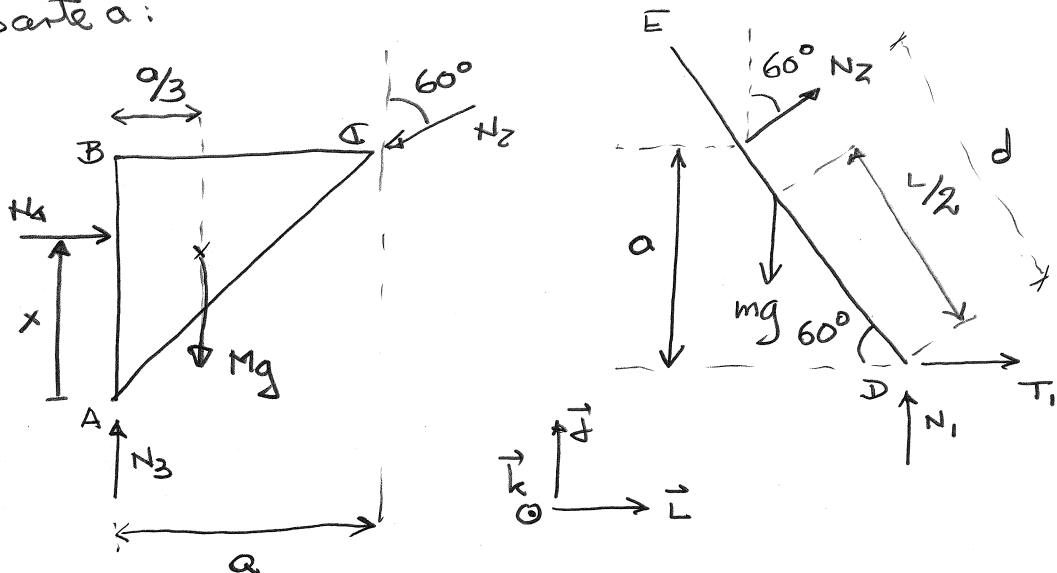


Ejercicio N° 1

parte a:



parte b: 1ª según  $\vec{i}$ )  $T_1 + N_2 \sin 60^\circ = 0$

1ª según  $\vec{j}$ )  $N_1 - mg + N_2 \cos 60^\circ = 0$

2ª en D:  $mg \frac{L}{2} \cos 60^\circ - N_2 d = 0$

$\frac{a}{d} = \sin 60^\circ = \frac{\sqrt{3}}{2} \Rightarrow d = \frac{2a}{\sqrt{3}} \Rightarrow N_2 = \frac{mgL}{4d} = \frac{mgL\sqrt{3}}{8a} > 0$

No hay desprendimiento en C.

$N_1 = mg - \frac{mgL\sqrt{3}}{16a} \geq 0 \Rightarrow L \leq \frac{16a}{\sqrt{3}}$

Primera condición  
Si no se cumple la barra desprende en D.

$T_1 = -\frac{mgL\sqrt{3}}{16a} < 0$

$|T_1| \leq f|N_1| \Rightarrow \frac{mgL\sqrt{3}}{16a} \leq f mg \left(1 - \frac{\sqrt{3}L}{16a}\right)$

$f \geq \frac{3L}{16a - \sqrt{3}L}$

Segunda condición.  
Si no se cumple la barra desliza en D.

$$L = \frac{4a}{\sqrt{3}}, f = \frac{1}{\sqrt{2}}$$

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parte c:  $\frac{4a}{\sqrt{3}} < \frac{16a}{\sqrt{3}} \rightarrow$  No hay desprendimiento en D

$$\frac{1}{\sqrt{2}} ? \frac{3L}{16a-4a} = \frac{\sqrt{3} \cdot 4}{12} = \frac{1}{\sqrt{3}}$$

