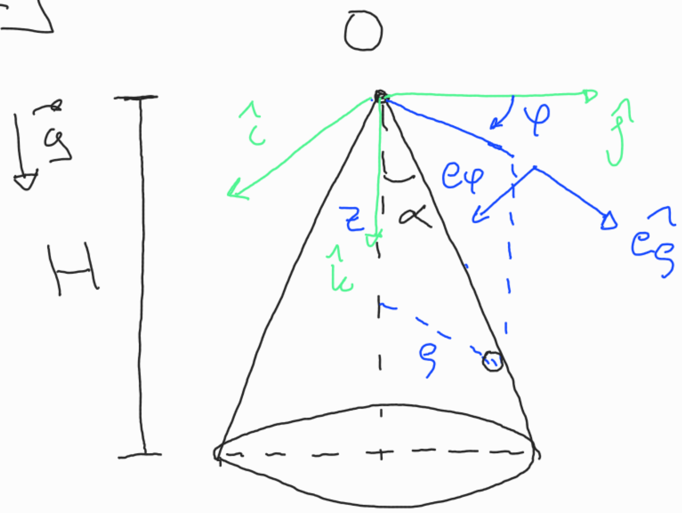
Examen Julio 2024. Mecánica Newtoniana

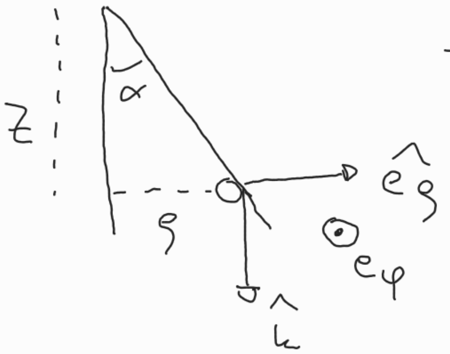
[1]



$\{\hat{e}_\rho, \hat{e}_\varphi, \hat{k}\}$ base de coordenadas similar a coordenadas cilíndricas

$\{\hat{i}, \hat{j}, \hat{k}\}$ sistema inercial

$\vec{r} = \rho\,\hat{e}_\rho + z\,\hat{k}$



$\tan\alpha = \dfrac{\rho}{z} \Rightarrow \boxed{\rho = z\tan\alpha}$

$r = z\tan\alpha\,\hat{e}_\rho + z\,\hat{k}$

$\vec{v} = \dot{\rho}\,\hat{e}_\rho + \rho\dot{\varphi}\,\hat{e}_\varphi + \dot{z}\,\hat{k} \rightarrow \vec{v} = \dot{z}\tan\alpha\,\hat{e}_\rho + z\tan\alpha\,\dot{\varphi}\,\hat{e}_\varphi + \dot{z}\,\hat{k}$

$\vec{a} = (\ddot{z}\tan\alpha - z\tan\alpha\,\dot{\varphi}^2)\,\hat{e}_\rho + (z\tan\alpha\,\ddot{\varphi} + 2\dot{z}\tan\alpha\,\dot{\varphi})\,\hat{e}_\varphi + \ddot{z}\,\hat{k}$

b) $\dfrac{d(\vec{L}_o \cdot \hat{k})}{dt} = \dfrac{d\vec{L}_o}{dt}\cdot\hat{k} + \vec{L}_o \cdot \cancel{\dfrac{d\hat{k}}{dt}}^{\,0}$

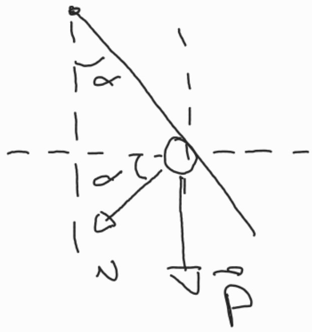
$\dfrac{d}{dt}(\vec{L}_o \cdot \hat{k}) = \dfrac{d\vec{L}_o}{dt}\cdot\hat{k}$ $\qquad \vec{L}_o = (\vec{r} \wedge m\vec{v})$

$\dfrac{d\vec{L}_o}{dt} = \cancel{\vec{v}\wedge m\vec{v}}^{\,0} + \vec{r}\wedge m\vec{a}$

