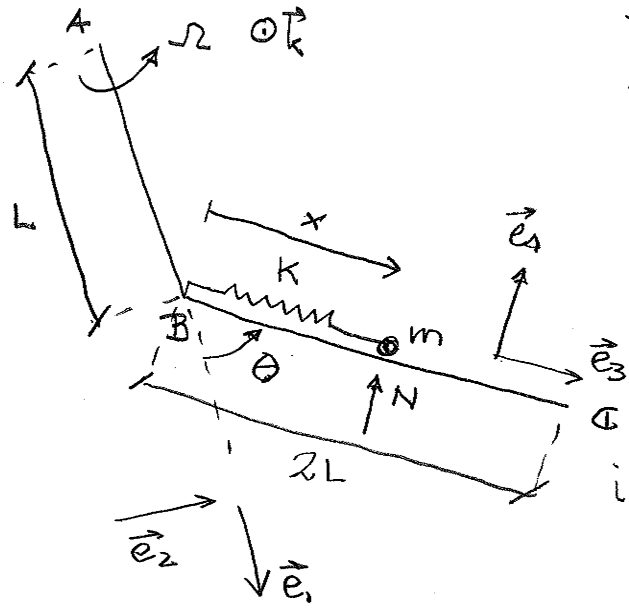
Ejercicio N° 1

Mecánica Newtoniana (1122)
Primer Parcial (29/4/2023)



$\vec{\Omega} = \Omega \vec{k}$, $\Omega > 0$

A fijo.

$N \geq 0$

$K = 2m\Omega^2$

$l_0 = 0$

$g = 0$

Por adición velocidades angulares o derivando ángulo $\Omega t + \theta(t)$

parte a: i) $\vec{\omega}_1 = (\Omega + \dot{\theta})\vec{k}$

ii) $S = \{B, \vec{e}_3, \vec{e}_4, \vec{k}\}$

$\vec{r}\,' = x\vec{e}_3$

$\boxed{\vec{v}\,' = \dot{x}\vec{e}_3}$

$\vec{v}_T = \underbrace{\vec{v}_B}_{= L\Omega\vec{e}_2} + \vec{\omega}_1 \wedge \vec{r}\,'$

$\vec{\omega}_1 = (\Omega + \dot{\theta})\vec{k}$

$\vec{\omega}_1 \wedge \vec{r}\,' = x(\Omega+\dot{\theta})\underbrace{\vec{k}\wedge\vec{e}_3}_{\vec{e}_4} = x(\Omega+\dot{\theta})\vec{e}_4$

$\boxed{\vec{v}_T = L\Omega\vec{e}_2 + x(\Omega+\dot{\theta})\vec{e}_4}$

$\boxed{\vec{v} = \dot{x}\vec{e}_3 + L\Omega\vec{e}_2 + x(\Omega+\dot{\theta})\vec{e}_4}$

$\boxed{\vec{a}\,' = \ddot{x}\vec{e}_3}$

$\vec{a}_C = 2\vec{\omega}_1\wedge\vec{v}\,' = 2(\Omega+\dot{\theta})\dot{x}\,\vec{k}\wedge\vec{e}_3 = \boxed{2(\Omega+\dot{\theta})\dot{x}\,\vec{e}_4 = \vec{a}_C}$

$\vec{a}_T = \underbrace{\vec{a}_B}_{= -L\Omega^2\vec{e}_1} + \dot{\vec{\omega}}_1\wedge\vec{r}\,' + \vec{\omega}_1\wedge[\vec{\omega}_1\wedge\vec{r}\,']$

$\dot{\vec{\omega}}_1 = \ddot{\theta}\vec{k}$

$\dot{\vec{\omega}}_1\wedge\vec{r}\,' = x\ddot{\theta}\,\vec{k}\wedge\vec{e}_3 = x\ddot{\theta}\vec{e}_4$

