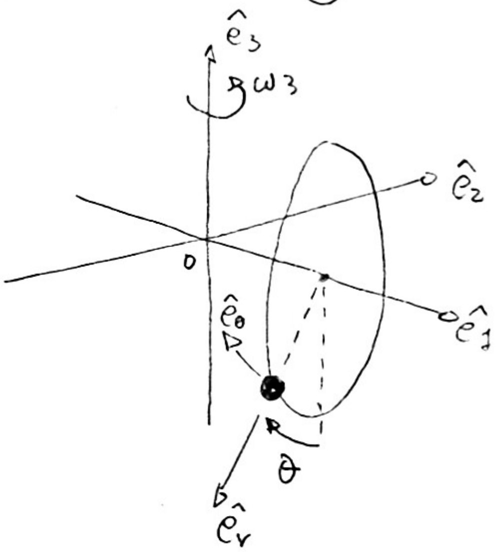
1er Parcial de Mecánica Newtoniana
05/05/2022

Ejercicio (1) (a)



$$\vec{r} = R\hat{e}_1 + R\hat{e}_r$$

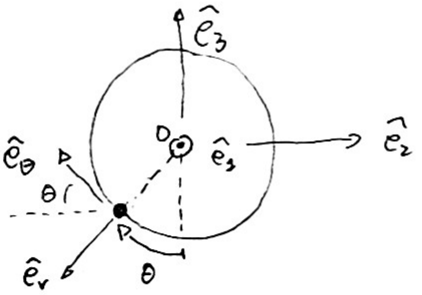
$$\vec{v} = R\dot{\hat{e}}_1 + R\dot{\hat{e}}_r = R\omega_3\hat{e}_2 + R\,\text{sen}\theta\,\omega_3\hat{e}_1 + R\dot{\theta}\hat{e}_\theta$$

$$\left(\dot{\hat{e}}_r = (\omega_3\hat{e}_3 - \dot{\theta}\hat{e}_1)\times\hat{e}_r = \omega_3\,\text{sen}\theta\,\hat{e}_1 + \dot{\theta}\hat{e}_\theta\right)$$

$$\vec{a} = R\omega_3\dot{\hat{e}}_2 + R\cos\theta\,\dot{\theta}\,\omega_3\hat{e}_1 + R\,\text{sen}\theta\,\omega_3\dot{\hat{e}}_1 + R\ddot{\theta}\hat{e}_\theta + R\dot{\theta}\dot{\hat{e}}_\theta =$$

$$= -R\omega_3^2\hat{e}_1 + R\cos\theta\,\dot{\theta}\,\omega_3\hat{e}_1 + R\,\text{sen}\theta\,\omega_3^2\hat{e}_2 + R\ddot{\theta}\hat{e}_\theta + R\dot{\theta}\omega_3\cos\theta\,\hat{e}_1 - R\dot{\theta}^2\hat{e}_r$$

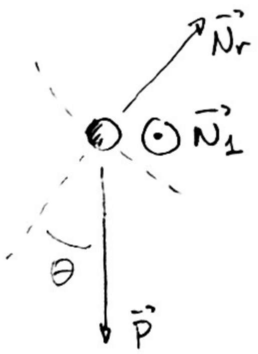
$$\left(\dot{\hat{e}}_\theta = (\omega_3\hat{e}_3 - \dot{\theta}\hat{e}_1)\times\hat{e}_\theta = \omega_3\cos\theta\,\hat{e}_1 - \dot{\theta}\hat{e}_r\right)$$



$$\boxed{\vec{v} = \underbrace{R\,\text{sen}\theta\,\omega_3\hat{e}_1 + R\omega_3\hat{e}_2}_{\text{velocidad de transporte}} + \underbrace{R\dot{\theta}\hat{e}_\theta}_{\text{velocidad relativa}}}$$

$$\boxed{\vec{a} = \underbrace{\left(R\ddot{\theta}\hat{e}_\theta - R\dot{\theta}^2\hat{e}_r\right)}_{\text{aceleración relativa}} + \underbrace{\left(-R\omega_3^2\hat{e}_1 + R\,\text{sen}\theta\,\omega_3^2\hat{e}_2\right)}_{\text{aceleración de transporte}} + \underbrace{\left(2R\omega_3\dot{\theta}\cos\theta\,\hat{e}_1\right)}_{\text{aceleración de Coriolis}}}$$

