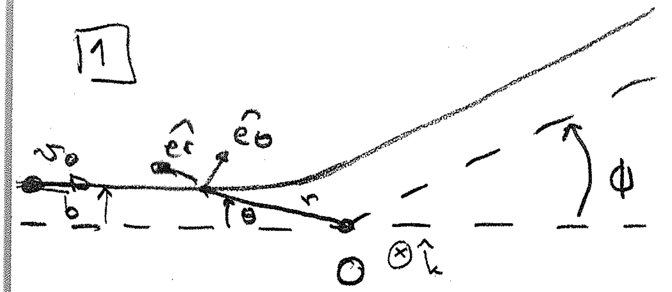
1



$$U(r) = \frac{A}{r} \rightarrow \vec{f}(r) = -\nabla U = \frac{A}{r^2}\hat{e}_r$$

$$\vec{r} = r\,\hat{e}_r$$

a) tenemos una fuerza central

$$\vec{L}_0 = \vec{r}\wedge\vec{p} = \vec{r}\wedge m\vec{v} \overset{\text{en } t=0}{=} -m v_0 b\,\hat{k} \quad \text{se conserva} \quad l = m v_0 b$$

$$\vec{v}(t) = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta \Rightarrow \vec{L}_0 = m r^2\dot{\theta}\,\hat{k} \Rightarrow l = m r^2 \dot{\theta}$$

para la trayectoria usamos Binet.

$$u(\theta) = \frac{1}{r} \Rightarrow a_r = -\frac{l^2 u^2}{m^2}\left[u + u''\right]$$

$$a_r = \frac{f(r)}{m} = \frac{A}{m r^2} = \frac{A u^2}{m} \Rightarrow \frac{A u^2}{m} = -\frac{l^2 u^2}{m^2}\left[u + u''\right]$$

$$u'' + u + \frac{Am}{l^2} = 0$$

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{du}\frac{du}{dt} = \frac{dr}{du}\frac{du}{d\theta}\frac{d\theta}{dt} = -\frac{1}{u^2}u'\dot{\theta} \rightarrow \dot{r} = -\frac{l u'}{m}$$

$$\dot{\theta} = \frac{l}{m r^2} = \frac{l u^2}{m}$$

pruebo $u(\theta) = \frac{1}{r} = \frac{e\cos(\theta-\alpha) - 1}{p}$

$$u' = -\frac{e}{p}\sin(\theta-\alpha)$$

$$u'' = -\frac{e}{p}\cos(\theta-\alpha) \qquad -\frac{e}{p}\cos(\theta-\alpha) + \frac{e\cos(\theta-\alpha)-1}{p} + \frac{Am}{l^2} = 0$$

$$\Rightarrow \boxed{p = \frac{l^2}{Am}}$$

resta determinar e y α

$u(0)=0$ (pues $r\to\infty$ en $t=0$) $\Rightarrow$ $\boxed{\cos(\alpha)=\frac{1}{e}}$ (ii)

$$\dot r(t=0) = -v_0 = -\frac{\ell\, u'(0)}{m} = -v_0 b\, \frac{e}{p} \sin(\alpha)$$

$$p = e b \sin\alpha = b\, tg(\alpha)$$

$$\frac{m(v_0 b)^2}{A} = b\, tg(\alpha) \Rightarrow \boxed{tg(\alpha) = \frac{m v_0^2 b}{A}}$$

$$\boxed{\sin(\alpha) = \frac{\ell^2}{Am}\frac{1}{eb}}$$

$$\cos^2\alpha + \sin^2\alpha = \frac{1}{e^2}\left(1 + \left(\frac{\ell^2}{Am}\right)^2\frac{1}{b^2}\right) = 1 \Rightarrow \boxed{e^2 = 1 + \left(\frac{\ell^2}{Am}\right)^2\frac{1}{b^2}}$$

b) $\theta_1 = \pi - \phi$

$r(\theta_1)\to\infty \Rightarrow u(\theta_1)=0 \Rightarrow \cos(\theta_1-\alpha) = \frac{1}{e}$

$\theta_1 = 0$ es la condición inicial.

$\theta_1 \neq 0$ es la que nos da ϕ $\Rightarrow$ usando (ii)