$\vec{\omega}_1\wedge[\vec{\omega}_1\wedge\vec{r}\,'] = x(\Omega+\dot{\theta})^2\,\underbrace{\vec{k}\wedge[\underbrace{\vec{k}\wedge\vec{e}_3}_{\vec{e}_4}]}_{-\vec{e}_3} = -x(\Omega+\dot{\theta})^2\vec{e}_3$

$\boxed{\vec{a}_T = -L\Omega^2\vec{e}_1 + x\ddot{\theta}\vec{e}_4 - x(\Omega+\dot{\theta})^2\vec{e}_3}$

$\boxed{\vec{a} = \left[\ddot{x} - x(\Omega+\dot{\theta})^2\right]\vec{e}_3 + \left[x\ddot{\theta} + 2(\Omega+\dot{\theta})\dot{x}\right]\vec{e}_4 - L\Omega^2\vec{e}_1}$

Otro sistema relativo: $\{A, \vec{e}_1, \vec{e}_2, \vec{k}\}$

$\vec{r}\,' = L\vec{e}_1 + x\vec{e}_3$

$\vec{v}\,' = \dot{x}\vec{e}_3 + x\underbrace{\frac{d'\vec{e}_3}{dt}}_{= \dot{\theta}\vec{e}_4} = \boxed{\dot{x}\vec{e}_3 + x\dot{\theta}\vec{e}_4 = \vec{v}\,'}$

$$\vec{v}_T = \underset{\overset{\shortparallel}{0}}{\vec{v}_A} + \vec{\omega}_2 \wedge \vec{r}\,'$$

$$\vec{\omega}_2 = \vec{\Omega} = \Omega \vec{k}$$

$$\vec{\omega}_2 \wedge \vec{r}\,' = \Omega \vec{k} \wedge \left(L\vec{e}_1 + x\vec{e}_3 \right) = \boxed{L\Omega \vec{e}_2 + x\Omega \vec{e}_4 = \vec{v}_T}$$

$$\boxed{\vec{v} = \dot{x}\vec{e}_3 + x(\dot{\theta}+\Omega)\vec{e}_4 + L\Omega\vec{e}_2}$$ Mismo resultado de antes

$$\vec{a}\,' = \ddot{x}\vec{e}_3 + \dot{x}\underbrace{\frac{d'\vec{e}_3}{dt}}_{\dot{\theta}\vec{e}_4} + \dot{x}\dot{\theta}\vec{e}_4 + x\ddot{\theta}\vec{e}_4 + x\dot{\theta}\underbrace{\frac{d'\vec{e}_4}{dt}}_{-\dot{\theta}\vec{e}_3} =$$

$$\boxed{\vec{a}\,' = \left(\ddot{x} - x\dot{\theta}^2\right)\vec{e}_3 + \left(2\dot{x}\dot{\theta} + x\ddot{\theta}\right)\vec{e}_4}$$

$$\vec{a}_C = 2\vec{\omega}_2 \wedge \vec{v}\,' = 2\Omega\vec{k} \wedge \left(\dot{x}\vec{e}_3 + x\dot{\theta}\vec{e}_4\right) = \boxed{2\Omega\dot{x}\vec{e}_4 - 2\Omega x\dot{\theta}\vec{e}_3 = \vec{a}_C}$$

$$\vec{a}_T = \underset{\overset{\shortparallel}{0}}{\vec{a}_A} + \underset{\overset{\shortparallel}{0}}{\dot{\vec{\omega}}_2} \wedge \vec{r}\,' + \vec{\omega}_2 \wedge \left[\vec{\omega}_2 \wedge \vec{r}\,'\right] = \Omega^2 \vec{k}\left[\vec{k} \wedge \left(L\vec{e}_1 + x\vec{e}_3\right)\right] =$$

$$= \Omega^2 \vec{k}\left[L\vec{e}_2 + x\vec{e}_4\right] = \Omega^2\left(-L\vec{e}_1 - x\vec{e}_3\right)$$

$$\boxed{\vec{a}_T = -\Omega^2\left(L\vec{e}_1 + x\vec{e}_3\right)}$$

$$\vec{a} = \left(\ddot{x} \underbrace{- x\dot{\theta}^2 - 2\Omega x\dot{\theta} - x\Omega^2}_{-x(\dot{\theta}+\Omega)^2}\right)\vec{e}_3 + \left[2\dot{x}\left(\dot{\theta}+\Omega\right) + x\ddot{\theta}\right]\vec{e}_4 - L\Omega^2\vec{e}_1$$

