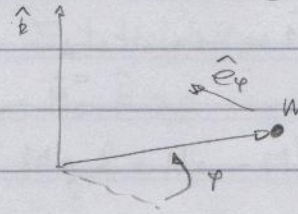
Ejercicio 1 -

(a)



$$\vec r = r\hat e_r + z\hat k \Rightarrow \vec v = \dot r\,\hat e_r + r\dot\varphi\,\hat e_\varphi + \dot z\,\hat k$$

$$\vec a = (\ddot r - r\dot\varphi^2)\,\hat e_r + (r\ddot\varphi + 2\dot r\dot\varphi)\,\hat e_\varphi + \ddot z\,\hat k$$

$$m\vec a\cdot\hat k = m\ddot z = \vec F\cdot\hat k = 0 \Rightarrow \dot z = \dot z_0 = 0 \,;\, \boxed{z = c^{te}}$$

$$m\vec a\cdot\hat e_\varphi = m(r\ddot\varphi + 2\dot r\dot\varphi) = b\dot r$$

$$\vec F = b\hat k\times\vec v = b\hat k\times(\dot r\,\hat e_r + r\dot\varphi\,\hat e_\varphi) = b\dot r\,\hat e_\varphi - br\dot\varphi\,\hat e_r$$

$$r\ddot\varphi + 2\dot r\dot\varphi = \frac{b\dot r}{m} \Rightarrow r\ddot\varphi + 2\dot r\left(\dot\varphi - \frac{b}{2m}\right) = 0$$

Si (H) $r^2\left(\dot\varphi - \frac{b}{2m}\right) = c^{te} \Rightarrow \frac{d}{dt}\left(r^2\left(\dot\varphi - \frac{b}{2m}\right)\right) = 0$

$$2r\dot r\left(\dot\varphi - \frac{b}{2m}\right) + r^2\ddot\varphi = 0$$

$$r\left(2\dot r\left(\dot\varphi - \frac{b}{2m}\right) + r\ddot\varphi\right) = 0.$$

Efectivamente la Hipotesis (H) se verifica.

(b) Observación $\vec F \perp \vec v \Rightarrow$ es de potencia nula $\Rightarrow$ la energía cinética es constante

$$K = \frac{1}{2}m\left(\dot r^2 + r^2\dot\varphi^2\right) = c^{te}$$

Por otro lado sea $r^2\left(\dot\varphi - \frac{b}{2m}\right) = \ell \; (= c^{te})$

$$\dot r^2 + r^2\left(\frac{\ell}{r^2} + \frac{b}{2m}\right)^2 = \mathbb{C} \; (= c^{te})$$

$$\dot r^2 + r^2\left(\frac{\ell^2}{r^4} + \frac{b^2}{4m^2} + \frac{b\ell}{mr^2}\right) = \mathbb{C}$$

$$\dot r^2 + \frac{\ell^2}{r^2} + \frac{b^2}{4m^2}r^2 = \mathbb{C}' \quad ; \quad \underset{C.I.}{\mathbb{C}'} = \frac{\ell^2}{r_0^2} + \frac{b^2 r_0^2}{4m^2}$$

$$\ell = r_0 v_0 - \frac{b r_0^2}{2m}$$

Ec. de movimiento para r (derivando):

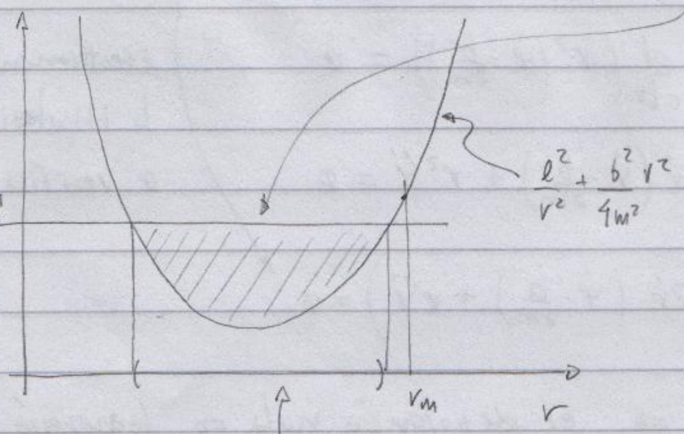
$$2\ddot{r} - \frac{2\ell^2}{r^3} + \frac{b^2}{2m^2} r = 0.$$

$$\boxed{\ddot{r} - \frac{\ell^2}{r^3} + \frac{b^2}{4m^2} r = 0}$$

(c) Utilizando la ecuación:

$$\dot{r}^2 + \frac{\ell^2}{r^2} + \frac{b^2}{4m^2} r^2 = C'$$

$$\dot{r}^2 = C' - \left(\frac{\ell^2}{r^2} + \frac{b^2}{4m^2} r^2\right) \geq 0$$