Por 2da Ley de Newton $\dfrac{d\vec{L}_o}{dt} = \vec{r}\wedge\vec{F}_N$

$$\vec{F}_N = mg\hat{k} - N\cos\alpha\,\hat{e}_\rho + N\sin\alpha\,\hat{k}$$

$$\vec{F}_N = (mg + N\sin\alpha)\hat{k} - N\cos\alpha\,\hat{e}_\rho$$



$$\frac{d\vec{L}_o}{dt} = (z\,tg\alpha\,\hat{e}_\rho + z\hat{k}) \wedge \left((mg + N\sin\alpha)\hat{k} - N\cos\alpha\,\hat{e}_\rho\right)$$

$$= (mg + N\sin\alpha)(z\,tg\alpha)(-\hat{e}_\varphi) + z N\cos\alpha(-\hat{e}_\varphi)$$

$$= \left[(mg + N\sin\alpha) z\,tg\alpha + N z\cos\alpha\right](-\hat{e}_\varphi)$$

$$\Rightarrow \frac{d\vec{L}_o}{dt}\cdot\hat{k} = 0 \quad \text{pues} \quad \hat{k}\cdot\hat{e}_\varphi = 0$$

$$\hookrightarrow \frac{d(\vec{L}_o\cdot\hat{k})}{dt} = 0 \quad \vec{L}_o\cdot\hat{k} \ \text{constante}$$

c) $$P_N = \vec{N}\cdot\vec{v} = (N\sin\alpha\,\hat{k} - N\cos\alpha\,\hat{e}_\rho)\cdot(\dot{z}\,tg\alpha\,\hat{e}_\rho + z\,tg\alpha\,\dot{\varphi}\,\hat{e}_\varphi + \dot{z}\hat{k})$$

$$= -N\cos\alpha\,\dot{z}\,tg\alpha + N\sin\alpha\,\dot{z} = 0$$

$$\hookrightarrow tg\alpha = \frac{\sin\alpha}{\cos\alpha}$$

El sistema es consevativo.

$$E = T + U \qquad T = \frac{1}{2} m \vec{v}^2$$

$$U_g = -mgz \qquad T = \frac{1}{2} m \left(\dot{z}\,tg\alpha\,\hat{e}_\rho + z\dot{\varphi}\,tg\alpha\,\hat{e}_\varphi + \dot{z}\hat{k}\right)^2$$

$$T = \frac{1}{2} m \left(\dot{z}^2(1 + tg^2\alpha) + z^2\dot{\varphi}^2 tg^2\alpha\right)$$

$$\vec{L}_o = (z\,tg\alpha\,\hat{e}_\rho + z\hat{k}) \wedge m(\dot{z}\,tg\alpha\,\hat{e}_\rho + z\dot{\varphi}\,tg\alpha\,\hat{e}_\varphi + \dot{z}\hat{k})$$

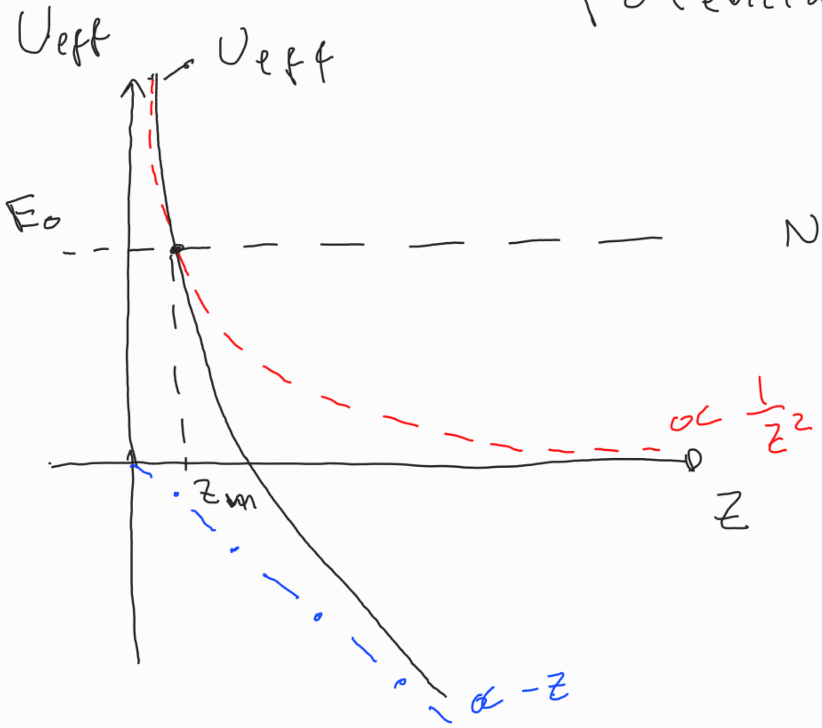
$$= m z^2 tg^2\alpha\,\dot{\varphi}\,\hat{k} + m z\,tg\alpha\,\dot{z}(-\hat{e}_\varphi) + m z\dot{z}\,tg\alpha\,\hat{e}_\varphi - m z^2\dot{\varphi}\,tg\alpha\,\hat{e}_\rho$$

$$\vec{L}_0 \cdot \hat{k} = m z^2 t_g^2\alpha\, \dot{\varphi} = l_0 \quad cte \Rightarrow \dot{\varphi} = \frac{l_0}{m z^2 t_g^2\alpha}$$

