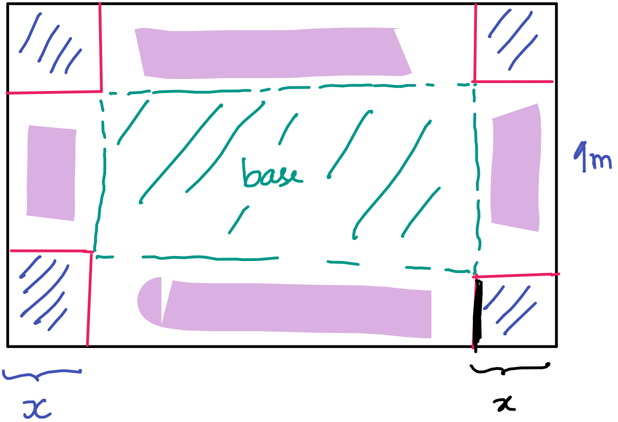
$$\underline{\int_a^b f = \int_a^c f + \int_c^b f}$$

Es siempre cierta sin importar la posición relativa de a,b,c.

$$\left(\int_{h(x)}^{g(x)} f(t)dt\right) = f(g(x)).g'(x) - f(h(x)).h'(x)$$

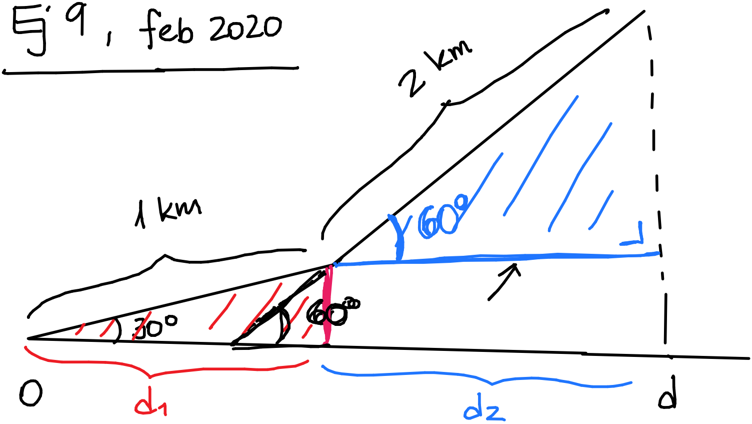
Ej 8, simulacro 2do parcial 2023.

2m



$$\mathrm{Vol} = (2-2x)(1-2x)x$$

Ej 9, feb 2020



Rojo:

$$\cos(30^\circ) = \frac{d_1}{1} = d_1$$

$$\cos(60^\circ) = \frac{d_2}{2} \Rightarrow d_2 = 2\cos(60^\circ) \quad \boxed{5}$$

Ej 5, feb 2018

$f: \mathbb{R} \to \mathbb{R}$ continua

$$\lim_{x\to 0} \frac{f(x) - f(-x)}{x} = a$$

$$\frac{f(x) - f(0) + f(0) - f(-x)}{x} =$$

Sabemos que ∃.

$$\frac{f(x) - f(0)}{x} + \frac{f(0) - f(-x)}{x} =$$

$$\underset{①}{\frac{f(x) - f(0)}{x}} + \underset{②}{\frac{f(-x) - f(0)}{-x}}$$

El lím ① existe sii el lím ② existe.

Si f es derivable en 0

$$\lim_{x\to 0} \frac{f(x)-f(0)}{x} = \underbrace{\lim_{x\to 0} \frac{f(-x)-f(0)}{-x}}_{\text{si } u=-x\text{, esto es } \lim\limits_{u\to 0} \frac{f(u)-f(0)}{u}}$$

$$\lim_{x\to 0} \frac{f(x)-f(-x)}{x} = 2\lim_{x\to 0} \frac{f(x)-f(0)}{x} =$$

$$= 2f'(0)$$

Si f es derivable en 0, $a = 2f'(0)$.