$$\cos(\theta_1-\alpha) = \cos(\alpha) \Rightarrow \theta_1 - \alpha = \alpha$$

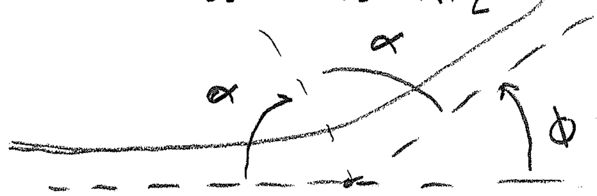
$$\theta_1 = 2\alpha$$

$$\hookrightarrow \pi - \phi = 2\alpha$$

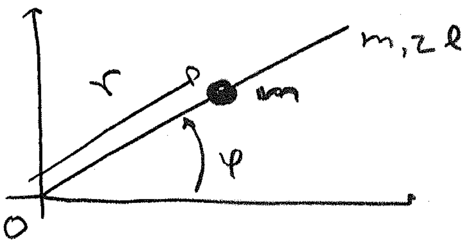
$$\boxed{\phi = \pi - 2\alpha}$$

además $\frac{\pi-\phi}{2} = \alpha$

α es la bisectriz



2

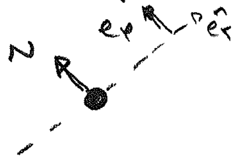


$r(0)=\ell$ y reposo relativo a la barra

$\dot\varphi(0)=\omega$

a) para la partícula 1ra Ley:

la partícula se mueve libremente



$$\boxed{\begin{aligned} m(\ddot r - r\dot\varphi^2) &= 0 \\ m(r\ddot\varphi + 2\dot r\dot\varphi) &= N \end{aligned}}$$

barra 2da cardinal desde O $\quad I_o\ddot\varphi = -Nr$

$$I_o = \frac{4}{3}m\ell^2 \qquad \boxed{\frac{4}{3}m\ell^2\ddot\varphi = -Nr}$$

b) $$\frac{4}{3}m\ell^2\ddot\varphi = -m(r^2\ddot\varphi + 2r\dot r\dot\varphi)$$

$$m\left(\left(\frac{4}{3}\ell^2 + r^2\right)\ddot\varphi + 2r\dot r\dot\varphi\right) = 0$$

$$\to \frac{d}{dt}\left\{\left(\frac{4}{3}\ell^2 + r^2\right)\dot\varphi\right\} = 0$$

$$\Rightarrow \left(\frac{4}{3}\ell^2 + r^2\right)\dot\varphi = C \xrightarrow[C.I]{} C = \left(\frac{4}{3}\ell^2 + \ell^2\right)\omega = \frac{7}{3}\ell^2\omega$$

$$\left(\frac{4}{3}\ell^2 + r^2\right)\dot\varphi = \frac{7}{3}\ell^2\omega \to (4\ell^2 + 3r^2)\dot\varphi = 7\ell^2\omega$$

$$\to \boxed{\dot\varphi = \frac{7\ell^2\omega}{4\ell^2 + 3r^2}}$$

c) $$m(\ddot r - r\dot\varphi^2) = 0 \Rightarrow m\ddot r - \frac{r(7\ell^2\omega)^2 m}{(4\ell^2 + 3r^2)^2} = 0$$ preintegro.

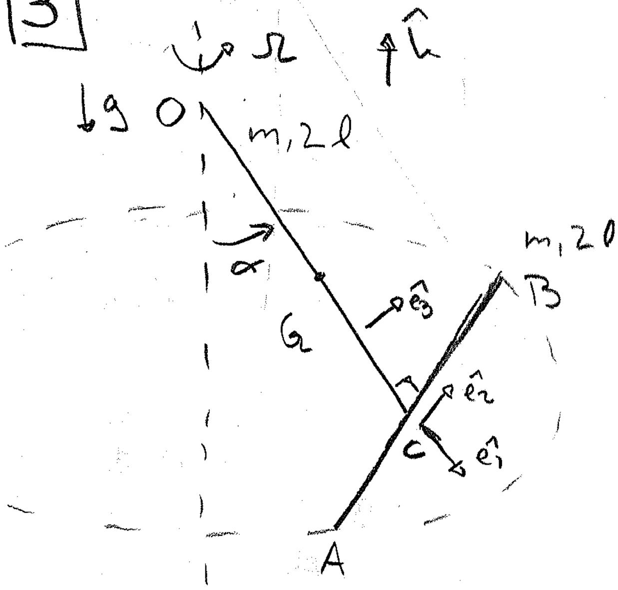
$$\frac{d}{dt}\left(\frac{1}{2}\dot r^2 m + \frac{(7\ell^2\omega)^2}{6}\frac{m}{(4\ell^2+3r^2)}\right) = 0 \to \frac{m}{2}\left(\dot r^2 + \frac{(7\ell^2\omega)^2}{3}\frac{1}{(4\ell^2+3r^2)}\right) = A$$

$$A = \frac{7\ell^2\omega^2}{3}\frac{m}{2}$$ es la energía mecánica del sistema.

$$\dot r^2(2l) = 7\frac{l^2\omega^2}{3} - \frac{(7l^2\omega)^2}{3}\frac{1}{16l^2} = \frac{7l^2\omega^2}{3}\left(1-\frac{7}{16}\right) = \frac{21}{16}(l\omega)^2$$

$$\boxed{\dot r(r=2l) = \frac{\sqrt{21}}{4}\, l\omega} \quad y \quad \vec v(r=2l) = \frac{\sqrt{21}}{4}\, l\omega\, \hat e_r + \frac{7}{8}\, l\omega\, \hat e_\varphi$$

$\boxed{3}$



$\{\hat e_1, \hat e_2, \hat e_3\}$ base solidaria barra 1 AB
barra 2 OC

a) $(m_1+m_2)\vec r_G = m_1 \vec r_{G_1} + m_2 \vec r_{G_2}$ centro de masa del sistema total.

