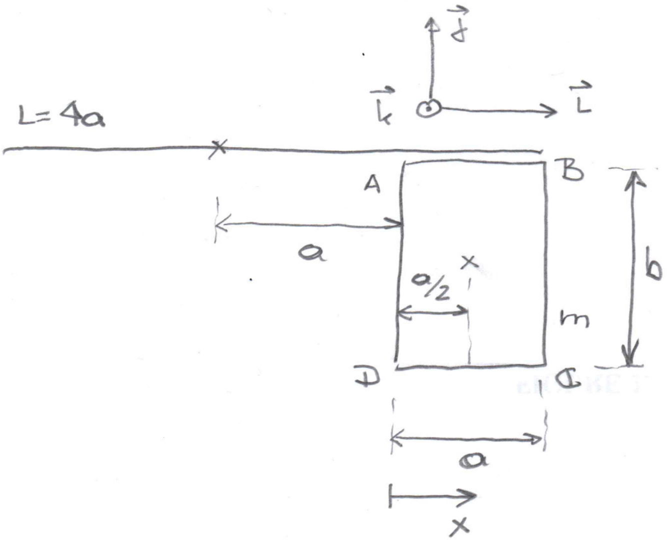
Mecánica Newtoniana (1122) Segundo Parcial (6/7/2023)

Ejercicio Nº 1



parte a: M masa de la barra

$$(M+m)\,x_G = m\frac{a}{2} - Ma = 0$$

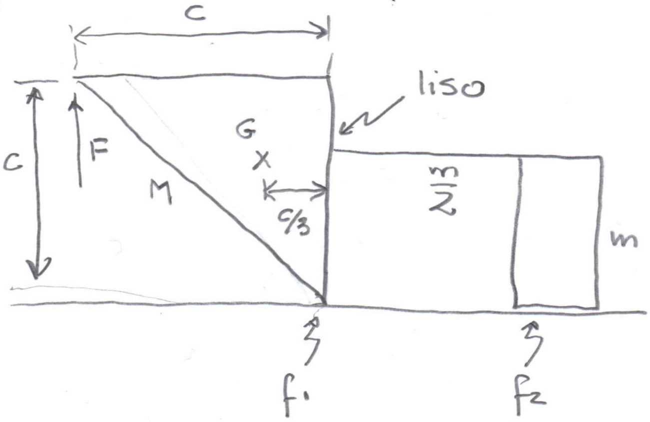
$$\Rightarrow \boxed{M = \frac{m}{2}}$$

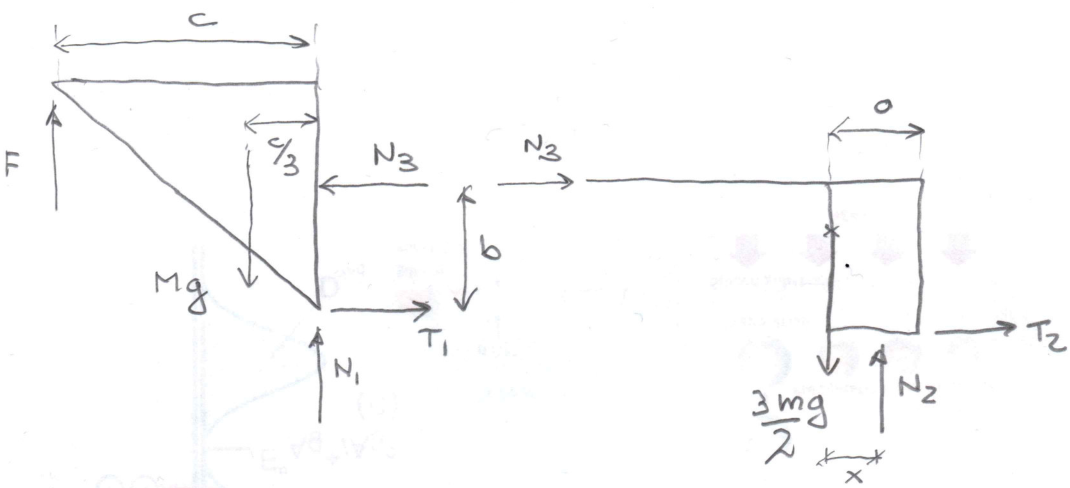
$$\frac{3m}{2}\,y_G = \frac{m}{2}b + \frac{m}{2}b = mb$$

$$\boxed{y_G = \frac{2b}{3}}$$

parte b:

