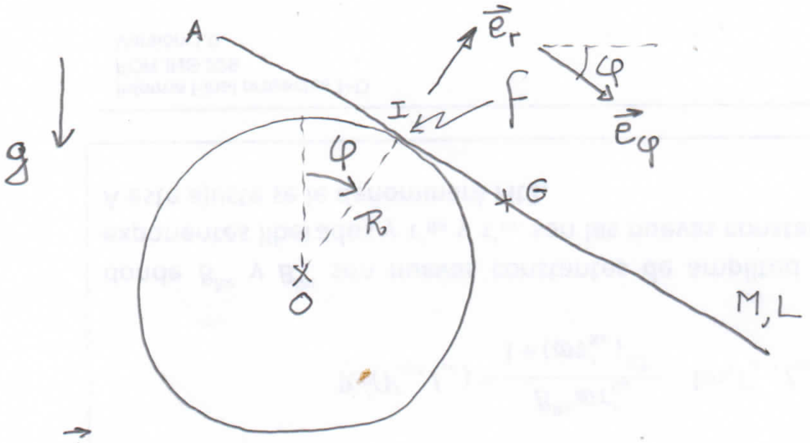
Ejercicio Nº1:

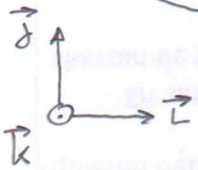
Segundo Parcial (10/7/2021)
Mecánica Newtoniana (1122)



$\varphi(0)=0$

$AI(0)=\frac{L}{4}$

$\omega(0)=\omega_0$



parte a:

$$\vec{r}_G = G-O = \underbrace{G-I}_{d\vec{e}_\varphi} + \underbrace{I-O}_{R\vec{e}_r}$$

$$G-I = d\vec{e}_\varphi = \underbrace{G-A}_{\frac{L}{2}\vec{e}_\varphi} + \underbrace{A-I}_{-|AI|\vec{e}_\varphi}$$

$$|AI| = \frac{L}{4}+R\varphi \Rightarrow d = \frac{L}{4}-R\varphi$$

$$\boxed{\vec{r}_G = \left(\frac{L}{4}-R\varphi\right)\vec{e}_\varphi + R\vec{e}_r}$$

$$\vec{v}_G = -R\dot\varphi\,\vec{e}_\varphi + \left(\frac{L}{4}-R\varphi\right)\underbrace{\dot{\vec{e}}_\varphi}_{-\dot\varphi\vec{e}_r} + R\underbrace{\dot{\vec{e}}_r}_{\dot\varphi\vec{e}_\varphi} \Rightarrow \boxed{\vec{v}_G = -\left(\frac{L}{4}-R\varphi\right)\dot\varphi\,\vec{e}_r}$$

parte b: ¿Sistema Conservativo?

Fuerzas: 1) Fuerzas Internas Barra: $P^{(int)}=0$

2) Peso: $U = Mg\,y_G$

$$y_G = \vec{r}_G\cdot\vec{j} = -\left(\frac{L}{4}-R\varphi\right)\operatorname{sen}\varphi + R\cos\varphi$$

3) Reacción en I: $P^{(reacc)} = \vec{R}\cdot\underbrace{\vec{v}_I}_{0} + \underbrace{\vec{M}_I}_{0}\cdot\vec{\omega} = 0$

x rodadura   fuerza aplicada en I.