$\vec r_{G_1} = 2l\,\hat e_1$

$\vec r_{G_2} = l\,\hat e_1$

$$\vec r_G = \frac{m}{2m}\left(2l\,\hat e_1 + l\,\hat e_1\right) = \frac{3}{2}\,l\,\hat e_1$$

Tensor en O

Suma de los tensores de c/barra en O.

barra 1 en C $\quad \mathbb{I}_1^C(\hat e_1, \hat e_2, \hat e_3) = \frac{ml^2}{3}\begin{pmatrix}1&0&0\\0&0&0\\0&0&1\end{pmatrix}$

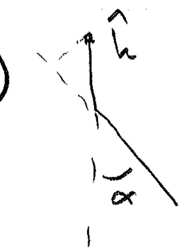
Steiner para llegar a O

$\vec r_G - \vec r_O = \vec r_C - \vec r_O = 2l\,\hat e_1$

$$\mathbb{I}_{1\,\{\hat e_1,\hat e_2,\hat e_3\}}^{(M,O)} = m4l^2\begin{pmatrix}0&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

$$\Rightarrow \mathbb{I}_1^O\{\hat e_1,\hat e_2,\hat e_3\} = ml^2\begin{pmatrix}\frac{1}{3}&0&0\\0&4&0\\0&0&13/3\end{pmatrix}$$

$$\mathbb{I}_2^O\{\hat{e}_1,\hat{e}_2,\hat{e}_3\} = \frac{4}{3}ml^2\begin{pmatrix} 0 & & 0 \\ & 1 & \\ 0 & & 1\end{pmatrix} \Rightarrow \underset{\text{total}}{\mathbb{I}_T^O}\{\hat{e}_1,\hat{e}_2,\hat{e}_3\} = \frac{ml^2}{3}\begin{pmatrix} 1 & & 0 \\ & 16 & \\ 0 & & 17\end{pmatrix}$$

b) 

$$\hat{k} = -\cos\alpha\,\hat{e}_1 + \sin\alpha\,\hat{e}_3$$

$$\vec{\omega} = \Omega\hat{k} = \Omega(-\cos\alpha\,\hat{e}_1 + \sin\alpha\,\hat{e}_3)$$

$$\vec{L}_o = \mathbb{I}_T^O\vec{\omega} = \frac{ml^2}{3}\begin{pmatrix} 1 & & 0 \\ & 16 & \\ 0 & & 17\end{pmatrix}\begin{pmatrix} -\Omega\cos\alpha \\ 0 \\ \Omega\sin\alpha\end{pmatrix}$$

$$\vec{L}_o = \frac{ml^2}{3}\left(-\Omega\cos\alpha\,\hat{e}_1 + 17\Omega\sin\alpha\,\hat{e}_3\right)$$

$$\frac{d\vec{L}_o}{dt} = \overbrace{\cancel{\frac{d'\vec{L}_o}{dt}}}^{0 \text{ pues } \Omega \text{ y } \alpha \text{ ctes}} + \vec{\omega}\wedge\vec{L}_o =$$

$$= \frac{ml^2}{3}\left(-\Omega\cos\alpha\,\hat{e}_1 + \Omega\sin\alpha\,\hat{e}_3\right)\wedge\left(-\Omega\cos\alpha\,\hat{e}_1 + 17\Omega\sin\alpha\,\hat{e}_3\right)$$

$$= \frac{ml^2}{3}\left(-17\Omega^2\sin\alpha\cos\alpha\,(-\hat{e}_2) - \Omega^2\sin\alpha\cos\alpha\,\hat{e}_2\right)$$

$$= \frac{16ml^2}{3}\Omega^2\sin\alpha\cos\alpha\,\hat{e}_2$$

$$\vec{M}_o^{ext} = \frac{3}{2}l\,\hat{e}_1\wedge(2mg\hat{k}) = 3lmg\left(\hat{e}_1\wedge(\cos\alpha\,\hat{e}_1 - \sin\alpha\,\hat{e}_3)\right)$$

$$= 3mgl\sin\alpha\,\hat{e}_2$$

para q' α se mantenga cte $\frac{16}{3}ml^2\Omega^2\sin\alpha\cos\alpha = 3mgl\sin\alpha$

$$\boxed{\cos\alpha = \frac{9}{16}\frac{g}{l\Omega^2}} \quad \exists \text{ si } 9g < 16l\Omega^2$$