Para que nunca llegue a $r_m = 0$

$$C' < \frac{\ell^2}{r_m^2} + \frac{b^2 r_m^2}{4m^2}$$

C.I.

$$\hookrightarrow C' = \frac{\ell^2}{r_0^2} + \frac{b^2 r_0^2}{4m^2}$$

$$\frac{\ell^2}{r_0^2} + \frac{b^2 r_0^2}{4m^2} < \frac{\ell^2}{r_m^2} + \frac{b^2 r_m^2}{4m^2}$$

$$\frac{b^2}{4m^2}\left(r_0^2 - r_m^2\right) \leq \ell^2\left(\frac{1}{r_m^2} - \frac{1}{r_0^2}\right)$$

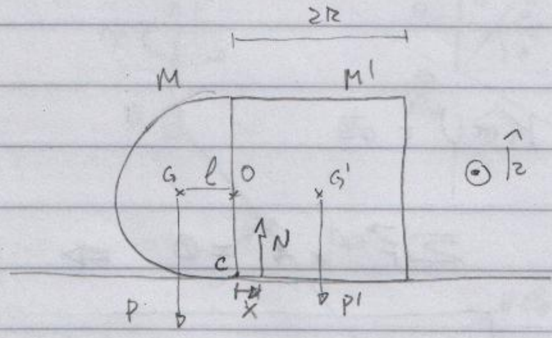
$$r_m^2 r_0^2 \leq \frac{4m^2\ell^2}{b^2}$$

$$r_m r_0 \leq \frac{2m\ell}{b} = \frac{2m}{b}\left(r_0 v_0 - \frac{b r_0^2}{2m}\right)$$

$$r_m \cancel{r_0} \leq \frac{2m \cancel{r_0} v_0}{b} - r_0^{\cancel{2}}$$

$$\boxed{r_m \leq \frac{2m v_0}{b} - r_0}$$

Ejercicio 2.—



ⓐ

Estaticidad $\Rightarrow \vec{M}_C = 0$

$\vec{M}_C \cdot \hat{k} = 0$

$Mg\,l + Nx - M'g\,R = 0$

$l = \frac{4R}{3\pi}$ $\longrightarrow$ $Nx = M'gR - Mg\frac{4R}{3\pi}$

Para impedir vuelco en sentido $\hat{k}$ (anti-horario) $\Rightarrow$ debo imponer $x \geq 0$.

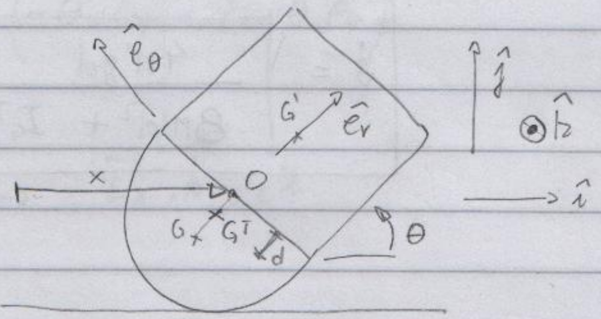
(como además $N > 0$) $\Rightarrow$ $M' - \frac{4M}{3\pi} \geq 0$.

$$\boxed{\frac{M'}{M} \geq \frac{4}{3\pi}} \qquad \frac{4}{3\pi} \simeq 0{,}42$$

ⓑ Si $M = 3M'$ $\Rightarrow$ no se cumple la condición de estaticidad.

$\vec{r}_G = x\,\hat{\imath} - d\,\hat{e}_r$

$\vec{v}_G = \dot{x}\,\hat{\imath} - d\dot{\theta}\,\hat{e}_\theta$



$4M'd = M'R - 3M'l$

$\boxed{d = R/4 - 3l/4}$

Observación: Sistema conservativo $\rightarrow$ la energía mecánica es constante.