$$T = \frac{1}{2} m \left(\dot{z}^2 (1 + t_g^2\alpha) + z^2 \frac{l_0^2}{m^2 z^4 t_g^4\alpha} t_g^2\alpha \right)$$

$$= \frac{1}{2} m \dot{z}^2 (1 + t_g^2\alpha) + \frac{l_0^2}{2 m t_g^2\alpha} \frac{1}{z^2}$$

$$E_0 = \frac{1}{2} m \dot{z}^2 (1 + t_g^2\alpha) + \underbrace{\frac{l_0^2}{2 m t_g^2\alpha} \frac{1}{z^2} - m g z}_{\text{Potencial Efectivo } U_{eff}(z)} \quad cte.$$



No es posible tener orbitas de altura acotada ya que $\forall E_0$ No hay soluciones con z cte.

Otra forma: las soluciones de altura constante z_0 ocurren si $\left.\frac{dU_{eff}}{dz}\right|_{z_0} = 0$

$$\frac{dU_{eff}}{dz} = -\frac{l_0^2}{2 m t_g^2\alpha} \frac{2}{z^3} - m g \stackrel{?}{=} 0 \Rightarrow z^3 = \frac{-m^2 g\, t_g^2\alpha}{l_0^2}$$

resultado absurdo pues $z > 0$

d) si en $t=0$ $\quad z(0)=\frac{H}{2}$ y $\vec{v}(0)=v_0\,\hat{e}_\varphi$ la velocidad es horizontal y tangente al cono

$\dot{z}(0)=0$

$$\vec{v}=\dot{z}\,tg\alpha\,\hat{e}_g+z\dot{\varphi}\,\hat{e}_\varphi+\dot{z}\,\hat{k}$$

$$\vec{v}(0)=\frac{H}{2}\dot{\varphi}_0\,tg\alpha\,\hat{e}_\varphi=v_0\,\hat{e}_\varphi \Rightarrow \boxed{\dot{\varphi}_0=\frac{2v_0}{H\,tg\alpha}}$$

$$l_0=m z_0^2\,tg^2\alpha\,\dot{\varphi}_0=m\frac{H}{2}tg\alpha\,v_0 \qquad l_0=\frac{mH}{2}tg\alpha\,v_0$$

$$\vec{F}_N=(mg+N\sin\alpha)\hat{k}-N\cos\alpha\,\hat{e}_g$$

2da Ley de Newton según $\hat{k}$ $\quad m\ddot{z}=mg+N\sin\alpha$

según $\hat{e}_g$) $\quad m\left(\ddot{z}\,tg\alpha-z\,tg\alpha\,\dot{\varphi}^2\right)=-N\cos\alpha$

$$mg\,tg\alpha+N\,tg\alpha\sin\alpha-m z\,tg\alpha\frac{l_0^2}{m^2z^4tg^4\alpha}=-N\cos\alpha$$

$$\dot{\varphi}=\frac{l_0}{mz^2tg^2\alpha}$$

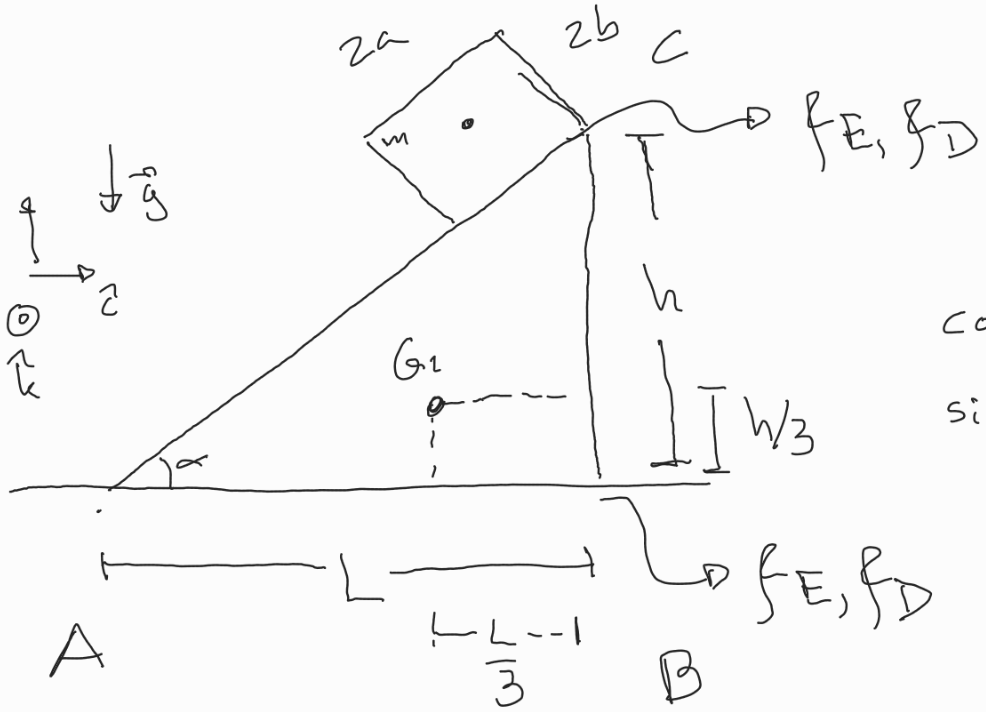
$$\rightarrow mg\,tg\alpha-m\,tg\alpha\frac{\left(\frac{mH}{2}tg\alpha\,v_0\right)^2}{m^2\,tg^4\alpha}\frac{1}{z^3}=-N\left(\cos\alpha+\frac{\sin^2\alpha}{\cos\alpha}\right)$$