$c > b$





parte c:
$$F - Mg + N_1 = 0$$
$$T_1 - N_3 = 0$$
$$-Fc + Mg\frac{c}{3} + N_3 b = 0$$

$$\left.\begin{array}{l} N_1 = Mg - F \geq 0 \Rightarrow F \leq Mg \\ N_3 = \frac{c}{b}\left(F - \frac{Mg}{3}\right) \geq 0 \Rightarrow F \geq \frac{Mg}{3} \end{array}\right\} \Rightarrow \frac{Mg}{3} \leq F \leq Mg$$

parte d: $F = \frac{Mg}{2}$

i) $N_1 = \frac{Mg}{2}$

$T_1 = N_3 = \frac{c}{b}\frac{Mg}{6}$

$\left.\right\} |T_1| \leq f_1 |N|$

$\frac{c}{b}\frac{Mg}{6} \leq f_1 \frac{Mg}{2} \Rightarrow \boxed{f_1 \geq \frac{c}{3b}}$

ii) $N_3 + T_2 = 0 \Rightarrow T_2 = -N_3 = -\frac{c}{b}\frac{Mg}{6}$

$N_2 - \frac{3mg}{2} = 0 \Rightarrow N_2 = \frac{3mg}{2}$

$x N_2 - N_3 b = 0 \Rightarrow x = \frac{N_3 b}{N_2} = \frac{c M \cancel{g} \cancel{b}}{6 \cancel{b} \frac{3m\cancel{g}}{2}} = \frac{M}{9m} c$

$\Rightarrow |T_2| \leq f_2 |N_2|$

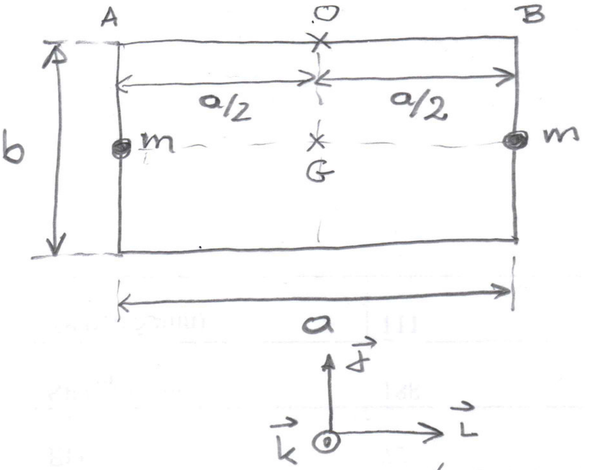
$\frac{cM\cancel{g}}{6b} \leq f_2 \frac{3m\cancel{g}}{2} \Rightarrow \boxed{f_2 \geq \frac{M}{9m}\frac{c}{b}}$

$0 \leq x \leq a$

$x \geq 0$ ✓

$\boxed{\frac{Mc}{9m} \leq a}$

Ejercicio N° 2



parte a:

Base $\vec i \, \vec j \, \vec k$

$$I_{G\vec i} = \frac{mb^2}{12}$$

EL eje $\vec k$ es eje principal en O porque es perpendicular al plano de simetría de la placa.