K:

$$I_{G_T}^T = I_{G_T}^D + I_{G_T}^{\square} = \underbrace{\overbrace{\left(\frac{MR^2}{2} - Ml^2\right)}^{I_G^D} + M(l-d)^2}_{I_{G_T}^D} + \underbrace{\frac{M'(2R)^2}{6} + M'(R+d)^2}_{I_{G_T}^{\square}} =$$

$$= \frac{3M'R^2}{2} + 3M(d^2 - 2ld) + \frac{2M'R^2}{3} + M'(R+d)^2$$

$$K = \frac{1}{2}(4M')\left(\dot{x}^2 + d^2\dot{\theta}^2 + 2\dot{x}d\dot{\theta}\,\mathrm{sen}\,\theta\right) + \frac{1}{2} I_{G_T}^T \dot{\theta}^2$$

$$U = 4M'g\vec{r}_G\cdot\hat{j} = 4M'g\,(x\hat{\imath} - d\hat{e}_r)\cdot\hat{j} = -4M'g\,d\,\text{sen}\,\theta.$$

$$E_{mec} = K + U = cte$$

Ademas $\quad \sum \vec{F}^{ext}\cdot\hat{\imath} = 0 \Rightarrow m\vec{a}_G\cdot\hat{\imath} = 0 \Rightarrow m\vec{v}_G\cdot\hat{\imath} = cte$

$$\vec{v}_G\cdot\hat{\imath} = \boxed{\dot{x} - d\dot{\theta}\,\text{sen}\,\theta = 0}$$

Cond. iniciales $(\dot{\theta} = 0,\ \dot{x} = 0,\ \theta = 0)$

$$\boxed{2M'(\dot{x}^2 + d^2\dot{\theta}^2 + 2d\dot{x}\dot{\theta}\,\text{sen}\,\theta)^2 + \frac{1}{2}I_{G_T}^{T}\dot{\theta}^2 - 4M'g\,d\,\text{sen}\,\theta = 0}$$

(c) $$2M'd^2\dot{\theta}^2(1 + 3\,\text{sen}^2\theta) + \frac{1}{2}\dot{\theta}^2 I_G^T - 4M'g\,d\,\text{sen}\,\theta = 0.$$

$$\dot{\theta} = \sqrt{\frac{4M'g\,d\,\text{sen}\,\theta}{2M'd^2(1 + 3\,\text{sen}^2\theta) + \frac{I_G^T}{2}}}$$

$\theta = \pi/2$

$$\boxed{\dot{\theta} = \sqrt{\frac{4M'gd}{8M'd^2 + I_G^T/2}}}$$

Ejercicio 3 -

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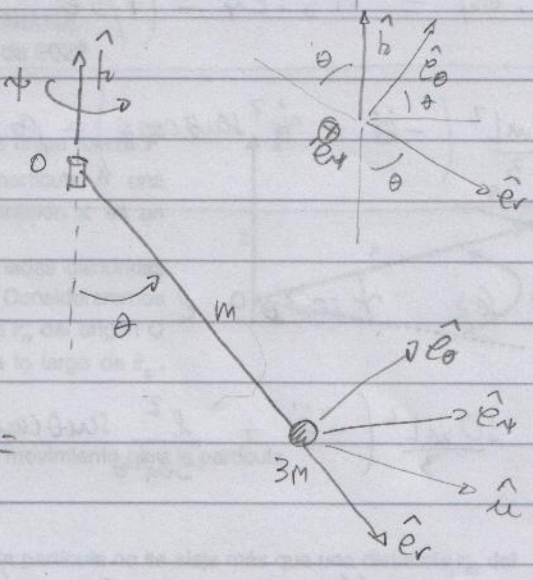

$$\mathbb{I}_G^{barra} = \frac{mL^2}{12}\begin{bmatrix} 0 & & \\ & 1 & \\ & & 1 \end{bmatrix} \quad (\hat{e}_r, \hat{e}_\psi, \hat{e}_\theta)$$

$$\mathbb{I}_O^{barra} = \frac{mL^2}{12}\begin{bmatrix} 0 & & \\ & 1 & \\ & & 1 \end{bmatrix} + \frac{mL^2}{4}\begin{bmatrix} 0 & & \\ & 1 & \\ & & 1 \end{bmatrix} =$$

$$= \frac{mL^2}{3}\begin{bmatrix} 0 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

$$\mathbb{I}_O^{Total} = \frac{mL^2}{3}\begin{bmatrix} 0 & & \\ & 1 & \\ & & 1 \end{bmatrix} + 3mL^2\begin{bmatrix} 0 & & \\ & 1 & \\ & & 1 \end{bmatrix} = \frac{10}{3} mL^2 \begin{bmatrix} 0 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

$$\vec{\omega} = -\dot{\theta}\,\hat{e}_\psi + \dot{\psi}\,\hat{k} = -\dot{\theta}\,\hat{e}_\psi + \dot{\psi}\cos\theta\,\hat{e}_r + \dot{\psi}\,\mathrm{sen}\,\theta\,\hat{e}_\theta$$

$$\vec{L}_O = \mathbb{I}_O^{Total}\vec{\omega} = \frac{10mL^2}{3}\left(-\dot{\theta}\,\hat{e}_\psi + \dot{\psi}\,\mathrm{sen}\,\theta\,\hat{e}_\theta\right)$$

