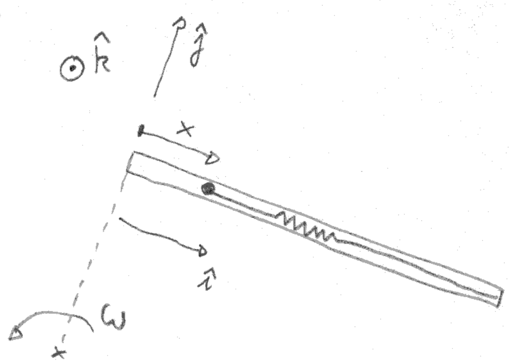
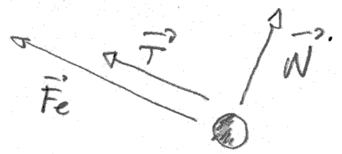
1

a





$$m\vec{a} = N\hat{j} - T\hat{i} - kx\hat{i}$$

$$(\hat{i}) \quad m(\ddot{x} - x\omega^2) = -T - kx$$

$$(\hat{j}) \quad m(-L\omega^2 + 2\dot{x}\omega) = N$$

$$\vec{r} = L\hat{j} + x\hat{i}$$

$$\vec{v} = -L\omega\hat{i} + \dot{x}\hat{i} + x\omega\hat{j}$$

$$\vec{a} = -L\omega^2\hat{j} + \ddot{x}\hat{i} + 2\dot{x}\omega\hat{j} - x\omega^2\hat{i}$$

$$\boxed{\vec{N} = m(2\dot{x}\omega - L\omega^2)\hat{j}}$$

$$\vec{T} = -\mu|N|\,\mathrm{sgn}(\dot{x})\,\hat{i}$$

$$\boxed{\vec{T} = -m\mu|2\dot{x}\omega - L\omega^2|\,\mathrm{sgn}(\dot{x})\,\hat{i}}$$

b

$$\ddot{x} - x\omega^2 = -\frac{k}{m}x - \mu\,|2\dot{x}\omega - L\omega^2|\,\mathrm{sgn}(\dot{x})$$

$$\ddot{x} + \mu|2\dot{x}\omega - L\omega^2|\,\mathrm{sgn}(\dot{x}) + (k/m - \omega^2)x = 0$$

$$\begin{cases} \ddot{x} + \mu(2\dot{x}\omega - L\omega^2) + (k/m - \omega^2)x = 0 & \dot{x} > \frac{L\omega}{2} \\ \ddot{x} - \mu(2\dot{x}\omega - L\omega^2) + (k/m - \omega^2)x = 0 & \dot{x} < \frac{L\omega}{2} \end{cases}$$

incluye $\dot{x} < 0$.

c "Al comienzo del movimiento" verifica $\dot{x} > \frac{L\omega}{2}$

$(\dot{x}(t=0) = L\omega \; ; \; x(0) = 0)$

$\mu = 1, \quad \frac{k}{m} = 3\omega^2$

$$\ddot{x} + 2\omega\dot{x} + 2\omega^2 x = L\omega^2$$

$$\gamma_i = \frac{-2\omega \pm \sqrt{4\omega^2 - 8\omega^2}}{2} = -\omega \pm i\omega$$

① ⓒ cont.

$$x_H = A e^{-\omega t} \cos(\omega t) + B e^{-\omega t} \operatorname{sen}(\omega t)$$

$$x_p = \frac{L}{2}$$

$$x(t) = x_H + x_p \qquad x(0) = \emptyset \Rightarrow A + \frac{L}{2} = \emptyset \Rightarrow \boxed{A = -L/2}$$

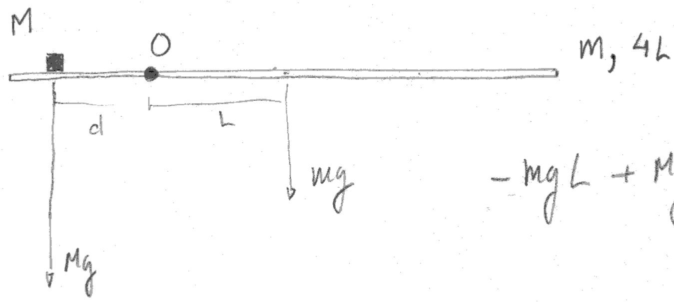
$$\dot{x}(t) = -A\omega e^{-\omega t} \cos(\omega t) - A\omega e^{-\omega t} \operatorname{sen}(\omega t)$$
$$- B\omega e^{-\omega t} \operatorname{sen}(\omega t) + B\omega e^{-\omega t} \cos(\omega t)$$