EL eje $\vec j$ es eje principal en O porque es eje de simetría:

$$\Rightarrow \mathbb{I}_O^{(\vec i,\vec j,\vec k)} = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix}$$

Por ser la figura plana $\underbrace{I_3}_{\sum_{i=1}^N m_i(x_i^2+y_i^2)} = \underbrace{I_1}_{\sum_{i=1}^N m_i(y_i^2+z_i^2) = \sum_{i=1}^N m_i y_i^2} + \underbrace{I_2}_{\sum_{i=1}^N m_i(x_i^2+z_i^2) = \sum_{i=1}^N m_i x_i^2}$

$z_i = 0 \ \forall i$

$$I_1 = I_1^{placa} + I_1^{masas} = 2m\left(-\frac{b}{2}\right)^2 = \frac{mb^2}{2}$$

$$I_1^{placa} = I_{O\vec i}^{placa} = I_{G\vec i}^{placa} + m \underbrace{d^2_{O\vec i\, G\vec i}}_{\left(\frac{b}{2}\right)^2} = \frac{mb^2}{12} + \frac{mb^2}{4} = \frac{mb^2}{4}\Big(\underbrace{\frac{1}{3}+1}_{4/3}\Big) = \frac{mb^2}{3}$$

$$I_1 = mb^2\left(\frac{1}{3}+\frac{1}{2}\right) = \frac{5mb^2}{6}$$

$$I_2 = I_2^{placa} + I_2^{masas} = m\left(\frac{a}{2}\right)^2 + m\left(-\frac{a}{2}\right)^2 = \frac{ma^2}{2}$$

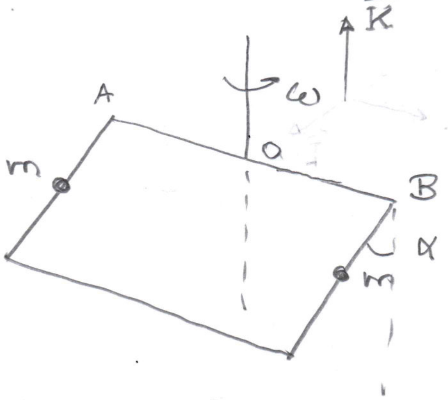
$$I_2^{placa} = I_{O\vec j}^{placa} = I_{G\vec j} + m \underbrace{d^2_{G\vec j\, O\vec j}}_{0} = \frac{ma^2}{12}$$ → a es al lado $\vec j$ como b es al lado $\vec i$

$$I_2 = \frac{ma^2}{2}\left(\frac{1}{6}+1\right) = \frac{7ma^2}{12}$$

$$I_3 = \frac{5mb^2}{6} + \frac{7ma^2}{12}$$

$$\mathbb{I}_O^{(\vec i,\vec j,\vec k)} = \begin{pmatrix} \frac{5mb^2}{6} & 0 & 0 \\ 0 & \frac{7ma^2}{12} & 0 \\ 0 & 0 & \frac{5mb^2}{6} + \frac{7ma^2}{12} \end{pmatrix}$$

Mecánica Newtoniana (1122) Segundo Parcial (6/7/2023) 4/7



parte b:

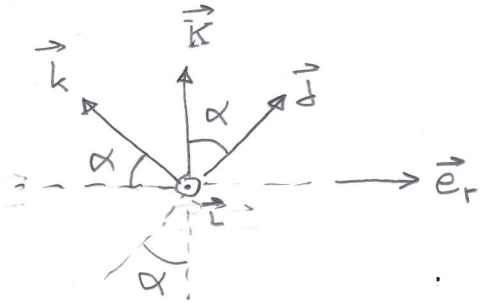
O fijo ($\vec{v}_O = 0$) $\Rightarrow \vec{L}_O = \mathbb{I}_O \vec{\omega}$