(b)



(Guía lisa) $\Rightarrow$ $m\vec{a} = -N_r\hat{e}_r + N_1\hat{e}_1 + \vec{P}$

$$m\vec{a}\cdot\hat{e}_\theta = -mg\,\text{sen}\theta$$

$$m(R\ddot{\theta} - R\,\text{sen}\theta\,\omega_3^2\cos\theta) = -mg\,\text{sen}\theta$$

$$\boxed{\ddot{\theta} + g/_R\,\text{sen}\theta - \omega_3^2\,\text{sen}\theta\cos\theta = 0}$$ Ecuación de Movimiento

$$m\vec{a}\cdot\hat{e}_r = -N_r + mg\cos\theta \quad \Rightarrow \quad N_r = mg\cos\theta - m\left(-R\dot{\theta}^2 - R\,\text{sen}^2\theta\,\omega_3^2\right)$$

$$\boxed{N_r = m\left(g\cos\theta + R\dot{\theta}^2 + R\,\text{sen}^2\theta\,\omega_3^2\right)}$$

$$m\vec{a}\cdot\hat{e}_1 = N_1 \quad \Rightarrow \quad \boxed{N_1 = m\left(-R\omega_3^2 + 2R\omega_3\dot{\theta}\cos\theta\right)}$$

$$\boxed{\vec{W} = N_1\,\hat{e}_1 - N_r\,\hat{e}_r}$$

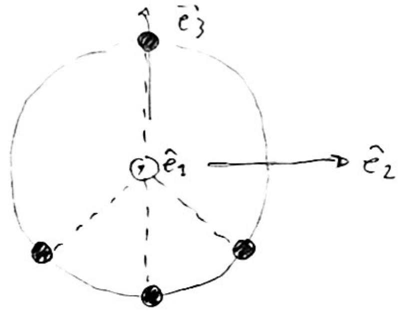
(c)

$$\ddot{\theta} + \text{sen}\,\theta\left(g/R - \omega_3^2\cos\theta\right) = 0 \quad \Rightarrow$$

ec. de movimiento

Posiciones de equilibrio del movimiento relativo $\Leftrightarrow \ddot{\theta} = 0 \ \forall t$

$$\text{sen}\,\theta_{eq}\left(g/R - \omega_3^2\cos\theta_{eq}\right) = 0.$$



$$\begin{cases} \text{sen}\,\theta_{eq} = 0 \Leftrightarrow & \theta_{eq} = 0 \\ & \theta_{eq} = \pi \\ \cos\theta_{eq} = \dfrac{g}{R\omega_3^2} & \exists \Leftrightarrow g < R\omega_3^2 \end{cases}$$

Estabilidad $\quad f(\theta) = \text{sen}\,\theta\,(g/R - \omega_3^2\cos\theta)$

$$\frac{\partial f}{\partial\theta} = \frac{\partial}{\partial\theta}\left(\text{sen}\,\theta\left(g/R - \omega_3^2\cos\theta\right)\right) = \cos\theta\left(g/R - \omega_3^2\cos\theta\right) + \omega_3^2\,\text{sen}^2\theta =$$

$$= g/R\cos\theta - \omega_3^2\cos^2\theta + \omega_3^2\,\text{sen}^2\theta = g/R\cos\theta - 2\omega_3^2\cos^2\theta + \omega_3^2$$

$$\left.\frac{\partial f}{\partial\theta}\right|_{\theta=0} = g/R - \omega_3^2 > 0 \Leftrightarrow g/R > \omega_3^2 \quad \text{Estable}$$

$$\left.\frac{\partial f}{\partial\theta}\right|_{\theta=\pi} = -g/R - \omega_3^2 < 0 \quad \text{inestable}$$