$\sqrt{3} ? \sqrt{2} \Rightarrow 3 > 2 \rightarrow$  Entonces se cumple la condición y no hay deslizamiento en D

$$N_2 = \frac{mg\sqrt{3}}{8a} \cdot \frac{4a}{\sqrt{3}} = \frac{mg}{2} = \frac{\sqrt{3}}{2}$$

parte d: 1ª según  $\vec{i}$ )  $N_4 - N_2 \sin 60^\circ = 0$

$$1ª \text{ " } \vec{j}) N_3 - Mg - N_2 \cos 60^\circ = 0$$

$$2 \text{ en } \odot: N_4(a-x) + Mg \frac{2a}{3} - N_3 a = 0$$

$$N_4 = \frac{mg}{2} \frac{\sqrt{3}}{2} = \frac{\sqrt{3}mg}{4} > 0 \text{ No hay desprendimiento de la pared.}$$

$$N_3 = Mg + \frac{mg}{4} > 0 \text{ No hay desprendimiento del piso}$$

$$x = a + \frac{Mg \frac{2a}{3} - (Mg + \frac{mg}{4})a}{\frac{\sqrt{3}mg}{4}} = a - \frac{1}{\sqrt{3}} \left( \frac{4M}{3m} + 1 \right) a$$

$$0 \leq x \leq a \quad x < a \Rightarrow \text{No hay vuelco en B.}$$

$$x \geq 0 \quad a \geq \frac{1}{\sqrt{3}} \left( \frac{4M}{3m} + 1 \right) a$$

$$3\sqrt{3} \geq \frac{4M}{m} + 3$$

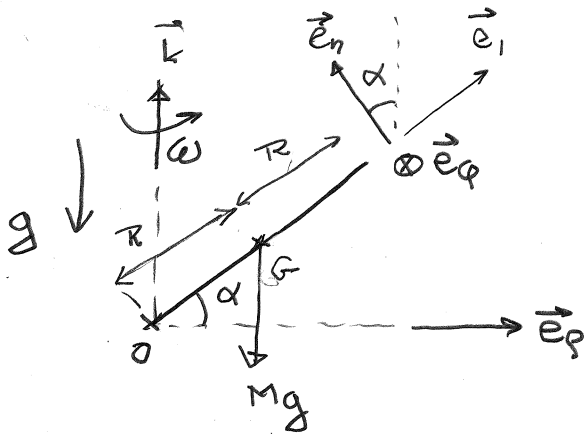
$$M \leq \frac{3(\sqrt{3}-1)m}{4}$$

Última condición.

Si no se cumple hay vuelco en A.

Ejercicio N° 2

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$\alpha > 0$  constante  
 $\omega$  constante

parte a: 0

$$\vec{v}_G = \vec{v}_O + \vec{\omega} \wedge (G-O)$$

$$\vec{\omega} = \omega \vec{k}$$

$$G-O = R \vec{e}_1$$

$$\vec{v}_G = \omega R \vec{k} \wedge \vec{e}_1 = \omega R \cos \alpha \vec{e}_\varphi = \vec{v}_G$$

$$\vec{a}_G = \vec{a}_O + \dot{\vec{\omega}} \wedge (G-O) + \vec{\omega} \wedge [\vec{\omega} \wedge (G-O)] = \omega^2 R \cos \alpha \underbrace{\vec{k} \wedge \vec{e}_\varphi}_{-\vec{e}_\varphi}$$

$$\vec{a}_G = -\omega^2 R \cos \alpha \vec{e}_\varphi$$

parte b:  $\vec{L}_O = \Pi_O \vec{\omega}$  porque O es fijo.

$$\vec{\omega} = \omega (\cos \alpha \vec{e}_n + \sin \alpha \vec{e}_1)$$

$$\vec{L}_O = \omega (\cos \alpha \Pi_O \vec{e}_n + \sin \alpha \Pi_O \vec{e}_1)$$

$\vec{e}_n$  eje principal porque es perpendicular al plano del disco que es plano de simetría y O pertenece a ese plano.