$\omega = \omega \vec{K}$

$\vec{L}_O = \omega \mathbb{I}_O \vec{K}$

$\vec{K} = \cos\alpha\, \vec{j} + \operatorname{sen}\alpha\, \vec{k}$

$$\vec{L}_O = \omega\Big(\cos\alpha \underbrace{\mathbb{I}_O \vec{j}}_{I_2 \vec{j}} + \operatorname{sen}\alpha \underbrace{\mathbb{I}_O \vec{k}}_{I_3 \vec{k}}\Big) =$$



$$= \omega\left[\frac{7ma^2}{12}\cos\alpha\, \vec{j} + \operatorname{sen}\alpha\left(\frac{5mb^2}{6} + \frac{7ma^2}{12}\right)\vec{k}\right]$$

$$\boxed{\vec{L}_O = \omega\left(\frac{7ma^2}{12}\vec{K} + \operatorname{sen}\alpha \frac{5mb^2}{6}\vec{k}\right)}$$

parte c: i) $\underbrace{M}_{3m}\vec{a}_G = \vec{R}^{(ext)} = -3mg\vec{K} + \vec{R}^{(react)}$

$G = O - \frac{b}{2}\vec{j}$

$\vec{v}_G = -\frac{b}{2}\dot{\vec{j}}$

$\vec{j} = \operatorname{sen}\alpha\, \vec{e}_r + \cos\alpha\, \vec{K} \Rightarrow \dot{\vec{j}} = \operatorname{sen}\alpha \underbrace{\dot{\vec{e}}_r}_{-\omega \vec{i}} \Rightarrow \vec{v}_G = \frac{b}{2}\omega \operatorname{sen}\alpha\, \vec{i}$

$\vec{a}_G = \frac{b}{2}\omega \operatorname{sen}\alpha \underbrace{\dot{\vec{i}}}_{\omega \vec{e}_r} = \frac{b\omega^2 \operatorname{sen}\alpha\, \vec{e}_r}{2}$

$$\Rightarrow \boxed{\vec{R}^{(react)} = 3mg\vec{K} + \frac{3mb\omega^2}{2}\operatorname{sen}\alpha\, \vec{e}_r}$$

parte ii) $\dot{\vec{L}}_O = \vec{M}_O^{(ext)} = \vec{M}_O^{(peso)} + \vec{M}_O^{(react)}$

$$\vec{M}_O^{(peso)} = \underbrace{(G-O)}_{-\frac{b}{2}\vec{j}} \wedge -3mg\vec{K} = \frac{3mgb}{2}\underbrace{\vec{j}\wedge\vec{K}}_{\operatorname{sen}\alpha\, \vec{i}}$$

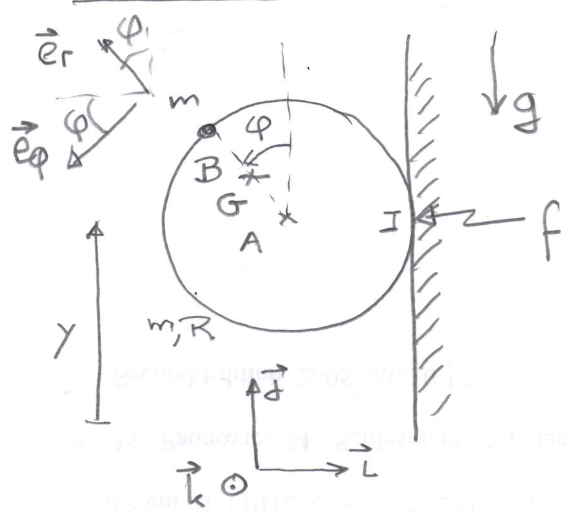
$\dot{\vec{L}}_O = \omega \operatorname{sen}\alpha \frac{5mb^2}{6}\dot{\vec{k}}$ $\qquad \dot{\vec{L}}_O = \omega^2 \operatorname{sen}\alpha \cos\alpha \frac{5mb^2}{6}\vec{i}$

$\Uparrow$

$\vec{k} = -\cos\alpha\, \vec{e}_r + \operatorname{sen}\alpha\, \vec{K} \Rightarrow \dot{\vec{k}} = -\cos\alpha\, \dot{\vec{e}}_r = \omega \cos\alpha\, \vec{i}$

$$\boxed{\vec{M}_O^{(react)} = \left(\frac{5mb^2}{6}\omega^2 \operatorname{sen}\alpha \cos\alpha - \frac{3mgb}{2}\operatorname{sen}\alpha\right)\vec{i}}$$