Mismo resultado de antes

Otro sistema relativo: $\{B, \vec{e}_1, \vec{e}_2, \vec{k}\}$

$$\vec{r}\,' = x\vec{e}_3 \Rightarrow \vec{v}\,' = \dot{x}\vec{e}_3 + x\underbrace{\frac{d'\vec{e}_3}{dt}}_{\dot{\theta}\vec{e}_4} = \boxed{\dot{x}\vec{e}_3 + x\dot{\theta}\vec{e}_4 = \vec{v}\,'}$$

$$\vec{v}_T = \underset{\overset{\shortparallel}{L\Omega\vec{e}_2}}{\vec{v}_B} + \underset{\overset{\shortparallel}{\Omega\vec{k}}}{\vec{\omega}_3} \wedge \vec{r}\,' = \boxed{L\Omega\vec{e}_2 + x\Omega\vec{e}_4 = \vec{v}_T}$$

Mismos $\vec{v}\,'$ y $\vec{v}_T$ caso anterior
$\Rightarrow \vec{v}$ será el mismo

$$\boxed{\vec{a}\,' = \left(\ddot{x} - x\dot{\theta}^2\right)\vec{e}_3 + \left(2\dot{x}\dot{\theta} + x\ddot{\theta}\right)\vec{e}_4}$$ Misma $\vec{a}\,'$ de antes

$$\vec{a}_C = 2\vec{\omega}_3 \wedge \vec{v}\,' = \boxed{2\Omega\dot{x}\vec{e}_4 - 2\Omega x\dot{\theta}e_3 = \vec{a}_C}$$

x q' es mismo $\vec{\omega}$ y $\vec{v}\,'$ de caso anterior

$$\vec{a}_T = \underset{\overset{\shortparallel}{-L\Omega^2\vec{e}_1}}{\vec{a}_B} + \underset{\overset{\shortparallel}{0}}{\dot{\vec{\omega}}_3} \wedge \vec{r}\,' + \underbrace{\vec{\omega}_3 \wedge \left[\vec{\omega}_3 \wedge \vec{r}\,'\right]}_{x\Omega^2 \vec{k} \wedge (\underbrace{\vec{k}\wedge\vec{e}_3}_{\vec{e}_4}) = -x\Omega^2\vec{e}_3}$$

$$\boxed{\vec a_T = -L\Omega^2 \vec e_1 - x\Omega^2 \vec e_3}$$

Misma $\vec a_T$ de caso anterior $\Rightarrow$ $\vec a$ será el mismo

iii) $\vec r = L\vec e_1 + x\vec e_3$

$$\vec v = L\underbrace{\dot{\vec e}_1}_{=\Omega \vec e_2} + \dot x\vec e_3 + x\underbrace{\dot{\vec e}_3}_{=(\Omega+\dot\theta)\vec e_4} = \boxed{L\Omega\vec e_2 + x(\Omega+\dot\theta)\vec e_4 + \dot x\vec e_3 = \vec v}$$

Mismo resultado parte anterior

$$\vec a = L\Omega\underbrace{\dot{\vec e}_2}_{=-\Omega\vec e_1} + \ddot x\vec e_3 + \dot x\underbrace{\dot{\vec e}_3}_{=(\Omega+\ddot\theta)\vec e_4} + \dot x(\Omega+\dot\theta)\vec e_4 + x\ddot\theta\vec e_4 + x(\Omega+\dot\theta)\underbrace{\dot{\vec e}_4}_{=-(\Omega+\dot\theta)\vec e_3} =$$

$$\boxed{\vec a = -L\Omega^2\vec e_1 + \left[\ddot x - x(\Omega+\dot\theta)^2\right]\vec e_3 + \left[2\dot x(\Omega+\dot\theta) + x\ddot\theta\right]\vec e_4}$$