$$\dot{x}(0) = L\omega \Rightarrow -A\omega + B\omega = L\omega$$
$$\frac{L}{2} + B = L \Rightarrow \boxed{B = L/2}$$

$$\boxed{x(t) = \frac{L}{2} e^{-\omega t} \left(\operatorname{sen}(\omega t) - \cos(\omega t) \right) + \frac{L}{2}}$$

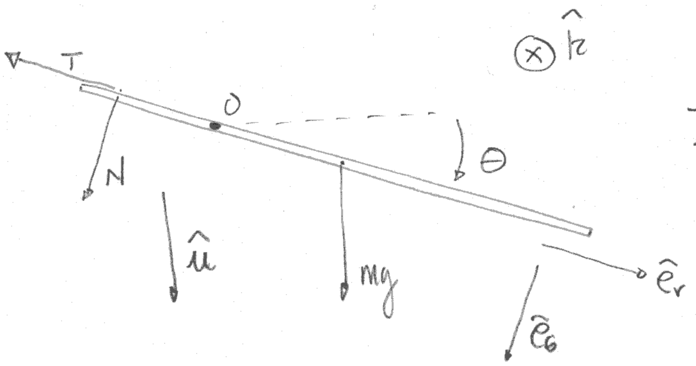
(2)

(a)



$$-mgL + Mgd = \vec{M}_0^{ext} = 0 \Rightarrow \boxed{m = \frac{M}{3}}$$

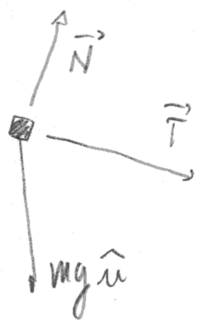
(b)



$$I_0^{\hat{k}} = I_G^{\hat{k}} + mL^2 = \frac{4mL^2}{3} + mL^2 = \frac{7mL^2}{3}$$

$$I_0^{\hat{k}}\,\ddot{\theta}\,\hat{k} = \vec{M}_0^{ext}\cdot\hat{k}$$

$$\frac{7mL^2}{3}\ddot{\theta} = mgL\cos\theta - N\frac{L}{3}$$



$$m\vec{a}_{Part.} = -N\hat{e}_\theta + T\hat{e}_r + mg\hat{u}$$

$$m\left(-\frac{L}{3}\ddot{\theta}\,\hat{e}_\theta + \frac{L}{3}\dot{\theta}^2\hat{e}_r\right) = -N\hat{e}_\theta + T\hat{e}_r + mg\hat{u}$$

$(\hat{e}_\theta)$ $\quad m\frac{L}{3}\ddot{\theta} = N - mg\cos\theta$

$(\hat{e}_r)$ $\quad m\frac{L}{3}\dot{\theta}^2 = T + mg\,\mathrm{sen}\theta$

$$\frac{7}{3}\cancel{m}L^{\cancel{2}}\ddot{\theta} = \cancel{m}g\cancel{L}\cos\theta - \frac{L}{3}\left(m\frac{L}{3}\ddot{\theta} + mg\cos\theta\right)$$

$$\frac{7}{3}L\ddot{\theta} + \frac{L}{9}\ddot{\theta} = g\cos\theta\left(1 - 1/3\right)$$

$$22\ddot{\theta} = \frac{6g}{L}\cos\theta$$

$$\boxed{\ddot{\theta} = \frac{3}{11}g/L\cos\theta} \Rightarrow \frac{\dot{\theta}^2}{2} = \frac{3}{11}g/L\,\mathrm{sen}\theta$$

$$\boxed{\begin{array}{l}\theta(0) = 0 \\ \dot{\theta}(0) = 0\end{array}}$$

② Cont.

$$T = \frac{mL}{3}\cdot\left(\frac{6}{11}\,g/_L\,\text{sen}\theta\right) - mg\,\text{sen}\theta = -\frac{9}{11}\,mg\,\text{sen}\theta$$

$$\vec{T} = -\frac{9}{11}\,mg\,\text{sen}\theta\,\hat{e}_r$$

$$N = \frac{mL}{3}\left(\frac{3}{11}\,g/_L\cos\theta\right) + mg\cos\theta = \frac{12}{11}\,mg\cos\theta$$