En $z=H$

$$mg\,tg\alpha-m\frac{v_0^2}{4H\,tg\alpha}=-\frac{N}{\cos\alpha} \rightarrow N>0 \Leftrightarrow m\left(\frac{v_0^2}{4H\,tg\alpha}-g\,tg\alpha\right)>0$$

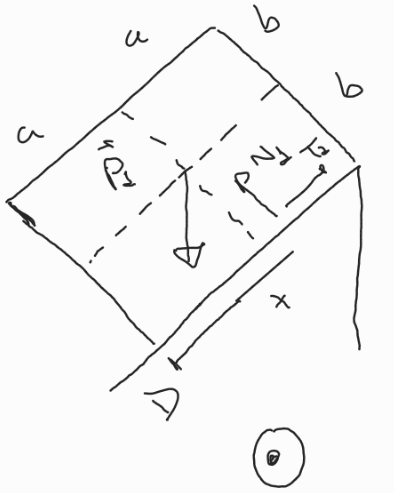
$$\Rightarrow v_0^2\geqslant 4Hg\,tg^2\alpha \qquad \boxed{v_0=2\sqrt{Hg}\,tg\alpha}$$

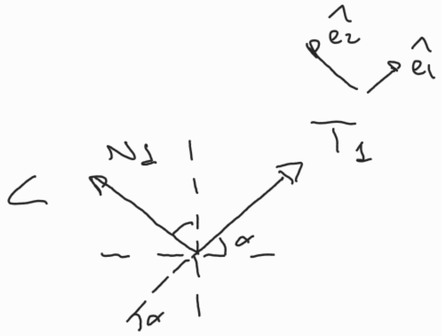
$\boxed{2}$



$\cos\alpha = \frac{\sqrt{3}}{2}$

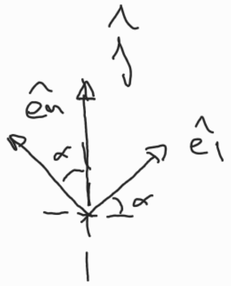
$\sin\alpha = \frac{1}{2}$





$\vec{T}_1 = T_1\cos\alpha\,\hat{i} + T_1\sin\alpha\,\hat{j}$

$\vec{N}_1 = -N_1\sin\alpha\,\hat{i} + N_1\cos\alpha\,\hat{j}$



$\hat{i})\quad T_1\cos\alpha = N_1\sin\alpha$

$\hat{j})\quad T_1\sin\alpha + N_1\cos\alpha = mg$

$\hat{j} = \cos\alpha\,\hat{e}_n + \sin\alpha\,\hat{e}_1$

Momentos nulos: $\vec{M}_D = 0$

$0 = N_1 x\,\hat{j} - mg\cos\alpha\, a\,\hat{j} + mg b\sin\alpha\,\hat{j}$

$N_1 x = mg(a\cos\alpha - b\sin\alpha)$

$$T_1 \frac{\sqrt{3}}{2} = N_1 \frac{1}{2} \rightarrow T_1 = \frac{1}{\sqrt{3}} N_1 \quad y \quad \frac{1}{2}\frac{1}{\sqrt{3}} N_1 + \frac{\sqrt{3}}{2} N_1 = mg$$

$$4 N_1 = 2\sqrt{3}\, mg \rightarrow \boxed{N_1 = \frac{\sqrt{3}}{2} mg} \quad \boxed{T_1 = \frac{1}{2} mg}$$

$$\frac{\sqrt{3}}{2} mg\, x = mg\left(a\frac{\sqrt{3}}{2} - \frac{b}{2}\right) \rightarrow \boxed{x = a - \frac{b}{\sqrt{3}}}$$