$\vec{e}_1$  es eje principal porque es eje de simetría y O pertenece a ese eje.

$$\Rightarrow \vec{L}_O = \omega (\cos \alpha I_{O, \vec{e}_n} \vec{e}_n + \sin \alpha I_{O, \vec{e}_1} \vec{e}_1)$$

$$\frac{3MR^2}{2} = \underbrace{I_{G, \vec{e}_n}}_{\frac{MR^2}{2}} + \underbrace{M d_{O, \vec{e}_n; G, \vec{e}_n}^2}_{R^2} \quad I_{G, \vec{e}_1} + \underbrace{M d_{O, \vec{e}_1; G, \vec{e}_1}^2}_0$$

Figura plana:  $I_{G, \vec{e}_1} + I_{G, \vec{e}_\varphi} = I_{G, \vec{e}_n}$

Por simetría:  $I_{G, \vec{e}_1} = I_{G, \vec{e}_\varphi} \Rightarrow 2 I_{G, \vec{e}_1} = I_{G, \vec{e}_n} = \frac{MR^2}{2} \Rightarrow I_{G, \vec{e}_1} = \frac{MR^2}{4}$

$$\vec{L}_O = \frac{MR^2 \omega}{4} (6 \cos \alpha \vec{e}_n + \sin \alpha \vec{e}_1)$$

Otra forma:  $\vec{L}_O = \vec{L}_G + \vec{P} \wedge (O-G)$

$$\vec{L}_G = \omega (\cos \alpha \Pi_G \vec{e}_n + \sin \alpha \Pi_G \vec{e}_1)$$

$\vec{e}_n$  y  $\vec{e}_1$  también son ejes principales en G por las mismas razones de antes, además  $\vec{e}_n$  es eje de simetría en G y  $\vec{e}_1$  es perpendicular a ese plano de simetría y G pertenece a ellos

$$\vec{L}_G = \omega \left( \cos \alpha I_G \vec{e}_n \vec{e}_n + \sin \alpha I_G \vec{e}_1 \vec{e}_1 \right)$$

$$\stackrel{M \vec{v}_G}{=} \frac{MR^2}{2} \omega \vec{e}_n + \frac{MR^2}{4} \omega \vec{e}_1$$

$$\vec{P}_\perp(O-G) = MR \cos \alpha \vec{e}_\varphi \wedge (-R \vec{e}_1) = MR^2 \omega \cos \alpha \vec{e}_n$$

$$\vec{L}_O = \frac{MR^2 \omega}{4} (2 \cos \alpha \vec{e}_n + \sin \alpha \vec{e}_1 + 4 \cos \alpha \vec{e}_n) \checkmark$$

parte c:  $M \vec{a}_G = \vec{R}^{(ext)} = \vec{R}^{(react)} - Mg \vec{k} \Rightarrow \vec{R}^{(react)} = M(g \vec{k} + \vec{a}_G)$

$$\boxed{\vec{R}^{(react)} = M(g \vec{k} - \omega^2 R \cos \alpha \vec{e}_\varphi)}$$

parte d:  $\vec{L}_O = \vec{M}_O^{(ext)} = \vec{M}_O^{(react)} + \vec{M}_O^g$

$$MgR \cos \alpha \vec{e}_\varphi$$

$$(G-O) \wedge (-Mg \vec{k}) = -MgR \vec{e}_1 \wedge \vec{k}$$

$$= -\cos \alpha \vec{e}_\varphi$$

$$\vec{L}_O = \frac{MR^2 \omega}{4} (6 \cos \alpha \vec{e}_n + \sin \alpha \vec{e}_1)$$

$$\vec{\omega} \wedge \vec{e}_n = \omega \vec{k} \wedge \vec{e}_n = -\omega \sin \alpha \vec{e}_\varphi$$

$$\vec{e}_1 = \vec{\omega} \wedge \vec{e}_1 = \omega \vec{k} \wedge \vec{e}_1 = \omega \cos \alpha \vec{e}_\varphi$$

$$\vec{L}_O = \frac{MR^2 \omega^2}{4} (-6 \cos \alpha \sin \alpha + \sin \alpha \cos \alpha) \vec{e}_\varphi = -\frac{5MR^2 \omega^2 \sin \alpha \cos \alpha}{4} \vec{e}_\varphi$$

$$\boxed{\vec{M}_O^{(react)} = -\frac{5MR^2 \omega^2 \sin \alpha \cos \alpha}{4} \vec{e}_\varphi - MgR \cos \alpha \vec{e}_\varphi}$$

parte e:  $|\vec{M}_O^{(react)}| \leq M_0^{\max}$

$$0 < \alpha < \frac{\pi}{2} \Rightarrow \frac{5MR^2 \omega^2 \sin \alpha \cos \alpha}{4} + MgR \cos \alpha \leq M_0^{\max}$$

