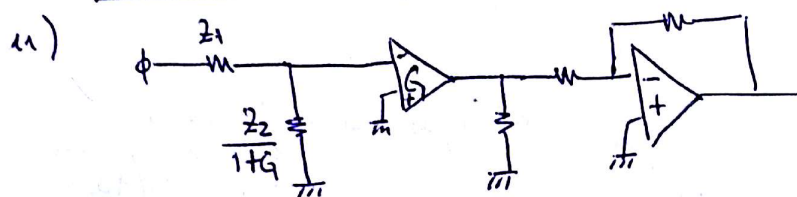Sistemas Lineales 2

Ex Feb 2017

Ej3)

a) i)

$$\begin{cases} \dot{x} = A x(t) + B u(t) \\ y = C x(t) \end{cases}$$

Estado $x = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}$ Entrada $u(t) = F(t)$

Salida $y = z$

$$A = \begin{bmatrix} 0 & 1 \\ -U/M & -B/M \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/M \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$H(s) = \frac{1}{Ms^2 + Bs + U}$$

ii)

$$Z_{in} = Z_1 + \frac{Z_2}{1+G}$$

$$Z_1 = \frac{R_1}{R_1 C_1 s + 1} \quad, \quad Z_2 = \frac{R_2}{R_2 C_2 s + 1}$$

iii) $\quad C_1(s) = \dfrac{R_2}{R_1} \dfrac{R_b}{R_a} \dfrac{R_1 C_1 s + 1}{R_2 C_2 s + 1}$ $\quad \left(C(s) = k \cdot C_1(s) \right)$

b) $A = 0,01 \, m^2$, $M = 1 \, kg$, $B = 1 \, \dfrac{Nseg}{m}$, $U = 10 \, \dfrac{N}{m}$, $R_1 C = 10 \, seg$, $\dfrac{R_2}{R_1} = 0,5$
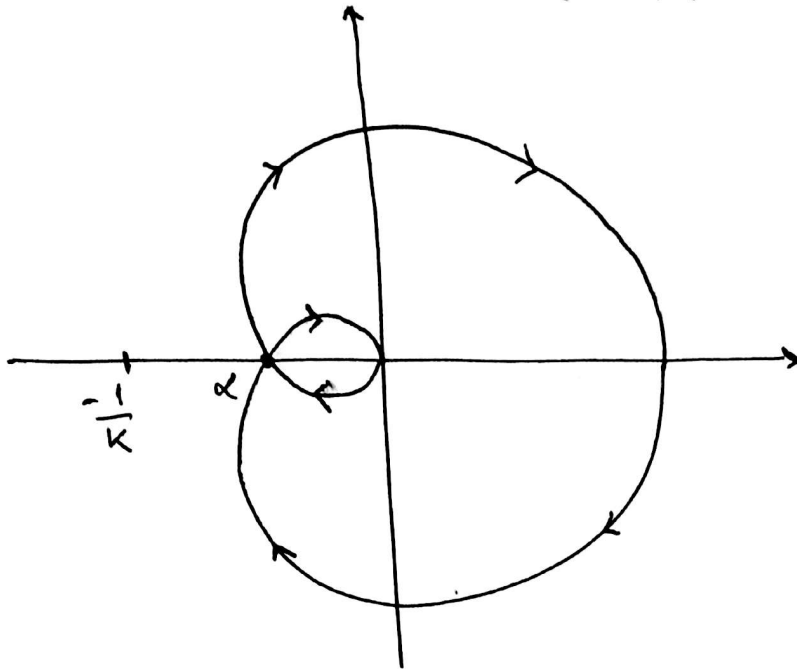
$R_a = 1 \, k\Omega$, $R_b = 100 \, k\Omega$

$$H_{ce} = \frac{A \cdot C(s) \cdot H_B(s) \cdot H(s)}{1 + A C(s) \cdot H_B(s) \cdot H(s)} = \frac{\dfrac{k}{(s+0,2)(s^2+s+10)}}{1 + \dfrac{k}{(s+0,2)(s^2+s+10)}} = \frac{k}{(s+0,2)(s^2+s+10)+k}$$

$\forall k$ / sistema estable: $\displaystyle \lim_{t \to \infty} z(t) = \lim_{s \to 0} s \, Z(s) = \frac{k}{2+k} \cdot R_o$

$$\left(Z(s) = \frac{R_o}{s} \cdot H_{ce} \right)$$

i) Error en régimen $\quad e = \left(1 - \dfrac{k}{2+k} \right) R_o = \dfrac{2}{2+k} R_o$

Para minimizar el error debemos elegir el mayor k que permite estabilidad.

b) ii) Nyquist $\qquad$ $L(s) = \dfrac{1}{(s+0,2)(s^2+s+10)}$

Debe ser $\alpha > -\dfrac{1}{k}$

para tener estabilidad.

$$L(j\omega) = \dfrac{1}{[0,2(10-\omega^2)-\omega^2]+j\omega(10,2-\omega^2)} \Rightarrow \operatorname{Im}\{L(j\omega)\}=0 \ \text{si} \left.\begin{array}{l} \omega=0 \\ \omega=\pm\sqrt{10,2} \end{array}\right.$$

$$\omega_1 = \sqrt{10,2} \simeq 3,19 \Rightarrow |L(j\omega_1)| = \left|\dfrac{1}{0,2(10-\omega_1^2)-\omega_1^2}\right| \simeq 0,098$$

$\alpha \simeq -0,098 \Rightarrow MG = 10,24 \qquad$ Estable si $0<k<10,24$

$\alpha > -\dfrac{1}{k}$

Elegimos $\underline{k=10}$ para minimizar error