$\Rightarrow T+U=E$

$$T = \frac{1}{2}M\underbrace{\vec{v}_G^2}_{\left(\frac{L}{4}-R\varphi\right)^2\dot\varphi^2} + \frac{1}{2}\vec{\omega}\cdot\mathbb{I}_G\vec{\omega}$$

$$\underbrace{I_{G\vec{k}}}_{} = \frac{ML^2}{12}$$

$$\vec{\omega} = -\dot\varphi\vec{k} \Rightarrow \vec{\omega}\cdot\mathbb{I}_G\vec{\omega} = \dot\varphi^2\,\vec{k}\cdot\mathbb{I}_G\vec{k} = \frac{ML^2}{12}\dot\varphi^2$$

$$\Rightarrow \left[\frac{M}{2}\left(\frac{L}{4}-R\varphi\right)^2 + \frac{ML^2}{24}\right]\dot\varphi^2 + Mg\left[R\cos\varphi - \left(\frac{L}{4}-R\varphi\right)\operatorname{sen}\varphi\right] = E$$

$$\frac{M}{\cancel{2}}\left(\frac{L}{4}-R\varphi\right)\left(-R\dot\varphi^3\right) + \left[\frac{M}{\cancel{2}}\left(\frac{L}{4}-R\varphi\right)^2 + \frac{ML^2}{12}\right]\dot\varphi\ddot\varphi +$$

$$+ Mg\left[\cancel{-R\operatorname{sen}\varphi\,\dot\varphi} + \cancel{R\dot\varphi\operatorname{sen}\varphi} - \left(\frac{L}{4}-R\varphi\right)\cos\varphi\,\dot\varphi\right] = 0$$

$$\boxed{\left[\left(\frac{L}{4}-R\varphi\right)^2 + \frac{L^2}{12}\right]\ddot\varphi - \left(\frac{L}{4}-R\varphi\right)R\dot\varphi^2 - g\left(\frac{L}{4}-R\varphi\right)\cos\varphi = 0}$$

Otra forma: 2ª Cardinal en I

$$\dot{\vec L}_I = \vec P \wedge \dot{\vec I} + \vec M_I^{(ext)}$$

$$\vec L_I = M(G-I)\wedge \underbrace{\vec v_I}_{0} + \mathbb{I}_I \vec\omega = -\dot\varphi I_{I,k}\vec k$$

$$I_{Ik} = I_{Gk} + Md^2 = \frac{ML^2}{12} + \left(\frac{L}{4}-R\varphi\right)^2 M$$

$$\vec L_I = -\left[\frac{ML^2}{12} + \left(\frac L4 - R\varphi\right)^2 M\right]\dot\varphi \vec k$$

$$\dot{\vec L}_I = \left\{2\left(\frac L4 - R\varphi\right) MR\dot\varphi^2 - \left[\frac{ML^2}{12}+M\left(\frac L4-R\varphi\right)^2\right]\ddot\varphi\right\}\vec k$$

$$\vec P\wedge\dot{\vec I} = M\left[-\left(\frac L4 - R\varphi\right)\dot\varphi\vec e_r\right]\wedge(R\dot\varphi\vec e_\varphi) = M\left(\frac L4-R\varphi\right)R\dot\varphi^2\vec k$$

$$\vec M_I^{(ext)} = (G-I)\wedge(-Mg\vec j) = -Mg\left[\frac L4 - R\varphi\right]\vec e_\varphi\wedge\vec j$$

$$\vec e_\varphi = \cos\varphi\,\vec i - \sin\varphi\,\vec j \Rightarrow \vec e_\varphi\wedge\vec j = \cos\varphi\,\vec k$$

$$\left(\frac L4 - R\varphi\right)MR\dot\varphi^2 - \left[\frac{ML^2}{12}+M\left(\frac L4-R\varphi\right)^2\right]\ddot\varphi = -Mg\left(\frac L4-R\varphi\right)\cos\varphi \quad ✓$$

parte c: $\vec R_v = T\vec e_\varphi + N\vec e_r \Rightarrow |T|\le f|N|$

$$\dot{\vec P} = M\vec a_G = \vec R - Mg\vec j$$

$$\vec a_G = \dot{\vec v}_G = R\dot\varphi^2\vec e_r - \left(\frac L4-R\varphi\right)\ddot\varphi\,\vec e_r - \left(\frac L4 - R\varphi\right)\dot\varphi\,\overset{\dot\varphi\vec e_\varphi}{\dot{\vec e}_r}$$