Condiciones: $N_1 \geq 0$ se cumple

$$|\vec{T}_1| \leq f_E |\vec{N}_1| \rightarrow \frac{1}{2} mg \leq f_E \frac{\sqrt{3}}{2} mg$$

$$\boxed{f_E \geq \frac{1}{\sqrt{3}}}$$

Si no se cumple la placa desliza hacia abajo

$0 < x < 2a \rightarrow 0 < x \Rightarrow a > \frac{b}{\sqrt{3}} \rightarrow \underline{\frac{a}{b} > \frac{1}{\sqrt{3}}}$

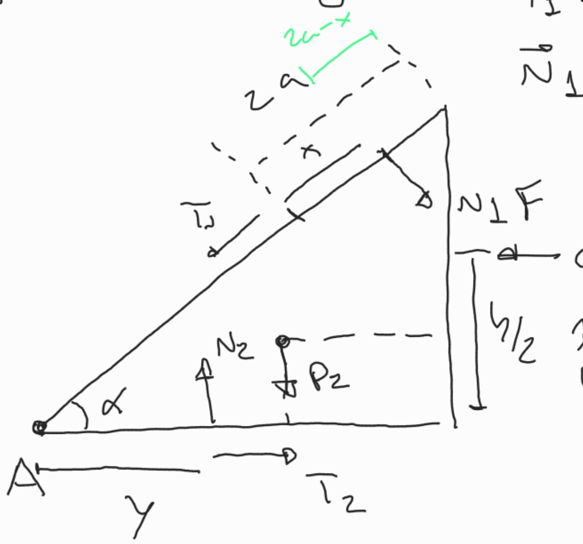
$\rightarrow x < 2a$ se cumple

si no se cumple vuelca en sentido antihorario

Equilibrio triángulo

$\vec{T}_1 = T_1 \cos\alpha\, \hat{i} + T_1 \sin\alpha\, \hat{j}$

$\vec{N}_1 = -N_1 \sin\alpha\, \hat{i} + N_1 \cos\alpha\, \hat{j}$



$\hat{i})\ T_2 - T_1\cos\alpha + N_1 \sin\alpha - F = 0$

$\hat{j})\ N_2 - Mg - T_1 \sin\alpha - N_1\cos\alpha = 0$

$$\boxed{N_1 = \frac{\sqrt{3}}{2} mg} \quad \boxed{T_1 = \frac{1}{2} mg}$$

$$T_2 - \frac{1}{2}\frac{\sqrt{3}}{2} mg + \frac{\sqrt{3}}{2}\frac{1}{2} mg - F = 0 \qquad \boxed{F = T_2}$$

$$N_2 - Mg - \frac{1}{2}\frac{1}{2} mg - \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2} mg \Rightarrow \boxed{N_2 = (M+m)g}$$

$N_2 > 0$ se cumple y

$$|\vec{T}_2| \leq f_E' |\vec{N}| \Rightarrow F \leq f_E' (M+m) g$$

$$tg\alpha = \frac{h}{L}$$

$$h = \frac{L}{\sqrt{3}}$$

$$\vec{M}_A^{ext} = 0 = N_2 y - Mg\frac{2}{3}L - N_1\left(\frac{L}{\cos\alpha} - (2a - x)\right) + \frac{h}{2} F = 0$$

$$= (M+m) g y - Mg\frac{2}{3}L - \frac{\sqrt{3}}{2} mg\left(\frac{2L}{\sqrt{3}} - 2a + a - \frac{b}{\sqrt{3}}\right) + \frac{L}{2\sqrt{3}} F = 0$$

$$= (M+m) g y - Mg\frac{2}{3}L - \frac{\sqrt{3}}{2} mg\left(\frac{2L}{\sqrt{3}} - a - \frac{b}{\sqrt{3}}\right) + \frac{L}{2\sqrt{3}} F = 0$$

$$y = \frac{1}{M+m}\left[\frac{2}{3} M L + \frac{\sqrt{3}}{2} m \left(\frac{2L}{\sqrt{3}} - a - \frac{b}{\sqrt{3}}\right) - \frac{L}{2\sqrt{3}}\frac{F}{g}\right]$$

$$y = \frac{1}{M+m}\left[L\left(\frac{2}{3}M + m\right) - \frac{\sqrt{3}}{2}\left(a + \frac{b}{\sqrt{3}}\right) m - \frac{L}{2\sqrt{3}}\frac{F}{g}\right]$$