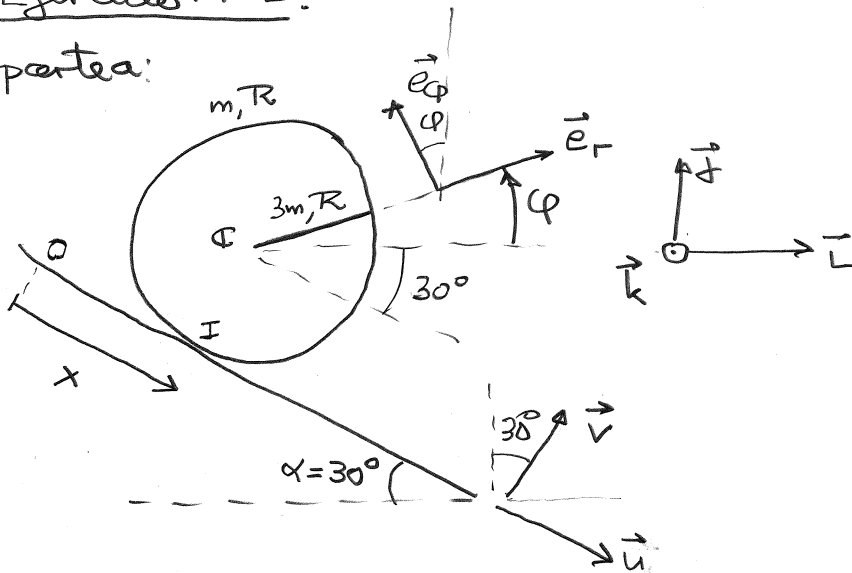
$$\boxed{\omega \leq \sqrt{\frac{4(M_0^{\max} - MgR \cos \alpha)}{5MR^2 \sin \alpha \cos \alpha}}}$$

Si  $M_0^{\max} < MgR \cos \alpha$  el radicando es negativo  $\Rightarrow \nexists \omega_{\max}$

Lo que sucede en este caso es que ya el momento del peso es mayor que el  $M_0^{\max} \Rightarrow$  la soldadura no alcanza a aguantar el momento del peso ni cuando el disco está quieto.

Ejercicio N° 3:

parte a:



$$\vec{v}_I = \vec{v}_C + \vec{\omega} \wedge (\mathbf{I} - \mathbf{C}) = 0 \Rightarrow \dot{x} \vec{u} + R \dot{\varphi} \vec{u} = 0 \Rightarrow \boxed{\dot{x} + R \dot{\varphi} = 0}$$

La distancia C-I es constante e igual a R

parte b:  $\vec{r}_G = \frac{1}{4m} (m \vec{r}_C + 3m \vec{r}_G^{(barra)}) = \vec{r}_C + \frac{3}{8} R \vec{e}_r$

$$\vec{r}_C = \vec{r}_0 + x \vec{u} + R \vec{v}$$

$$\vec{e}_r = \cos(\varphi + 30^\circ) \vec{u} + \sin(\varphi + 30^\circ) \vec{v}$$

$$\Rightarrow \boxed{\vec{r}_G = \vec{r}_0 + x \vec{u} + R \vec{v} + \frac{3R}{8} [\cos(\varphi + 30^\circ) \vec{u} + \sin(\varphi + 30^\circ) \vec{v}]}$$

parte c: 1) Internas  $\Rightarrow$  Potencia Nula  $\vec{F}_G \cdot \dot{\vec{r}}$