$$N - Mg\underbrace{\vec j\cdot\vec e_r}_{\cos\varphi} = MR\dot\varphi^2 - \left(\frac L4-R\varphi\right)\ddot\varphi M$$

$$T - Mg\underbrace{\vec j\cdot\vec e_\varphi}_{-\sin\varphi} = -M\left(\frac L4 - R\varphi\right)\dot\varphi^2$$

$$\varphi(0)=0,\quad \dot\varphi(0) = -\omega_0$$

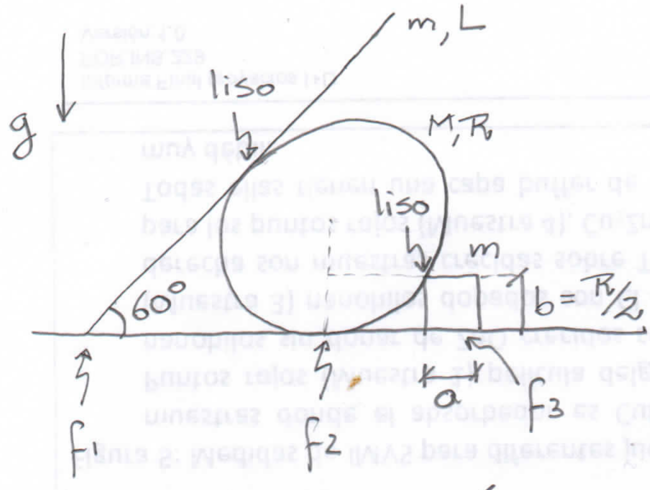
$$\underbrace{\left(\frac{L^2}{16}+\frac{L^2}{12}\right)}\ddot\varphi(0) = \frac L4 R\omega_0^2 + g\frac L4 \Rightarrow \ddot\varphi(0) = \frac{R\omega_0^2+g}{\frac{7L}{12}} = \frac{12(R\omega_0^2+g)}{7L}$$

$$\frac{L^2}{4}\left(\frac14+\frac13\right) = \frac{L^2}{4}\,\frac{7}{12}$$

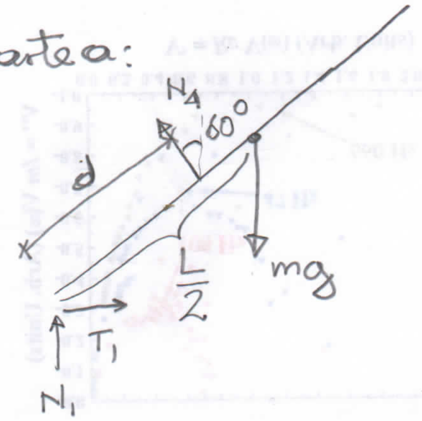
$$N = Mg + MR\omega_0^2 - \frac{\cancel{L}}{\cancel{4}}\,\frac{\cancel{12}^{\,3}(R\omega_0^2+g)M}{7\cancel{L}} = \frac{4M(g+R\omega_0^2)}{7} > 0$$

$$T = -\frac{ML\omega_0^2}{4} \Rightarrow \frac{\cancel{M}L\omega_0^2}{4} \le f\,\frac{4\cancel{M}(g+R\omega_0^2)}{7} \Rightarrow \boxed{f\ge\frac{7L\omega_0^2}{16(g+R\omega_0^2)}}$$

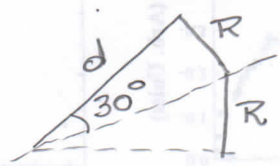
Ejercicio Nº2



parte a:



2ª en extremo de la barra en contacto con el piso: $N_4 d - \frac{L}{2}\underbrace{\cos 60^\circ}_{=\frac{1}{2}} mg = 0$



$\frac{R}{d} = \operatorname{tg} 30^\circ = \frac{\operatorname{sen} 30^\circ}{\cos 30^\circ} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

$d = R\sqrt{3}$ $\quad L \geq d = \sqrt{3} R$ para que la barra esté apoyada en el disco.