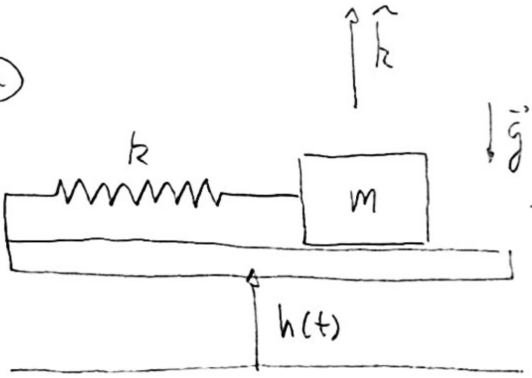
$$\left.\frac{\partial f}{\partial\theta}\right|_{\theta=\text{Arccos}\left(g/R\omega_3^2\right)} = g/R\left(\frac{g}{R\omega_3^2}\right) - 2\omega_3^2\left(\frac{g^2}{R^2\omega_3^4}\right) + \omega_3^2 =$$

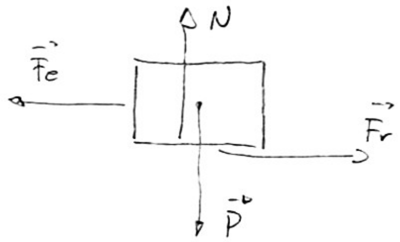
$$= \frac{g^2}{R^2\omega_3^2} - \frac{2g^2}{R^2\omega_3^2} + \omega_3^2 =$$

$$= \omega_3^2 - \frac{g^2}{R^2\omega_3^2} > 0 \Leftrightarrow \frac{g}{R} < \omega_3^2 \quad \text{estable}$$

Ejercicio (2)

(a)





$$m\vec{a} = \sum \vec{F}^{ext} \Rightarrow m\ddot{h}\hat{k} = \vec{N} + \vec{P} + \vec{F_e} + \vec{F_r}.$$

$$-m\omega^2 H \operatorname{sen}\omega t\,\hat{k} = N\hat{k} - mg\hat{k} - kL\,\hat{\imath} + F_r\,\hat{\imath}$$

El bloque se mantiene apoyado siempre que $N > 0 \Rightarrow N = mg - m\omega^2 H \operatorname{sen}\omega t > 0$

$$\boxed{g > H\omega^2} \Leftarrow (g > H\omega^2 \operatorname{sen}\omega t)\ \forall t$$

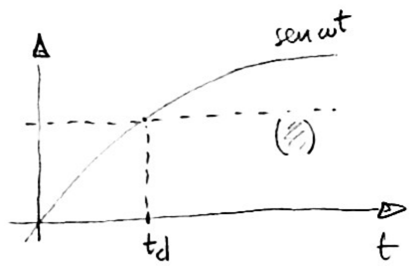
(b) $(\hat{\imath})$ $\quad 0 = -kL + F_r$

$F_r = kL$ $\qquad$ No hay deslizamiento mientras $|F_r| \leq \mu_s |N|$

$$kL \leq \mu_s (mg - m\omega^2 H \operatorname{sen}\omega t)$$

$$\mu_s m\omega^2 H \operatorname{sen}\omega t \leq \mu_s mg - kL$$

$$\operatorname{sen}\omega t \leq \left(\frac{\mu_s mg - kL}{\mu_s m\omega^2 H}\right)$$



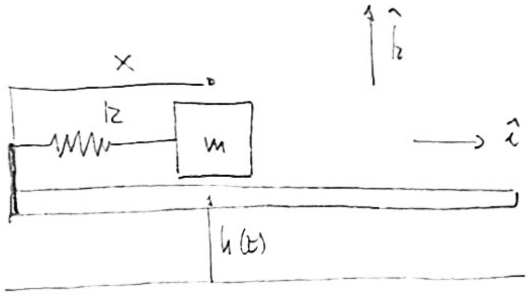
$\mu_s mg > kL$
(cond. de no deslizamiento para la superficie inmóvil)

$$\boxed{1 < \frac{\mu_s mg - kL}{\mu_s m\omega^2 H}}$$ Condición no desliza nunca.