2) Pero  $\Rightarrow$  Conservativo  $\Rightarrow U = 4m g y_G$

$$\vec{r}_C \cdot \dot{\vec{r}} + \frac{3R}{8} \underbrace{\vec{e}_r \cdot \dot{\vec{r}}}_{\sin \varphi}$$

$$x \dot{u} \cdot \dot{\vec{r}} + R \dot{v} \cdot \dot{\vec{r}}$$

$$-\sin 30^\circ = -\frac{1}{2} \quad \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$\Rightarrow$  A menos de una constante  $U = -2mgx + \frac{3mgR}{2} \sin \varphi$

$$x = -R\varphi \Rightarrow \boxed{U = mgR \left( 2\varphi + \frac{3}{2} \sin \varphi \right)}$$

3) Reacción en I:  $P = \vec{R}^{(reac)} \cdot \vec{v}_I + \vec{\omega} \cdot \vec{M}_I^{(reac)} = 0$

$\Rightarrow$  de Potencia Nula

Pongo fuerza aplicada en I

parte d: Sist. Conservativo  $\Rightarrow T+U=E=cte.$

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$$T = \frac{M\vec{v}_P^2}{2} + M\vec{v}_P \cdot [\vec{\omega} \wedge (G-P)] + \frac{1}{2} \vec{\omega} \cdot \mathbb{I}_P \vec{\omega}$$

Aplicamos en  $P=C$ :  $T = \frac{M\vec{v}_C^2}{2} + M\vec{v}_C \cdot [\vec{\omega} \wedge (G-C)] + \frac{1}{2} \vec{\omega} \cdot \mathbb{I}_C \vec{\omega}$

$$\vec{v}_C = \dot{x} \vec{u} \Rightarrow \frac{M\vec{v}_C^2}{2} = \frac{M\dot{x}^2}{2} = \frac{MR^2\dot{\varphi}^2}{2} = \frac{4mR^2\dot{\varphi}^2}{2} = 2mR^2\dot{\varphi}^2$$

$$M\vec{v}_C \cdot [\vec{\omega} \wedge (G-C)] = M\dot{x}\dot{\varphi} \frac{3R}{8} [\vec{u} \cdot (\vec{k} \wedge \vec{e}_r)]$$

$$\vec{u} \cdot \vec{e}_\varphi = \cos(\varphi + 30^\circ + 90^\circ) = -\sin(\varphi + 30^\circ) = -\cos(60^\circ - \varphi)$$

$$M\vec{v}_C \cdot [\vec{\omega} \wedge (G-C)] = -M\dot{x}\dot{\varphi} \frac{3R}{8} \cos(60^\circ - \varphi) = \frac{3mR^2\dot{\varphi}^2 \cos(60^\circ - \varphi)}{2}$$

$$\frac{1}{2} \vec{\omega} \cdot \mathbb{I}_C \vec{\omega} = \frac{\dot{\varphi}^2}{2} \vec{k} \cdot \mathbb{I}_C \vec{k}$$

$$\mathbb{I}_{C\vec{k}} = \overset{\text{disco}}{\mathbb{I}_{G,\vec{k}}} + \overset{\text{barra}}{\mathbb{I}_{C,\vec{k}}} = \frac{3mR^2}{2}$$

$$\begin{aligned} & \overset{\text{disco}}{\frac{mR^2}{2}} + \overset{\text{barra}}{\mathbb{I}_{G,\vec{k}} + 3m d_{G,\vec{k}}^2} \\ & \frac{mR^2}{2} + \frac{3mR^2}{12} + \frac{3mR^2}{4} \\ & = \frac{3mR^2}{12} (3+1) = mR^2 \end{aligned}$$

$$T = 2mR^2\dot{\varphi}^2 + \frac{3mR^2\dot{\varphi}^2 \cos(60^\circ - \varphi)}{2} + \frac{3mR^2\dot{\varphi}^2}{4}$$

$$T = \frac{mR^2\dot{\varphi}^2}{4} [11 + 6\cos(60^\circ - \varphi)]$$

Otra forma:  $P = I \dot{\varphi} / \vec{v}_I = 0 \Rightarrow T = \frac{1}{2} \vec{\omega} \cdot \mathbb{I}_I \vec{\omega} = \frac{\dot{\varphi}^2}{2} \vec{k} \cdot \mathbb{I}_I \vec{k}$