Obs. $f(x) = |x|$

No es derivable en 0.

$$\lim_{x\to 0} \frac{f(x)-f(-x)}{x} = \lim_{x\to 0} \frac{\overset{0}{\boxed{|x|-|-x|}}}{x}$$

$$= \lim 0 = 0.$$

$x \to 0$

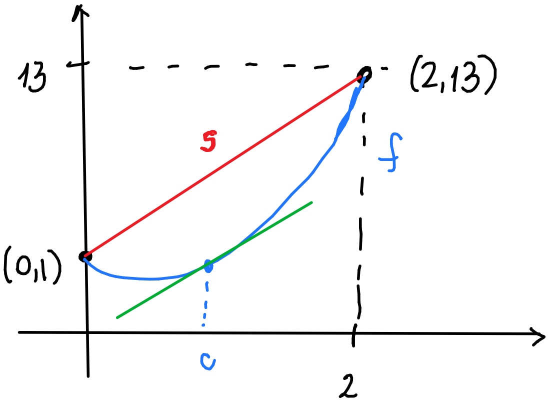
Existe este límite y no existe

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x} . \qquad \boxed{B}$$

Ej 8, 2020 agosto matutino

$f : [0,2] \longrightarrow \mathbb{R}$

$f(x) = x^2 + 4x + 1 \; , \; f(0) = 1, \; f(2) = 4 + 8 + 1 = 13$



Pendiente de s:

$$\frac{f(2)-f(0)}{2-0} = 6$$

Calculemos c:

Sabemos $f'(c) = 6$

$f(x) = x^2+4x+1$, $f'(x) = 2x+4$

$2c+4 = 6 \Longleftrightarrow c = 1$

$f(c) = 6$

Buscamos la recta de pendiente 6 que pasa por $(1,6)$. $\boxed{A}$

Ej 2, julio 2018

$$f(x) = \frac{x+2}{x^2+4} \quad , \quad x \in [-2,2]$$

$$f'(x) = \frac{x^2+4-2x(x+2)}{(x^2+4)^2} =$$

$$= \frac{-x^2-4x+4}{(x^2+4)^2}$$

$$-x^2-4x+4=0 \iff$$

$$x = \frac{4\pm\sqrt{16+16}}{-2} = \boxed{-2\pm2\sqrt{2}}$$

$$f'(x)=0 \iff x=-2+2\sqrt{2}$$

Por el tma de Weierstrass, f tiene máximo y mín. abs en $[-2,2]$.

Éstos pueden darse en $-2, 2$ o $-2+2\sqrt{2}$

$$f(-2)=0, \quad f(2)=\frac{4}{8}=\frac{1}{2}$$

$$f(-2+\sqrt{2}) = \frac{-\cancel{2}+2\sqrt{2}+\cancel{2}}{(-2+2\sqrt{2})^2+4} =$$

$$= \frac{2\sqrt{2}}{4+8-8\sqrt{2}+4} = \frac{2\sqrt{2}}{2(8-4\sqrt{2})} =$$

$$= \frac{\sqrt{2}}{4(2-\sqrt{2})}$$

Como $f(-2) \geq 0$, $f(2) \geq 0$,

$f(-2+2\sqrt{2}) \geq 0$, el mínimo es $f(-2)=0$.

$(m=0)$.

El máximo es

$$\frac{1}{2} \text{ o } \frac{\sqrt{2}}{4(2-\sqrt{2})}$$

$$\frac{1}{2} > \frac{\sqrt{2}}{4(2-\sqrt{2})} \iff$$

$$4(2-\sqrt{2}) > \sqrt{2}\cdot 2 \iff$$

$8 > \sqrt{2} \cdot 6 \iff 64 > 2 \cdot 36$

NO.

Entonces $M = \dfrac{\sqrt{2}}{4(2-\sqrt{2})}$.

Ⓐ