Ⓑ

$$\left.\begin{aligned} \dot{\vec{L}}_O \cdot \hat{k} &= \vec{M}_O^{ext} \cdot \hat{k} = 0 \\ \hat{k} &= c^{te} \end{aligned}\right\} \Rightarrow \vec{L}_O \cdot \hat{k} = c^{te}$$

$$\vec{L}_O \cdot \hat{k} = \frac{10mL^2}{3}\dot{\psi}\,\mathrm{sen}^2\theta = c^{te} \Rightarrow \boxed{\dot{\psi}\,\mathrm{sen}^2\theta = c^{te}}$$

$$\dot{\vec{L}}_O = \frac{10mL^2}{3}\Big(-\ddot{\theta}\,\hat{e}_\psi + \left(\ddot{\psi}\,\mathrm{sen}\,\theta + \dot{\psi}\dot{\theta}\cos\theta\right)\hat{e}_\theta + \dot{\theta}\dot{\psi}\,\hat{u} +$$
$$+ \dot{\psi}\,\mathrm{sen}\,\theta\left(\dot{\psi}\cos\theta\,\hat{e}_\psi - \dot{\theta}\,\hat{e}_r\right)\Big)$$

$$\dot{\hat{e}}_\psi = \vec{\omega} \times \hat{e}_\psi = -\dot{\psi}\,\hat{u}$$
$$\dot{\hat{e}}_\theta = \vec{\omega} \times \hat{e}_\theta = -\dot{\theta}\,\hat{e}_r + \dot{\psi}\cos\theta\,\hat{e}_\psi$$

$$\dot{\vec{L}}_0 \cdot \hat{e}_\psi = \vec{M}_0^{ext} \cdot \hat{e}_\psi = \left(+\beta\dot{\theta} + mg\frac{L}{2}\operatorname{sen}\theta + 3mgL\operatorname{sen}\theta\right)$$

$$\boxed{\frac{10mL^2}{3}\left(-\ddot{\theta} + \dot{\psi}^2 \operatorname{sen}\theta\cos\theta\right) = \beta\dot{\theta} + \frac{7}{2}mgL\operatorname{sen}\theta}$$

c) Seja $\dot{\psi}\operatorname{sen}^2\theta = \ell$

$$\frac{10mL^2}{3}\left(-\ddot{\theta} + \frac{\ell^2}{\operatorname{sen}^4\theta}\operatorname{sen}\theta\cos\theta\right) - \beta\dot{\theta} - \frac{7}{2}mgL\operatorname{sen}\theta = \emptyset$$

$$\ddot{\theta} + \frac{3\beta}{10mL^2}\dot{\theta} + \left(\frac{21}{20}\,g/L\,\operatorname{sen}\theta - \frac{\ell^2\cos\theta}{\operatorname{sen}^3\theta}\right) = \emptyset$$

Movimento circular $\Rightarrow$ $\theta = \theta_0 = c^{te} \Rightarrow \dot{\theta} = \ddot{\theta} = \emptyset$

$\dot{\psi} = \dot{\psi}_0 = c^{te}$

$$\dot{\psi}_0 \operatorname{sen}^2\theta_0 = \ell$$

$$\frac{21}{20}\,g/L\,\operatorname{sen}\theta_0 - \dot{\psi}_0^2\operatorname{sen}\theta_0\cos\theta_0 = \emptyset$$

$$\operatorname{sen}\theta_0\left(\frac{21}{20}\,g/L - \dot{\psi}_0^2\cos\theta_0\right) = \emptyset$$

$$\cos\theta_0 = \frac{21}{20}\,\frac{g}{L\dot{\psi}_0^2}$$

Raio do giro $= L\operatorname{sen}\theta_0 = L\sqrt{1 - \left(\frac{21}{20}\,\frac{g}{L\dot{\psi}_0^2}\right)^2}$