$$\mathbb{I}_{I\vec{k}} = \mathbb{I}_{G\vec{k}} + 4m d_{I,\vec{k}}^2$$

$$\begin{aligned} \mathbb{I}_{G\vec{k}} &= \overset{\text{(disco)}}{\mathbb{I}_{G,\vec{k}}} + \overset{\text{(barra)}}{\mathbb{I}_{G,\vec{k}}} = \overset{\text{(barra)}}{\mathbb{I}_{G,\vec{k}}} + 3m d_{G,\vec{k}}^2 \\ &= \overset{\text{(disco)}}{\mathbb{I}_{C,\vec{k}}} + m d_{G,\vec{k}}^2 + \overset{\text{(barra)}}{\mathbb{I}_{C,\vec{k}}} + 3m d_{G,\vec{k}}^2 \\ &= \frac{mR^2}{2} + \frac{9R^2}{64} + \frac{3mR^2}{12} + \left(\frac{3R}{8} - \frac{R}{2}\right)^2 = \frac{R^2}{64} \\ &= \frac{mR^2}{4} \end{aligned}$$

$$\mathbb{I}_{G\vec{k}} = \frac{3mR^2}{4} + \frac{12mR^2}{64} = \frac{mR^2}{16} (12+3) = \frac{15mR^2}{16}$$

$$d_{I\vec{k}, G\vec{k}}^2 = (G-I)^2 = \left( \underset{\substack{\text{"} \\ \frac{3R}{8}\vec{e}_r}}{G} - \underset{\substack{\text{"} \\ R\vec{v}}}{I} \right)^2 = \frac{9R^2}{64} + R^2 + \frac{3R^2}{4} \vec{e}_r \cdot \vec{v} \quad \left( \frac{7}{8} \right)$$

$\cos(60^\circ - \varphi)$

$$I_{I\vec{k}} = \frac{15mR^2}{16} + \frac{9mR^2}{16} + 4mR^2 + 3mR^2 \cos(60^\circ - \varphi) =$$

$$= mR^2 \left( \frac{24}{16} + 4 + 3 \cos(60^\circ - \varphi) \right) = \frac{mR^2}{2} (11 + 6 \cos(60^\circ - \varphi))$$

$\frac{3}{2}$

$$\Rightarrow T = \frac{mR^2}{4} [11 + 6 \cos(60^\circ - \varphi)] \dot{\varphi}^2 \quad \checkmark$$

Otra forma:  $T = \frac{M\vec{v}_G^2}{2} + \frac{1}{2} \vec{\omega} \cdot \mathbb{I}_G \vec{\omega}$

$$\vec{v}_G = \vec{v}_C + \frac{3}{8} R \dot{\varphi} \vec{e}_\varphi = \dot{x} \vec{u} + \frac{3}{8} R \dot{\varphi} \vec{e}_\varphi$$

$$\frac{M\vec{v}_G^2}{2} = \frac{4m}{2} \left[ \dot{x}^2 + \frac{9}{64} R^2 \dot{\varphi}^2 + \frac{3\dot{x}R\dot{\varphi}}{4} \vec{u} \cdot \vec{e}_\varphi \right] =$$

$-\cos(60^\circ - \varphi)$

$$= 2m \left[ \frac{73R^2}{64} \dot{\varphi}^2 + \frac{3R^2}{4} \dot{\varphi}^2 \cos(60^\circ - \varphi) \right] =$$

$$= mR^2 \dot{\varphi}^2 \left[ \frac{73}{32} + \frac{3}{2} \cos(60^\circ - \varphi) \right]$$

$$\frac{1}{2} \vec{\omega} \cdot \mathbb{I}_G \vec{\omega} = \frac{\dot{\varphi}^2}{2} \vec{k} \cdot \mathbb{I}_G \vec{k} = \frac{\dot{\varphi}^2}{2} I_{G\vec{k}} = \frac{15mR^2 \dot{\varphi}^2}{32}$$

$$T = mR^2 \dot{\varphi}^2 \left[ \frac{88}{32} + \frac{3}{2} \cos(60^\circ - \varphi) \right] = \frac{mR^2 \dot{\varphi}^2}{4} [11 + 6 \cos(60^\circ - \varphi)]$$