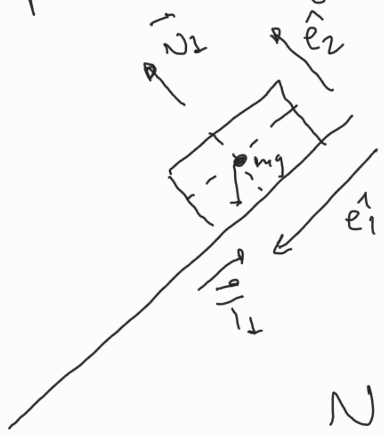
$$y = L \underbrace{\left(\frac{\frac{2}{3}M + m}{M+m}\right)}_{<1} - \frac{\sqrt{3}}{2}\frac{m}{M+m} \underbrace{\left(a + \frac{b}{\sqrt{3}}\right)}_{<2a} - L \frac{1}{2\sqrt{3}} \frac{F}{\underbrace{(M+m)g}}_{\leq f_E'}$$

pues $\frac{b}{\sqrt{3}} < a$

$0 < y < L \Rightarrow y < L$ se cumple

$$0 < L\left(\frac{\frac{2}{3}M + m}{M+m}\right) - \frac{\sqrt{3}}{2}\frac{m}{M+m}\left(a + \frac{b}{\sqrt{3}}\right) - \frac{L}{2\sqrt{3}}\frac{F}{(M+m)g}$$

b) La placa triangular en equilibrio, la rectangular desliza



$\hat{j} = \cos\alpha\,\hat{e}_2 - \sin\alpha\,\hat{e}_1$

$N_1 - mg\cos\alpha = 0 \qquad N_1 = \frac{\sqrt{3}}{2}mg$

$mg\sin\alpha - T_1 = m\ddot{x}$

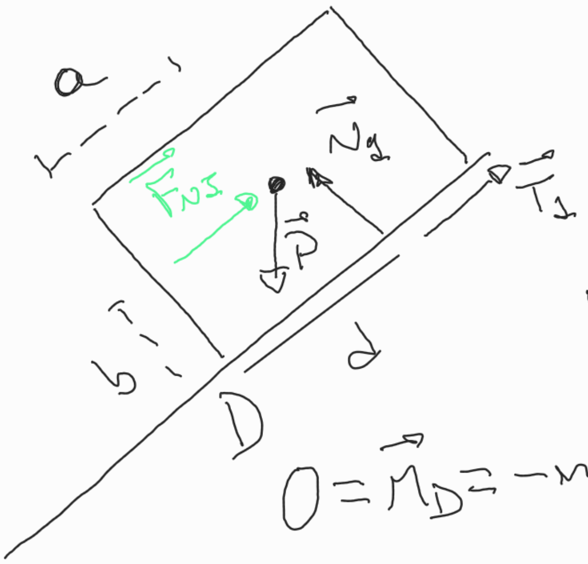
$|\vec{T}_1| = f_D|\vec{N}_1| \Rightarrow T_1 = f_D\, mg\cos\alpha \to \ddot{x} = g(\sin\alpha - f_D\cos\alpha)$

$\ddot{x} = \frac{g}{2}(1-\sqrt{3}f_D)$

Observación:

si $f_D < f_E < \frac{1}{\sqrt{3}}$

$\Rightarrow \ddot{x} > 0$



$F_{NI} = -m\ddot{x}\,\hat{e}_1$

$0 = \vec{M}_D = -m\ddot{x}\,b\hat{k} - mg\cos\alpha\, a\hat{k} + mg\sin\alpha\, b\hat{k} + N_2 d\hat{k}$

$0 = -\frac{mg}{2}(1-\sqrt{3}f_D)b + mg(b\sin\alpha - a\cos\alpha) + \frac{\sqrt{3}}{2}mg\,d$

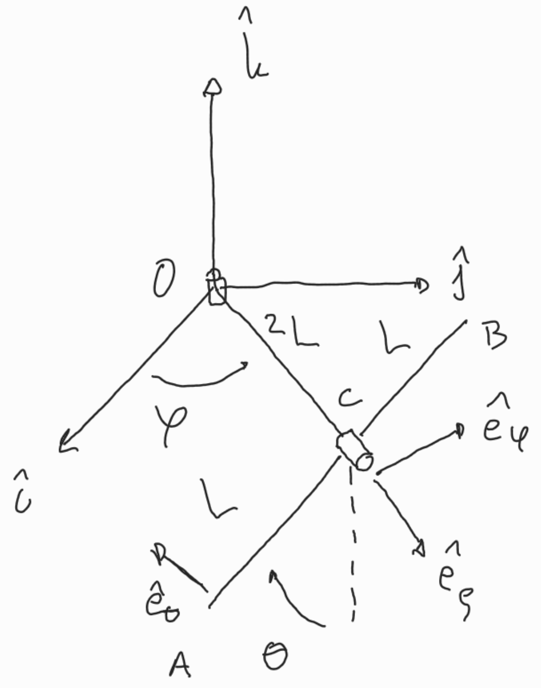
$0 = mg\left[b\left(\frac{1}{2} - \frac{1}{2} + \frac{\sqrt{3}}{2}f_D\right) - \frac{\sqrt{3}}{2}a + \frac{\sqrt{3}}{2}d\right]$