Ejercicio N° 3



$$\vec v_I = 0 = \underbrace{\vec v_A}_{\dot y \vec j} + \underbrace{\vec\omega}_{\dot\varphi \vec k} \wedge \underbrace{(I-A)}_{R\vec i} = (\dot y + R\dot\varphi)\vec j$$

$$\Rightarrow \dot y + R\dot\varphi = 0$$

parte a: $G = A + \frac{R}{2}\vec e_r$

$$\vec v_G = \vec v_A + \frac{R}{2}\dot{\vec e}_r = \dot y \vec j + \frac{R}{2}\dot\varphi \vec e_\varphi$$

$$\vec a_G = \underbrace{\ddot y}_{-R\ddot\varphi} \vec j + \frac{R}{2}\ddot\varphi \vec e_\varphi - \frac{R\dot\varphi^2}{2}\vec e_r$$

$$\boxed{\vec a_G = -R\ddot\varphi \vec j + \frac{R}{2}\ddot\varphi \vec e_\varphi - \frac{R\dot\varphi^2}{2}\vec e_r}$$

parte b: Fuerzas: 1) Internas rígido $\Rightarrow$ potencia nula

2) Peso: conservativo: $U = 2mg\left(y + \frac{R}{2}\cos\varphi\right)$

3) Reacción en I:

$$P = \vec R \cdot \underbrace{\vec v_I}_{\vec 0} + \underbrace{\vec M_I}_{\vec 0} \cdot \vec\omega = 0 \Rightarrow \text{potencia nula}$$

$\Rightarrow$ Sistema Conservativo: $T + U = E$

$$T = T^{disco} + T^{masa}$$

$$T^{disco} = \frac{1}{2}\vec\omega \cdot \mathbb{I}_I \vec\omega = \frac{1}{2}\dot\varphi^2 \underbrace{\vec k \cdot \mathbb{I}_I \vec k}$$

$$I_{Ik} = \underbrace{I_{Ak}}_{\frac{mR^2}{2}} + m \underbrace{d^2_{Ik,Ak}}_{R^2} = \frac{3mR^2}{2}$$

$$T^{masa} = \frac{1}{2} m \vec v_B^2$$

$$B = A + R\vec e_r \Rightarrow \vec v_B = \vec v_A + R\dot{\vec e}_r = \dot y \vec j + R\dot\varphi \vec e_\varphi$$

$$T^{masa} = \frac{m\dot y^2}{2} + \frac{mR^2\dot\varphi^2}{2} + m\underbrace{\dot y}_{-R\dot\varphi} R\dot\varphi \underbrace{\vec j \cdot \vec e_\varphi}_{-\operatorname{sen}\varphi} = mR^2\dot\varphi^2(1+\operatorname{sen}\varphi)$$

$$T = \frac{3mR^2\dot\varphi^2}{4} + mR^2\dot\varphi^2(1+\operatorname{sen}\varphi) = \frac{mR^2\dot\varphi^2}{4}(7 + 4\operatorname{sen}\varphi)$$

Otro método: $T = \frac{1}{2}\dot\varphi^2 I^{total}_{Ik}$

$$I^{total}_{Ik} = \underbrace{I^{(disco)}_{Ik}}_{\frac{3mR^2}{2}} + I^{(masa)}_{Ik}$$

$$I_{I\vec{k}}^{(\text{masa})} = m\underbrace{(B-I)}_{=R\vec{e}_r - R\vec{i}}{}^2 = m\Big(2R^2 - 2R^2\underbrace{\vec{e}_r\cdot\vec{i}}_{=-\operatorname{sen}\varphi}\Big)$$