$\frac{11}{4}$

$$\Rightarrow \frac{mR^2 \dot{\varphi}^2}{4} [11 + 6 \cos(60^\circ - \varphi)] + mgR \left( 2\varphi + \frac{3}{2} \operatorname{sen} \varphi \right) = E = \text{cte}$$

$$\frac{mR^2 \dot{\varphi}^2}{2} [11 + 6 \cos(60^\circ - \varphi)] + \frac{mR^2 \dot{\varphi}^2}{4} 6 \operatorname{sen}(60^\circ - \varphi) + mgR \left( 2\dot{\varphi} + \frac{3}{2} \cos \varphi \dot{\varphi} \right) = 0$$

$$\boxed{R \dot{\varphi} [11 + 6 \cos(60^\circ - \varphi)] + 3 \operatorname{sen}(60^\circ - \varphi) R \dot{\varphi}^2 + g(4 + 3 \cos \varphi) = 0}$$

Por coordenadas:  $\dot{\vec{L}}_I = M \vec{v}_G \wedge \dot{\vec{I}} + \vec{M}_I^{(ext)}$

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$$\vec{L}_I = M(G - I) \wedge \vec{v}_I + I_I \vec{\omega} = \dot{\varphi} I_I \vec{k} = \dot{\varphi} I_I \vec{r} \vec{k}$$

$$\vec{L}_I = \frac{mR^2 \dot{\varphi}}{2} [11 + 6 \cos(60^\circ - \varphi)] \vec{k}$$

$$\dot{\vec{L}}_I = \frac{mR^2 \ddot{\varphi}}{2} [11 + 6 \cos(60^\circ - \varphi)] + mR^2 \dot{\varphi}^2 3 \operatorname{sen}(60^\circ - \varphi)$$

$$I = G - R \vec{v} \Rightarrow \dot{I} = \dot{G} = \dot{x} \vec{u}$$

$$M \vec{v}_G \wedge \dot{I} = 4m \left( \dot{x} \vec{u} + \frac{3}{8} R \dot{\varphi} \vec{e}_\varphi \right) \wedge \dot{x} \vec{u} = \frac{3mR \dot{\varphi}}{2} \vec{e}_\varphi \wedge \vec{u} \dot{x}$$

$$M \vec{v}_G \wedge \dot{I} = \frac{3mR^2 \dot{\varphi}^2}{2} \operatorname{sen}(60^\circ - \varphi) \vec{k} \left[ \begin{array}{l} \operatorname{sen}(60^\circ - \varphi) = \cos(90^\circ - 60^\circ + \varphi) = \cos(\varphi + 30^\circ) \\ \operatorname{sen}(120^\circ + \varphi) (-\vec{k}) \end{array} \right]$$

$$\vec{M}_I^{(ext)} = (G - I) \wedge (-4mg \vec{j}) = \left( \frac{3R}{8} \vec{e}_r + R \vec{v} \right) (-4mg \vec{j}) =$$

$$= -4mgR \left( \frac{3}{8} \vec{e}_r \wedge \vec{j} + \vec{v} \wedge \vec{j} \right) = -mgR \left( 2 + \frac{3}{2} \cos \varphi \right) \vec{k}$$

$\cos \varphi \vec{k} \quad \operatorname{sen} 30^\circ \vec{k} = \frac{\vec{k}}{2}$

$$\frac{mR^2 \ddot{\varphi}}{2} [11 + 6 \cos(60^\circ - \varphi)] + 3mR^2 \dot{\varphi}^2 \operatorname{sen}(60^\circ - \varphi) =$$

$$= \frac{3mR^2 \dot{\varphi}^2}{2} \operatorname{sen}(60^\circ - \varphi) - mgR \left( 2 + \frac{3}{2} \cos \varphi \right)$$

$$R \dot{\varphi} [11 + 6 \cos(60^\circ - \varphi)] + 3R \dot{\varphi}^2 \operatorname{sen}(60^\circ - \varphi) + g(4 + 3 \cos \varphi) = 0 \checkmark$$