Mismo resultado parte anterior

parte b: $\dot\theta(t) = \Omega = cte$

$\theta(0) = \frac{\pi}{2}$

$x(0) = L$

$\dot x(0) = 0$

$\dot x(t) = 0 \Rightarrow |T| \leq \mu|\vec N|$ $\quad \vec T = T\vec e_3 \quad \vec N = N\vec e_4 + B\vec k$

$$\underbrace{m\vec a}_{=-L\Omega^2\vec e_1 - L4\Omega^2\vec e_3} = T\vec e_3 + N\vec e_4 + B\vec k + \underbrace{\vec F_{res}}_{=-Kx\vec e_3}$$

$\vec e_1(0) = -\vec e_4$

Proyecto según $\vec e_3 \Rightarrow -mL4\Omega^2 = T - \overbrace{K}^{2m\Omega^2}L \Rightarrow T = -2mL\Omega^2$

" " $\vec e_4 \Rightarrow mL\Omega^2 = N$

" " $\vec k \Rightarrow B = 0$

$\Rightarrow 2mL\Omega^2 \leq \mu mL\Omega^2 \Rightarrow \boxed{\mu \geq 2}$

parte c: $\theta(t) = \frac{\pi}{2}$ cte $\quad \vec T = 0$

$\dot x(0) = 0 \quad \Rightarrow \dot\theta(t) = 0,\ \ddot\theta = 0$

$x(0) = L$

Nuevamente $\vec e_1 = -\vec e_4$

$$\vec a = (\ddot x - x\Omega^2)\vec e_3 + (L\Omega^2 + 2\dot x\Omega)\vec e_4$$

según $\vec{e}_3$: $m(\ddot{x} - x\Omega^2) = -\underset{\substack{\parallel \\ 2m\Omega^2}}{K}x \Rightarrow \ddot{x} + x\Omega^2 = 0$