$$I_{I\vec{k}}^{(\text{total})} = \frac{3mR^2}{2} + 2mR^2(1+\operatorname{sen}\varphi) = \frac{mR^2}{2}(7+4\operatorname{sen}\varphi)$$

$$\Rightarrow T = \frac{mR^2\dot{\varphi}^2}{4}(7+4\operatorname{sen}\varphi)$$

$$\Rightarrow \frac{mR^2\dot{\varphi}^2}{4}(7+4\operatorname{sen}\varphi) + 2mg\left(y+\frac{R}{2}\cos\varphi\right) = E$$

$$0 = \frac{\cancel{m}R^{\cancel{2}}\cancel{\dot{\varphi}}\ddot{\varphi}}{2}(7+4\operatorname{sen}\varphi) + \cancel{m}R^{\cancel{2}}\dot{\varphi}^2\cos\varphi\cancel{\dot{\varphi}} + 2\cancel{m}g\underbrace{\dot{y}}_{=-R\cancel{\dot{\varphi}}} - \cancel{m}gR\operatorname{sen}\varphi\cancel{\dot{\varphi}}$$

$$\boxed{\frac{R\ddot{\varphi}}{2}(7+4\operatorname{sen}\varphi) + R\dot{\varphi}^2\cos\varphi = 2g + g\operatorname{sen}\varphi}$$

Otro método: Segunda Cardinal en I

$$\dot{\vec{L}}_I = M\vec{v}_G\wedge\dot{\vec{I}} + \vec{M}_I^{(\text{ext})}$$

$$\vec{M}_I^{(\text{ext})} = M_I^{(\text{peso})} = \underbrace{(G-I)}_{=\frac{R}{2}\vec{e}_r - R\vec{i}}\wedge(-2mg\vec{j}) =$$

$$= -2mgR\Big(\frac{\overbrace{\vec{e}_r\wedge\vec{j}}^{-\operatorname{sen}\varphi\vec{k}}}{2} - \overbrace{\vec{i}\wedge\vec{j}}^{\vec{k}}\Big) = mgR(\operatorname{sen}\varphi+2)$$

$$\vec{L}_I = \vec{L}_I^{(\text{disco})} + \vec{L}_I^{(\text{masa})}$$

$$\vec{L}_I^{(\text{disco})} = M(G-I)\wedge\underbrace{\vec{v}_I}_{=0} + \mathbb{I}_I\vec{\omega} = I_{I\vec{k}}\dot{\varphi}\vec{k} = \frac{3mR^2}{2}\dot{\varphi}\vec{k}$$

$$\vec{L}_I^{(\text{masa})} = m\underbrace{(B-I)}_{=R\vec{e}_r - R\vec{i}}\wedge\underbrace{\vec{v}_B}_{=-R\dot{\varphi}\vec{j} + R\dot{\varphi}\vec{e}_\varphi}$$

$$= mR^2\dot{\varphi}\Big(\underbrace{-\vec{e}_r\wedge\vec{j}}_{-\operatorname{sen}\varphi\vec{k}} + \underbrace{\vec{i}\wedge\vec{j}}_{=\vec{k}} + \underbrace{\vec{e}_r\wedge\vec{e}_\varphi}_{=\vec{k}} - \underbrace{\vec{i}\wedge\vec{e}_\varphi}_{=-(-\vec{i})\wedge\vec{e}_\varphi = \operatorname{sen}\varphi\vec{k}}\Big) =$$

$$= mR^2\dot{\varphi}(2+2\operatorname{sen}\varphi)\vec{k}$$

$$\vec{L}_I = \frac{mR^2\dot{\varphi}}{2}(7+4\operatorname{sen}\varphi)\vec{k}$$

Mecánica Newtoniana (1122) Segundo Parcial (6/7/2023)

7/7

$$\dot{\vec{L}}_I = \left[\frac{mR^2\ddot{\varphi}}{2}\left(7+4\operatorname{sen}\varphi\right) + 2mR^2\dot{\varphi}^2\cos\varphi\right]\vec{k}$$