$N_4 = \frac{mgL}{4\sqrt{3}R} > 0$ ✓

1ª cardinal según dirección vertical: $N_1 - mg + N_4 \underbrace{\cos 60^\circ}_{=\frac{1}{2}} = 0$

$N_1 = mg\left(1 - \frac{L}{8\sqrt{3}R}\right) \geq 0 \Rightarrow L \leq 8\sqrt{3}R$

$\Rightarrow \boxed{\sqrt{3}R \leq L \leq 8\sqrt{3}R}$

$|T_1| \leq f_1 |N_1|$

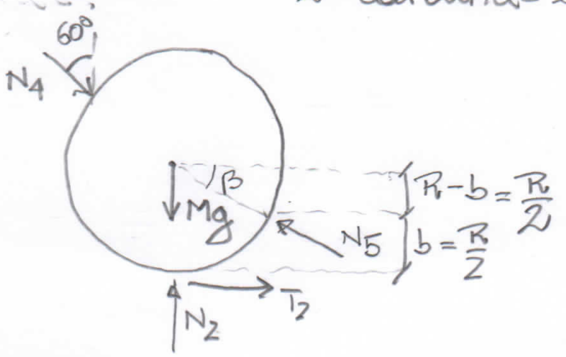
1ª cardinal horizontal a la barra: $T_1 - N_4 \underbrace{\operatorname{sen} 60^\circ}_{=\frac{\sqrt{3}}{2}} = 0$

$T_1 = \frac{\sqrt{3}}{2} N_4 = \frac{mgL}{8R} \Rightarrow \frac{mgL}{8R} \leq f_1 mg\left(1 - \frac{L}{8\sqrt{3}R}\right)$

$\boxed{f_1 \geq \frac{\sqrt{3}L}{8\sqrt{3}R - L}}$

parte b: $L = 4\sqrt{3}R \Rightarrow N_4 = mg,\ N_1 = mg/2$

2ª cardinal en el centro del disco: $\boxed{T_2 = 0}$



1ª cardinal horizontal

$N_4 \underbrace{\operatorname{sen} 60^\circ}_{=\frac{\sqrt{3}}{2}} - N_5 \cos\beta = 0$

$\operatorname{sen}\beta = \frac{R}{2}\cdot\frac{1}{R} = \frac{1}{2}$

$\Rightarrow \beta = 30^\circ$

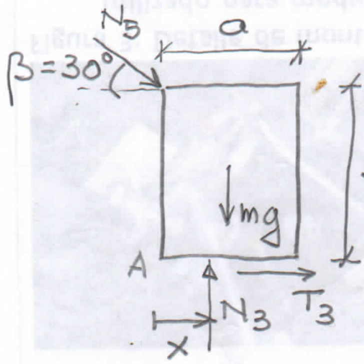
$\cos\beta = \frac{\sqrt{3}}{2} \Rightarrow N_5 = N_4 = mg$

1ª cardinal vertical: $N_2 - Mg - N_4 \underbrace{\cos 60°}_{=1/2} + N_5 \underbrace{\operatorname{sen}\beta}_{=1/2} = 0$

$\Rightarrow \boxed{N_2 = Mg}$

parte a: 1ª horizontal a la placa: $T_3 + N_5 \underbrace{\cos 30°}_{=\frac{\sqrt{3}}{2}} = 0$

$T_3 = -\frac{\sqrt{3}}{2} mg$



$b = \frac{R}{2}$

1ª vertical a la placa: $N_3 - mg - N_5 \underbrace{\operatorname{sen} 30°}_{=\frac{1}{2}} = 0$