" $\vec{e}_4$: $m(L\Omega^2 + 2\dot{x}\Omega) = N$

$\downarrow$

$x(t) = A\cos\Omega t + B\,\mathrm{sen}\,\Omega t$

$x(0) = L = A$

$\dot{x}(t) = -L\Omega\,\mathrm{sen}\,\Omega t + B\Omega\cos\Omega t$

$\dot{x}(0) = 0 \Rightarrow B = 0 \Rightarrow x(t) = L\cos\Omega t$

$\dot{x}(t) = -L\Omega\,\mathrm{sen}\,\Omega t$

$N(t) = mL\Omega^2(1 - 2\,\mathrm{sen}\,\Omega t) \geq 0$

$\mathrm{sen}\,\Omega t \leq \frac{1}{2}$

$\Omega t \leq \frac{\pi}{6} \Rightarrow \boxed{t_{desp} = \frac{\pi}{6\Omega}}$

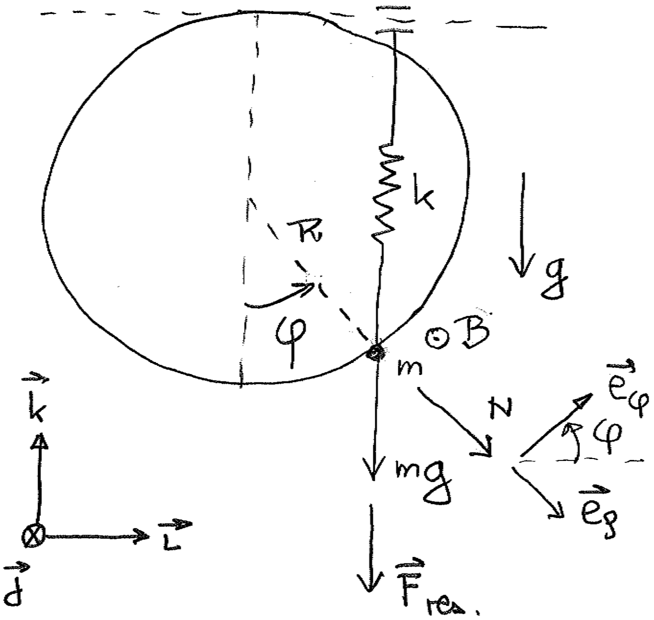
$x(t_{desp}) = L\cos\frac{\pi}{6} = \frac{\sqrt{3}\,L}{2} < 2L \Rightarrow$ la partícula no se escapa de la barra AB porque $x(t)$ disminuye

¿Y la partícula tampoco llega al extremo B?

$x(t) = 0 \Rightarrow \cos\Omega t = 0 \Rightarrow \Omega t = \frac{\pi}{2}$

$t_B = \frac{\pi}{2\Omega} > t_{desp}$.

Ejercicio N° 2



Guía lisa $\Rightarrow \vec{T} = 0$

$$\vec{F}_{res} = -k(\ell - \underset{0}{\ell_0})(-\vec{k}) = k\ell\vec{k}$$

$$\ell = R + R\cos\varphi$$

Vínculo Bilateral: $N \gtrless 0$

$$kR = mg$$

parte a:

1) Reacción de la guía lisa:

$$\vec{R} = N\vec{e}_\rho - B\vec{\jmath}$$

$$\vec{r} = R\vec{e}_\rho$$

$$\vec{v} = R\dot{\varphi}\vec{e}_\varphi$$

$\vec{R}\cdot\vec{v} = 0 \Rightarrow$ Reacción de Potencia Nula

2) Peso $\Rightarrow$ conservativo: $U_g = mgz \qquad z = \vec{r}\cdot\vec{k} = R\underbrace{\vec{e}_\rho\cdot\vec{k}}_{-\cos\varphi} = -R\cos\varphi$

$$\Rightarrow \boxed{U_g = -mgR\cos\varphi}$$

3) Resorte $\Rightarrow$ conservativo: $U_k = \dfrac{k\ell^2}{2} = \dfrac{k(R + R\cos(\varphi))^2}{2}$

$$\boxed{U_k = \frac{kR^2(1+\cos(\varphi))^2}{2}}$$

No hay fuerzas residuales $\Rightarrow$ sistema conservativo.

parte b: Sist. conservativo $\Rightarrow$ se conserva $E = T + U$

$$T = \frac{m\vec{v}^2}{2} = \frac{mR^2\dot{\varphi}^2}{2} \qquad U = -mgR\cos\varphi + \frac{kR^2(1+\cos(\varphi))^2}{2}$$

$$\frac{mR^2\dot{\varphi}^2}{2} - mgR\cos\varphi + \frac{kR^2(1+\cos(\varphi))^2}{2} = E$$

Ec. de mov: $mR^{\cancel{2}}\cancel{\dot{\varphi}}\ddot{\varphi} + mg\cancel{R}\,\text{sen}\varphi\,\cancel{\dot{\varphi}} + kR^{\cancel{2}}(1+\cos(\varphi))(-\text{sen}\varphi\,\cancel{\dot{\varphi}}) = 0$

$$\boxed{mR\ddot{\varphi} + mg\,\text{sen}\varphi - kR(1+\cos\varphi)\,\text{sen}\varphi = 0}$$