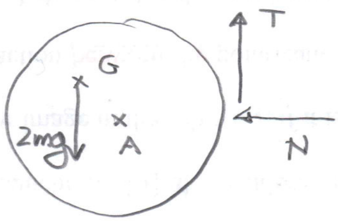
$$\underset{2m}{M}\,\underset{-R\dot{\varphi}\vec{j}+\frac{R}{2}\dot{\varphi}\vec{e}_\varphi}{\vec{v}_G} \wedge \underset{\dot{y}\vec{j}=-R\dot{\varphi}\vec{j}}{\dot{I}} = -2mR^2\frac{\dot{\varphi}^2}{2}\underbrace{\vec{e}_\varphi\wedge\vec{j}}_{-\vec{e}_\varphi\wedge(-\vec{j})=-\cos\varphi\vec{k}} = mR^2\dot{\varphi}^2\cos\varphi\,\vec{k}$$

$$\frac{\cancel{m}R^{\cancel{2}}\ddot{\varphi}}{2}\left(7+4\operatorname{sen}\varphi\right) + \cancel{2}\cancel{m}R^{\cancel{2}}\dot{\varphi}^2\cos\varphi = \cancel{mR^2\dot{\varphi}^2\cos\varphi} + \cancel{m}gR\left(2+\operatorname{sen}\varphi\right)$$

$$\frac{R\ddot{\varphi}}{2}\left(7+4\operatorname{sen}\varphi\right) + R\dot{\varphi}^2\cos\varphi = g\left(2+\operatorname{sen}\varphi\right)$$

parte c: $|T| < f|N|$

$$M\vec{a}_G = -2mg\vec{j} + T\vec{j} - N\vec{i} \quad \text{c/ } N>0 \text{ para que no desprenda}$$



$$N = -\underset{2m}{M}\vec{a}_G\cdot\vec{i} = -\frac{MR\ddot{\varphi}}{2}\underbrace{\vec{e}_\varphi\cdot\vec{i}}_{-\cos\varphi} + \frac{MR\dot{\varphi}^2}{2}\vec{e}_r\cdot\vec{i}$$

$$\dot{\varphi}(0) = 0$$

$$N(0) = mR\ddot{\varphi}(0)\cos\varphi(0)$$

$$\frac{R\ddot{\varphi}(0)}{2}\left(7+4\operatorname{sen}\varphi(0)\right) = g\left(2+\operatorname{sen}\varphi(0)\right)$$

$$R\ddot{\varphi}(0) = \frac{2g\left(2+\operatorname{sen}\varphi(0)\right)}{7+4\operatorname{sen}\varphi(0)} = \frac{2g\left(2+1/2\right)}{9} = \frac{5g}{9}$$

$$N(0) = \frac{5mg}{9}\frac{\sqrt{3}}{2} = \frac{5\sqrt{3}\,mg}{18} > 0$$

$$T = 2mg + \underset{2m}{M}\,\underset{-R\ddot{\varphi}+\frac{R\ddot{\varphi}}{2}\vec{e}_\varphi\cdot\vec{j}-\frac{R\dot{\varphi}^2}{2}\vec{e}_r\cdot\vec{j}}{\vec{a}_G}\cdot\vec{j} \qquad \vec{e}_\varphi\cdot\vec{j} = -\operatorname{sen}\varphi$$

$$T(0) = 2mg - 2mR\ddot{\varphi}(0)\left(1+\frac{\operatorname{sen}\varphi(0)}{2}\right) = 2mg - 2m\frac{5g}{9}\frac{5}{4} =$$

$$= 2mg\left(1-\frac{25}{36}\right) = 2mg\frac{11}{36} = \frac{11mg}{18} > 0$$

$$\frac{11\cancel{mg}}{\cancel{18}} \leq f\frac{5\sqrt{3}\,\cancel{mg}}{\cancel{18}} \Rightarrow \boxed{f \geq \frac{11}{5\sqrt{3}}}$$