$N_3 = \frac{3}{2} mg$

$|T_3| \leq f_3 |N_3|$

$\frac{\sqrt{3}}{2} \leq f_3 \frac{3}{2} \Rightarrow \boxed{f_3 \geq \frac{1}{\sqrt{3}}}$

2ª cardinal en A a la placa: $x N_3 - mg\frac{a}{2} - N_5 \underbrace{\cos 30°}_{=\frac{\sqrt{3}}{2}} b = 0$

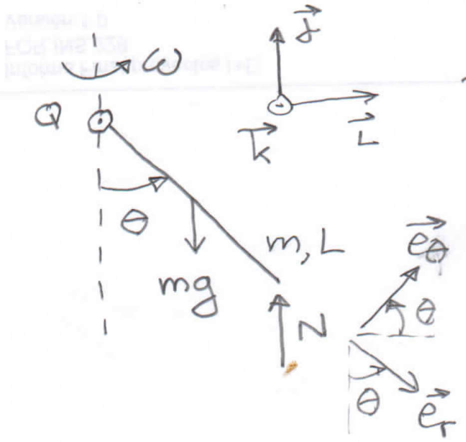
$x = \left(\frac{\cancel{mg}a}{2} + \cancel{mg}\frac{\sqrt{3}}{2}b\right) \Big/ \frac{3}{2}\cancel{mg}$

$0 \leq x \leq a$ $\quad \frac{a}{3} + \frac{b}{\sqrt{3}} \leq a$

$\frac{1}{\sqrt{3}}\frac{R}{2} \leq \frac{2a}{3} \Rightarrow \boxed{a \geq \frac{\sqrt{3}}{4}R}$

Ejercicio Nº 3

$\theta = \theta_0$



parte a:

$$\vec L_Q = m(G-Q)\wedge \underbrace{\vec v_Q}_{=0} + \mathbb{I}_Q \vec\omega$$

$$\vec\omega = \omega \vec j$$

$$\vec L_Q = \omega\, \mathbb{I}_Q \vec j$$

$$\vec j = \sin\theta\, \vec e_\theta - \cos\theta\, \vec e_r$$

$$\vec L_Q = \omega \sin\theta\, \underbrace{\mathbb{I}_Q \vec e_\theta}_{=I_{Q,\vec e_\theta}\vec e_\theta} - \omega\cos\theta\, \mathbb{I}_Q \vec e_r$$

$$I_{Q,\vec e_\theta} = I_{G,\vec e_\theta} + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{12} + \frac{mL^2}{4} = \frac{mL^2}{4}\left(\frac{1}{3}+1\right) = \frac{mL^2}{3}$$

$$\boxed{\vec L_Q = \frac{mL^2}{3}\omega \sin\theta\, \vec e_\theta}$$

parte b:

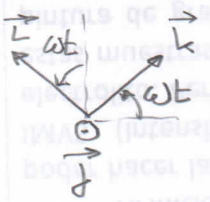
$$\dot{\vec L}_Q = \vec P \wedge \underbrace{\dot Q}_{=0} + \vec M_Q^{(ext)} = \left(-mg\frac{L}{2}\sin\theta + NL\sin\theta\right)\vec k + \vec M_Q^{(react)}$$

$$\vec M_O^{(react)} \cdot \vec k = 0$$

$$\theta = \theta_0 \Rightarrow \dot{\vec L}_Q = \frac{mL^2\omega}{3}\sin\theta\, \dot{\vec e}_\theta$$

$$\vec e_\theta = \cos\theta\, \vec i + \sin\theta\, \vec j$$

$$\dot{\vec e}_\theta = -\sin\theta\,\dot\theta\, \vec i + \cos\theta\, \underbrace{\dot{\vec i}}_{=-\omega\vec k} + \cos\theta\,\dot\theta\, \vec j \quad (\dot{\vec j} = 0)$$