(c)

$$\boxed{t_d = \frac{1}{\omega} \operatorname{Arsen}\left(\frac{\mu_s mg - kL}{\mu_s m\omega^2 H}\right)}$$

(d)



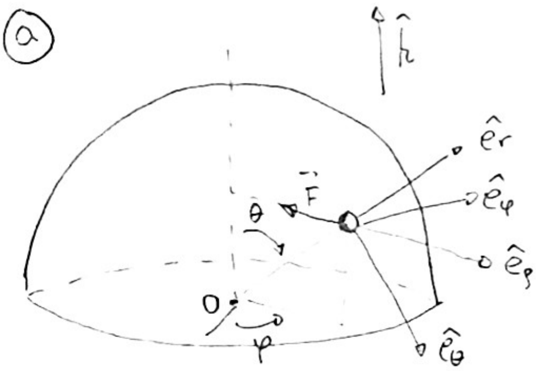
$$m(\ddot{x}\,\hat{i} + \ddot{h}\,\hat{k}) = N\,\hat{k} - mg\,\hat{k} - kx\,\hat{i} + F_r\,\hat{i}$$

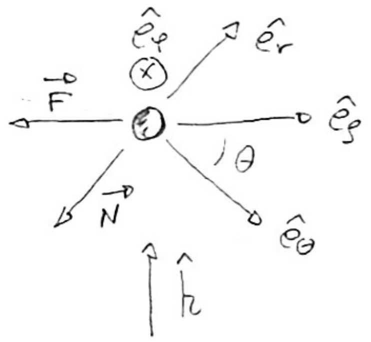
$$m\ddot{x} = -kx - \mu_k\left(mg - m\omega^2 H \operatorname{sen}\omega t\right).\ \text{signo}(\dot{x})$$

$$\boxed{m\ddot{x} = -kx - \mu_k\left(mg - m\omega^2 H \operatorname{sen}\omega t\right)\frac{\dot{x}}{|\dot{x}|}}$$

③

ⓐ





$$\vec{L}_O = \vec{r} \times \vec{p} \Rightarrow \frac{d\vec{L}_O}{dt} = \underbrace{\vec{v} \times \vec{p}}_{=\emptyset} + \vec{r} \times \dot{\vec{p}} = \vec{r} \times m\vec{a} = \vec{r} \times \Sigma\vec{F}$$

$$\frac{d}{dt}\vec{L}_O = R\hat{e}_r \times \left(-N\hat{e}_r - F\hat{e}_\rho\right) = RF\cos\theta\,\hat{e}_\varphi$$

$$\frac{d}{dt}\left(\vec{L}_O \cdot \hat{k}\right) = \left(\frac{d}{dt}\vec{L}_O\right) \cdot \hat{k} = \emptyset \quad \Rightarrow \quad \boxed{\vec{L}_O \cdot \vec{k} = c^{te}}$$

$$\vec{L}_O = R\hat{e}_r \times m\left(R\dot{\theta}\,\hat{e}_\theta + R\,\text{sen}\theta\,\dot{\varphi}\,\hat{e}_\varphi\right) = mR^2\dot{\theta}\,\hat{e}_\varphi - mR^2\,\text{sen}\theta\,\dot{\varphi}\,\hat{e}_\theta$$

$$\vec{L}_O \cdot \hat{k} = -mR^2\,\text{sen}\theta\,\dot{\varphi}\left(\hat{e}_\theta \cdot \hat{k}\right) = mR^2\,\text{sen}^2\theta\,\dot{\varphi} \Rightarrow \boxed{mR^2\,\text{sen}^2\theta\,\dot{\varphi} = \ell = c^{te}}$$