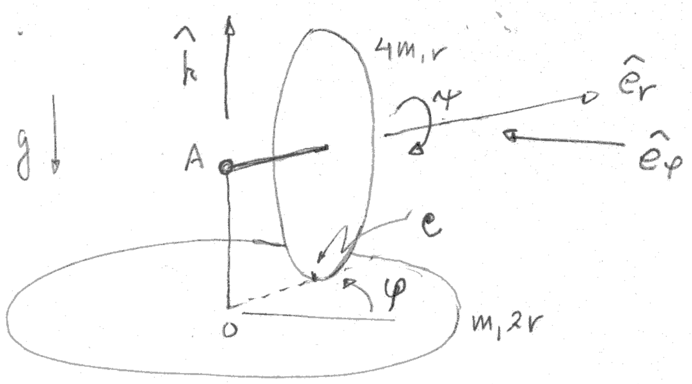
$$\vec{N} = -\frac{12}{11}\,mg\cos\theta\,\hat{e}_\theta$$

$$¿\,|\vec{T}| = \mu|\vec{N}|\,? \Rightarrow \quad \frac{9}{11}\,\text{sen}\theta_d = \frac{12}{11}\cos\theta_d$$

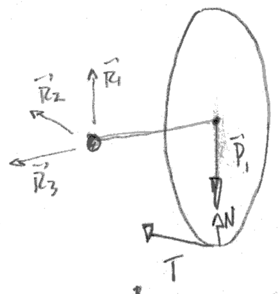
$$\boxed{tg\theta_d = \frac{4}{3}} \qquad \theta_d \simeq 53^{\circ}$$

La partícula desliza acercándose a O.

③



ⓐ



Esta elección de T implica asumir $\dot{\psi}(0)<0$.

$$\mathbb{I}_A = \underset{(\hat{e}_r,\hat{e}_\varphi,\hat{k})}{\mathbb{I}_G} + 4mr^2\begin{bmatrix} 0 & & \\ & 1 & \\ & & 1 \end{bmatrix} = mr^2\begin{bmatrix} 2 & & \\ & 5 & \\ & & 5 \end{bmatrix}$$

$$\mathbb{I}_G = \frac{(4m)r^2}{2}\begin{bmatrix} 1 & & \\ & \frac{1}{2} & \\ & & \frac{1}{2} \end{bmatrix}$$

$$\left\{\begin{array}{ll} \mathbb{I}_A = mr^2\begin{bmatrix} 2 & & \\ & 5 & \\ & & 5 \end{bmatrix} ; & \mathbb{I}_A \dot{\vec{\omega}}_{Disco} = \vec{M}^{ext}_A \\ \vec{\omega}_{Disco} = \dot{\varphi}\hat{k} + \dot{\psi}\hat{e}_r ; & \vec{M}^{ext}_A = (4mgr - Nr)\hat{e}_\varphi + (r\hat{e}_r - r\hat{k}) \times T\hat{e}_\varphi = \\ & = (4mgr - Nr)\hat{e}_\varphi + Tr\hat{k} + Tr\hat{e}_r \end{array}\right.$$

$$\to \frac{d}{dt} mr^2\left(2\dot{\psi}\hat{e}_r + 5\dot{\varphi}\hat{k}\right) = (4mgr - Nr)\hat{e}_\varphi + Tr\hat{k} + Tr\hat{e}_r$$

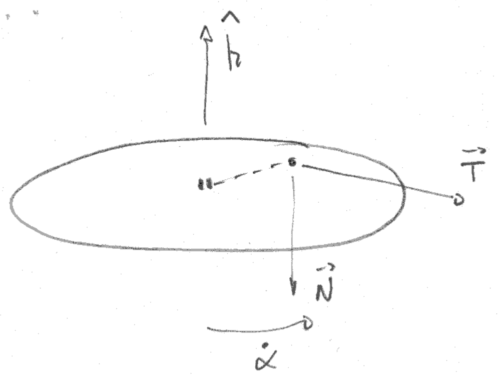
$$mr^2\left(2\ddot{\psi}\hat{e}_r + 2\dot{\psi}\dot{\varphi}\hat{e}_\varphi + 5\ddot{\varphi}\hat{k}\right) = (4mgr - Nr)\hat{e}_\varphi + Tr\hat{k} + Tr\hat{e}_r$$