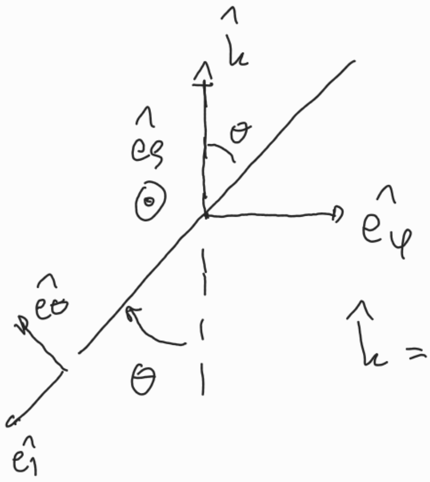
$0 = \frac{\sqrt{3}}{2}mg\left[d + f_D b - a\right] \qquad d = a - f_D b$

$0 < d < 2a \to 0 < d \Leftrightarrow a - f_D b > 0 \Rightarrow \boxed{\frac{a}{b} > f_D}$

$\to d < 2a$ se verifica.

3





$$\hat{k} = -\cos\theta\,\hat{e}_1 + \sin\theta\,\hat{e}_\theta$$

a) $\vec{L}_O \cdot \hat{k}$ magnitud conservada $\Rightarrow \dfrac{d(\vec{L}_O \cdot \hat{k})}{dt} = 0$

$$\frac{d(\vec{L}_O \cdot \hat{k})}{dt} = \frac{d\vec{L}_O}{dt} \cdot \hat{k} + \vec{L}_O \cdot \cancel{\frac{d\hat{k}}{dt}}^{\,"0}$$

$$\Rightarrow \frac{d(\vec{L}_O \cdot \hat{k})}{dt} = 0 \Leftrightarrow \underbrace{\frac{d\vec{L}_O}{dt}}_{\vec{M}_O^{ext}} \cdot \hat{k} = 0$$

$$\vec{M}_O^{ext} \cdot \hat{k} = 0$$

$\vec{M}_O^{ext} = \vec{M}_O^{react} + \vec{M}_O^{Act.}$; $\vec{M}_O^{react} \cdot \hat{k} = 0$ por articulación cilíndrica en O

$\vec{M}_O^{Act} = 2L\,\hat{e}_\rho \wedge (-mg\hat{k}) = 2mgL\,\hat{e}_\varphi$

$\hat{e}_\varphi \cdot \hat{k} = 0$

$\Rightarrow \vec{M}_O^{ext} \cdot \hat{k} = 0$ $\hookrightarrow$ $\vec{L}_O \cdot \hat{k}$ magnitud conservada.

b) Sobre el sistema tenemos: - El peso conservativo

- Reactivas en O que son de potencia nula

- Articulaciones lisas para los giros en O y C

La energía se conserva.

$U_g = 0$ pues el centro de masa en C no cambia su altura.

c) ¿$\vec{L}_o \cdot \hat{k}$? $\vec{L}_o = \mathbb{I}_o \vec{\omega}$

$\vec{\omega} = \dot{\varphi}\hat{k} - \dot{\theta}\hat{e}_\xi$ $\vec{\omega} = -\dot{\varphi}\cos\theta\,\hat{e}_1 + \dot{\varphi}\sin\theta\,\hat{e}_\theta - \dot{\theta}\,\hat{e}_\xi$

$\hat{k} = -\cos\theta\,\hat{e}_1 + \sin\theta\,\hat{e}_\theta$

$$\mathbb{I}_O^{\{\hat{e}_1,\hat{e}_\theta,\hat{e}_\xi\}} = \mathbb{I}_C^{\{\hat{e}_1,\hat{e}_\theta,\hat{e}_\xi\}} + \mathbb{J}_O^{M,C}$$

$$\vec{r}_C - \vec{r}_O = 2L\,\hat{e}_\xi \Rightarrow \mathbb{J}_O^{M,C} = M4L^2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ en } \hat{e}_1, \hat{e}_\theta, \hat{e}_\xi$$

$$\mathbb{I}_C^{\{\hat{e}_1,\hat{e}_\theta,\hat{e}_\xi\}} = \frac{M4L^2}{12}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbb{I}_O = ML^2\begin{pmatrix} 4 & & \\ & \frac{1}{3}+4 & \\ & & 1/3 \end{pmatrix} = \frac{ML^2}{3}\begin{pmatrix} 12 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\vec{L}_o = \frac{ML^2}{3}\begin{pmatrix} 12 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -\dot{\varphi}\cos\theta \\ \dot{\varphi}\sin\theta \\ -\dot{\theta} \end{pmatrix} = \frac{ML^2}{3}\left(-12\dot{\varphi}\cos\theta\,\hat{e}_1 + 13\dot{\varphi}\sin\theta\,\hat{e}_\theta - \dot{\theta}\,\hat{e}_\xi\right.$$