ⓑ $\vec{N} \cdot \vec{v} = \emptyset \Rightarrow$ fuerza reactiva de potencia nula.

$$\vec{F} = -F\,\hat{e}_\rho = -\vec{\nabla}U \;/\; U = F\rho = FR\,\text{sen}\theta \qquad \vec{F} = \text{conservativa}$$

$$E_{mec} = c^{te} \Rightarrow \quad K + U = c^{te}$$

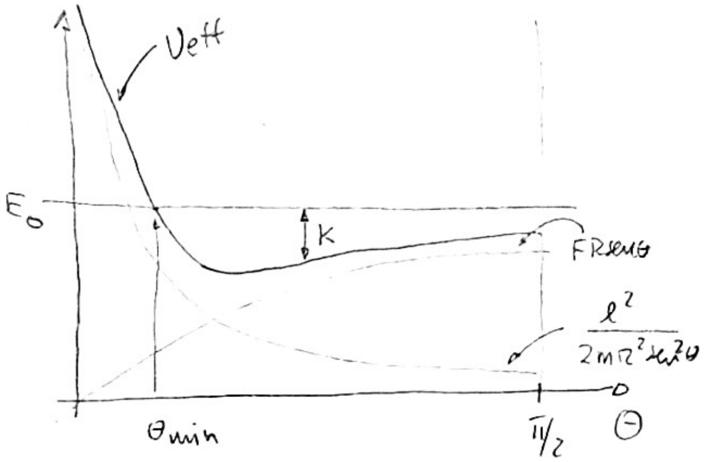
$$\frac{1}{2}m\left(R^2\dot{\theta}^2 + R^2\,\text{sen}^2\theta\,\dot{\varphi}^2\right) + FR\,\text{sen}\theta = E_0$$

$$E_0 = \frac{mR^2\dot{\theta}^2}{2} + \frac{mR^2\,\text{sen}^2\theta}{2}\left(\frac{\ell^2}{m^2R^4\,\text{sen}^4\theta}\right) + FR\,\text{sen}\theta$$

$$\boxed{E_0 = \frac{mR^2\dot{\theta}^2}{2} + \frac{\ell^2}{2mR^2\,\text{sen}^2\theta} + FR\,\text{sen}\theta} \qquad U_{eff} = \frac{\ell^2}{2mR^2\,\text{sen}^2\theta} + FR\,\text{sen}\theta$$

(c)

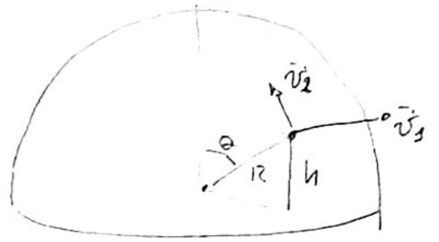
$h_{max} \Rightarrow \theta_{min}$



$$E_0 = \frac{\ell^2}{2mR^2 sen^2\theta_{min}} + FR sen\theta_{min}$$

$$\begin{cases} \ell = m R sen\theta_0 v_1 = m\sqrt{R^2-h^2}\, v_1 \\ E_0 = \frac{m}{2}\left(v_1^2+v_2^2\right) + F\sqrt{R^2-h^2} \end{cases}$$

en el instante inicial



$$\boxed{E_0 = \frac{\ell^2}{2m\left(R^2-h_{max}^2\right)} + F\sqrt{R^2-h_{max}^2}}$$

Dependiendo del potencial y las condiciones iniciales podrían haber 2 raíces físicamente válidas. Obviamente se debe optar por la que corresponda al mínimo valor de θ o máximo valor de h.