Otra forma: Newton: $m\vec{a} = N\vec{e}_\rho - B\vec{\jmath} - mg\vec{k} + kR(1+\cos(\varphi))\vec{k}$

$$\vec{a} = R\ddot{\varphi}\vec{e}_\varphi - R\dot{\varphi}^2\vec{e}_\rho$$

Proyecto según $\vec{e}_\varphi$: $mR\ddot{\varphi} = -mg\underbrace{\vec{k}\cdot\vec{e}_\varphi}_{\text{sen}\varphi} + kR(1+\cos(\varphi))\underbrace{\vec{k}\cdot\vec{e}_\varphi}_{\text{sen}\varphi}$

Mismo resultado

parte b:

Equilibrio: $\frac{\partial U}{\partial \varphi} = 0$

$$\frac{\partial U}{\partial \varphi} = mgR\,\text{sen}\,\varphi - \underbrace{kR}_{mg}{}^{2}(1+\cos\varphi)\,\text{sen}\,\varphi$$

$$\frac{\partial U}{\partial \varphi} = mg\,\text{sen}\,\varphi\,(1-1-\cos\varphi) = -mg\,\text{sen}\,\varphi\cos\varphi$$

1) $\text{sen}\,\varphi = 0 \Rightarrow \boxed{\varphi = 0 \text{ o } \pi}$

2) $\cos\varphi = 0 \Rightarrow \boxed{\varphi = \pm\frac{\pi}{2}}$

$$\frac{\partial^2 U}{\partial \varphi^2} = -mg\left[\cos^2\varphi - \text{sen}^2\varphi\right]$$

1) $\varphi = 0, \pi \quad \frac{\partial^2 U}{\partial \varphi^2} = -mg < 0 \Rightarrow$ Inestable

2) $\varphi = \pm\frac{\pi}{2} \quad \frac{\partial^2 U}{\partial \varphi^2} = mg > 0 \Rightarrow$ Estable

parte d: $\varphi(0) = \frac{\pi}{2}$

$$R\dot\varphi(0) = v_0 \Rightarrow \dot\varphi(0) = \frac{v_0}{R}$$

$$\frac{mR^2\dot\varphi^2}{2} \underbrace{- mgR\cos\varphi + \frac{kR^2(1+\cos\varphi)^2}{2}}_{-mgR\cos\varphi + \frac{mgR}{2}(1+2\cos\varphi+\cos^2\varphi)} = \frac{mv_0^2}{2} + \underbrace{\frac{kR^2}{2}}_{\frac{mgR}{2}}$$

$$\frac{\cancel{m}R^2\dot\varphi^2}{\cancel{2}} = \frac{\cancel{m}v_0^2}{\cancel{2}} - \frac{\cancel{m}gR}{\cancel{2}}\cos^2\varphi \geq 0$$

$v_0^2 \geq gR\cos^2\varphi \Rightarrow$ Retroceso en $\boxed{\cos\varphi_{ret} = \pm\frac{v_0}{\sqrt{gR}}}$

Para que de una vuelta entera $|\cos\varphi_{ret}| \geq 1$

$$\boxed{|v_0| > \sqrt{gR}}$$

parte e: $N \geq 0$

Proyecto ec. de Newton según $\vec{e}_\rho$

$$-mR\dot{\varphi}^2 = N - mg\underbrace{\vec{k}\cdot\vec{e}_\rho}_{-\cos\varphi} + \underbrace{kR}_{=mg}(1+\cos\varphi)\underbrace{\vec{i}\cdot\vec{e}_\rho}_{-\cos\varphi}$$

$$N = mg(\cancel{1}+\cos\varphi)\cos\varphi - \cancel{mg\cos\varphi} - R\dot{\varphi}^2 m$$

$$N = mg\cos^2\varphi - R\dot{\varphi}^2 m$$

$$\left.\begin{array}{l} \varphi(0) = 0 \\ \dot{\varphi}(0) = \frac{v_1}{R} \end{array}\right\} \Rightarrow N(0) = mg - \frac{v_1^2}{R} m$$

Hay desprendimiento $N < 0 \Rightarrow \boxed{g < \frac{v_1^2}{R}} \Rightarrow \boxed{|v_1| > \sqrt{gR}}$