$$\vec e_r = \sin\theta\, \vec i - \cos\theta\, \vec j \Rightarrow \dot{\vec e}_\theta = -\dot\theta\, \vec e_r - \omega\cos\theta\, \vec k$$

$$\dot\theta = 0 \Rightarrow \dot{\vec L}_Q = -\frac{mL^2\omega^2}{3}\sin\theta\cos\theta\, \vec k$$

$$-\frac{mL^2\omega^2}{3}\sin\theta\cos\theta = -mg\frac{\cancel{L}}{2}\cancel{\sin\theta} + N\cancel{L}\,\cancel{\sin\theta}$$

$$N = \frac{mg}{2} - \frac{mL\omega^2\cos\theta}{3} \geq 0 \qquad \frac{g}{2} \geq \frac{L\omega^2\cos\theta}{3}$$

$$\boxed{\omega \leq \sqrt{\frac{3g}{2L\cos\theta}}}$$

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partec: $\dot{\vec{L}}_Q = \vec{P}\wedge\underbrace{\dot{\vec{Q}}}_{0} + \vec{M}_Q^{(ext)} = -\frac{mgL}{2}\operatorname{sen}\theta\,\vec{k} + \vec{M}_Q^{(react)}$

$$\vec{L}_Q = \mathbb{I}_Q\vec{\omega}$$

$$\vec{\omega} = \omega\vec{j} + \dot{\theta}\vec{k} \qquad \mathbb{I}_Q\vec{k} = \frac{mL^2}{3}\vec{k}$$

$$\vec{L}_Q = \omega\underbrace{\mathbb{I}_Q\vec{j}}_{\frac{mL^2}{3}\operatorname{sen}\theta\,\vec{e}_\theta} + \dot{\theta}\,\mathbb{I}_Q\vec{k}$$

$$\vec{L}_Q = \frac{mL^2}{3}\left(\omega\operatorname{sen}\theta\,\vec{e}_\theta + \dot{\theta}\vec{k}\right)$$

$$\dot{\vec{L}}_Q = \frac{mL^2}{3}\left(\omega\cos\theta\dot{\theta}\vec{e}_\theta + \omega\operatorname{sen}\theta\,\dot{\vec{e}}_\theta + \ddot{\theta}\vec{k} + \dot{\theta}\overbrace{\dot{\vec{k}}}^{\omega\vec{\imath}}\right) =$$

$$\vec{\imath} = \operatorname{sen}\theta\,\vec{e}_r + \cos\theta\,\vec{e}_\theta$$

$$\dot{\vec{L}}_Q = \frac{mL^2}{3}\Big(\omega\cos\theta\dot{\theta}\vec{e}_\theta - \cancel{\omega\operatorname{sen}\theta\dot{\theta}\vec{e}_r} - \omega^2\operatorname{sen}\theta\cos\theta\vec{k} +$$

$$+\ddot{\theta}\vec{k} + \cancel{\dot{\theta}\omega\operatorname{sen}\theta\,\vec{e}_r} + \dot{\theta}\omega\cos\theta\dot{\theta}\vec{e}_\theta\Big)$$

$$\dot{\vec{L}}_Q = \frac{mL^2}{3}\left[2\omega\dot{\theta}\cos\theta\,\vec{e}_\theta + \left(\ddot{\theta} - \omega^2\operatorname{sen}\theta\cos\theta\right)\vec{k}\right]$$

$$\dot{\vec{L}}_Q\cdot\vec{k} = \frac{\cancel{m}L^{\cancel{2}}}{3}\left(\ddot{\theta} - \omega^2\operatorname{sen}\theta\cos\theta\right) = -\frac{\cancel{m}g\cancel{L}}{2}\operatorname{sen}\theta + \underbrace{\vec{M}_Q^{(react)}\cdot\vec{k}}_{0}$$

$$\boxed{\ddot{\theta} = \omega^2\operatorname{sen}\theta\cos\theta - \frac{3g}{2L}\operatorname{sen}\theta}$$