$$\left\{\begin{array}{l} 2mr\ddot{\psi} = T \\ 2mr\dot{\psi}\dot{\varphi} = 4mg - N \\ 5mr\ddot{\varphi} = T \end{array}\right.$$

$$2\ddot{\psi} = 5\ddot{\varphi}$$

$$2(\dot{\psi} - \dot{\psi}_0) = 5\dot{\varphi}$$

$$\boxed{\frac{2}{5}(\dot{\psi} - \dot{\psi}_0) = \dot{\varphi}}$$



$$I_0^k \ddot{\alpha} \hat{k} = -Tr\hat{k}$$

$$\frac{m(2r)^2}{2}\ddot{\alpha} = -Tr$$

$$2mr^2\ddot{\alpha} = -Tr$$

$$2mr\ddot{\alpha} = -T \Rightarrow \ddot{\alpha} = -\ddot{\psi}$$

$$\boxed{\dot{\alpha} = -(\dot{\psi} - \dot{\psi}_0)}$$

Condición de rodadura sin deslizamiento =

$$\vec{v}_c = r\dot{\varphi}\hat{e}_\varphi + r\dot{\psi}\hat{e}_\varphi = r\dot{\alpha}\hat{e}_\varphi \Rightarrow \dot{\varphi} + \dot{\psi} = \dot{\alpha}$$

$$\frac{2}{5}(\dot{\psi} - \dot{\psi}_0) + \dot{\psi} = -\dot{\psi} + \dot{\psi}_0 \Rightarrow 2\dot{\psi} - 2\dot{\psi}_0 + 5\dot{\psi} = -5\dot{\psi} + 5\dot{\psi}_0$$

$$12\dot{\psi} = 7\dot{\psi}_0$$

$$\dot{\psi}_{RSD} = \frac{7}{12}\dot{\psi}_0$$

$$\dot{\alpha}_{RSD} = \dot{\alpha}_\infty = -\dot{\psi}_{RSD} + \dot{\psi}_0 = -\frac{7}{12}\dot{\psi}_0 + \dot{\psi}_0$$

$$\dot{\varphi}_\infty = \frac{2}{5}\left(-\frac{5}{12}\right)\dot{\psi}_0$$

$$\dot{\varphi}_\infty = -\frac{\dot{\psi}_0}{6}$$

$$\boxed{\frac{5}{12}\dot{\psi}_0 = \dot{\alpha}_{RSD}}$$

Obs: signo $\dot{\psi}_0$ igual al signo de $\dot{\alpha}_\infty \Rightarrow$ consistente.

(b)

$$E_{mec\,0} = K_{\text{Disco } 4m} = \frac{1}{2}\vec{\omega}_0 \mathbb{I}_A \vec{\omega}_0 = mr^2\dot{\psi}_0^2$$

$$\vec{\omega}_0 = \dot{\psi}_0 \vec{e}_r$$

$$\boxed{\Delta E_{mec} = -\frac{5}{12}mr^2\dot{\psi}_0^2}$$

$$E_{mec\,\infty} = \frac{1}{2}\vec{\omega}_\infty \mathbb{I}_A \vec{\omega}_\infty + \frac{1}{2}I_0^k\dot{\alpha}_\infty^2 =$$

$$= \frac{1}{2}\left(\frac{7}{12}\dot{\psi}_0\hat{e}_r - \frac{\dot{\psi}_0}{6}\hat{k}\right)mr^2\begin{bmatrix}2 & & \\ & 5 & \\ & & 5\end{bmatrix}\left(\frac{7}{12}\dot{\psi}_0\hat{e}_r - \frac{\dot{\psi}_0}{6}\hat{k}\right) + mr^2\frac{25}{144}\dot{\psi}_0^2 =$$

$$= \frac{mr^2}{2}\left(\frac{49}{72}\dot{\psi}_0^2 + \frac{5}{36}\dot{\psi}_0^2\right) + \frac{mr^2}{2}\frac{25}{72}\dot{\psi}_0^2 = \frac{7mr^2\dot{\psi}_0^2}{12}$$