$$\vec{L}_o \cdot \hat{k} = \vec{L}_o \cdot \left(-\cos\theta\,\hat{e}_1 + \sin\theta\,\hat{e}_\theta\right) = \frac{ML^2}{3}\left(12\dot{\varphi}\cos^2\theta + 13\dot{\varphi}\sin^2\theta\right)$$

$$\vec{L}_o \cdot \hat{k} = \frac{ML^2}{3}\left(12 + \sin^2\theta\right)\dot{\varphi}$$

d) $\dot{\varphi}(0) = \dot{\varphi}_0 \quad \dot{\theta}(0) = \dot{\theta}_0 \approx 0 \quad \varphi(0) = \theta(0) = 0$

$$T = \frac{1}{2}\vec{\omega}\cdot \mathbb{I}_o \vec{\omega}$$

$$T = \frac{1}{2}\left(-\dot{\varphi}\cos\theta\,\hat{e}_1 + \dot{\varphi}\sin\theta\,\hat{e}_\theta - \dot{\theta}\,\hat{e}_\xi\right)\cdot\frac{ML^2}{3}\left(-12\dot{\varphi}\cos\theta\,\hat{e}_1 + 13\dot{\varphi}\sin\theta\,\hat{e}_\theta - \dot{\theta}\,\hat{e}_\xi\right)$$

$$T = \frac{1}{2}\frac{ML^2}{3}\left(12\dot{\varphi}^2\cos^2\theta + 13\dot{\varphi}^2\sin^2\theta + \dot{\theta}^2\right)$$

$$T = \frac{ML^2}{6}\left(\left[12 + \sin^2\theta\right]\dot{\varphi}^2 + \dot{\theta}^2\right)$$

$$E = T + U = T$$

Lo defino $U_g = 0$ pues el c.m. tiene altura constante

$$\vec{L}_o \cdot \hat{k} = \frac{ML^2}{12}\left(12 + \sin^2\theta\right)\dot{\varphi} = l_0 \qquad \text{por C.I. } \varphi = \theta = 0, \ \dot{\varphi} = \dot{\varphi}_0$$

$$l_0 = ML^2\dot{\varphi}_0$$

$$E_0 = \frac{ML^2}{6}\left(12\dot{\varphi}_0^2 + \dot{\theta}_0^2\right) \approx 2ML^2\dot{\varphi}_0^2$$

y $\quad \dot{\varphi} = \frac{12}{ML^2}\,\frac{l_0}{12 + \sin^2\theta} = \frac{12\dot{\varphi}_0}{12 + \sin^2\theta}$

Sustituyo en $E = E_0$

$$\underbrace{2ML^2\dot{\varphi}_0^2}_{E_0} = \frac{ML^2}{6}\left(\left[12 + \sin^2\theta\right]\overbrace{\left(\frac{12\dot{\varphi}_0}{12 + \sin^2\theta}\right)^2}^{\dot{\varphi}^2} + \dot{\theta}^2\right)$$

$$12\,\dot{\varphi}_0^2 = \frac{(12)^2\,\dot{\varphi}_0^2}{12+\sin^2\theta} + \dot{\theta}^2$$

$$\theta_f = \frac{\pi}{2} \Rightarrow \sin\theta_f = 1$$

$$12\,\dot{\varphi}_0^2 = \frac{(12)^2}{13}\,\dot{\varphi}_0^2 + \dot{\theta}_f^2$$

$$12\left(1-\frac{12}{13}\right)\dot{\varphi}_0^2 = \dot{\theta}_f^2 \Rightarrow \boxed{\dot{\theta}_f^2 = \frac{12}{13}\,\dot{\varphi}_0^2}$$

$$\dot{\varphi}_f = \frac{12}{13}\,\dot{\varphi}_0 \Rightarrow \boxed{\vec{\omega} = \frac{12}{13}\,\dot{\varphi}_0\,\hat{k} - \sqrt{\frac{12}{13}}\,\dot{\varphi}_0\,\hat{e}_S}$$

$$\vec{v}_c = 2L\,\dot{\varphi}\,\hat{e}_\varphi$$

$$\vec{v}_c(t=0) = 2L\,\dot{\varphi}_0\,\hat{e}_\varphi$$

$$\vec{v}_c(t_f) = 2L\,\frac{12}{13}\,\dot{\varphi}_0\,\hat{e}_\varphi$$