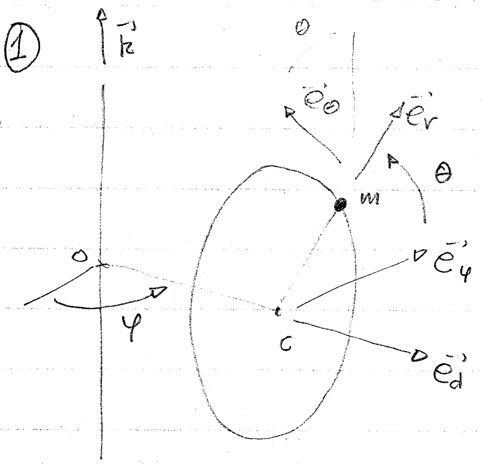
① 

(a)

$$\vec r_m = d\,\vec e_d + r\,\vec e_r$$
$$\vec v = d\dot\varphi\,\vec e_\varphi + r\,\dot{\vec e}_r$$

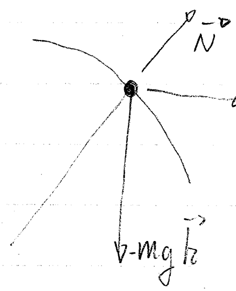
$$\dot{\vec e}_r = \left(\dot\varphi\,\vec k + \dot\theta\,\vec e_d\right)\times \vec e_r = -\dot\varphi\cos\theta\,\vec e_d + \dot\theta\,\vec e_\theta$$

$$\vec v = d\dot\varphi\,\vec e_\varphi + r\dot\theta\,\vec e_\theta - r\dot\varphi\cos\theta\,\vec e_d$$

$$\vec a = \cancel{d\ddot\varphi\,\vec e_\varphi} + d\dot\varphi\,\dot{\vec e}_\varphi + r\ddot\theta\,\vec e_\theta + r\dot\theta\,\dot{\vec e}_\theta - \cancel{r\ddot\varphi\cos\theta\,\vec e_d} + r\dot\varphi\dot\theta\,\text{sen}\,\theta\,\vec e_d - r\dot\varphi\cos\theta\,\dot{\vec e}_d$$

$$\vec a = \cancel{d\ddot\varphi\,\vec e_\varphi} - d\dot\varphi^2\,\vec e_d + r\ddot\theta\,\vec e_\theta + r\dot\theta\dot\varphi\,\text{sen}\,\theta\,\vec e_d - r\dot\theta^2\,\vec e_r - \cancel{r\ddot\varphi\cos\theta\,\vec e_d} + r\dot\varphi\dot\theta\,\text{sen}\,\theta\,\vec e_d - r\dot\varphi^2\cos\theta\,\vec e_\varphi$$

$$\left(\dot{\vec e}_\theta = \left(\dot\varphi\,\vec k + \dot\theta\,\vec e_d\right)\times\vec e_\theta = \dot\varphi\,\text{sen}\,\theta\,\vec e_d - \dot\theta\,\vec e_r\right)$$



Ec. mov. ⇒ $m\vec a\cdot\vec e_\theta = -mg\,\vec k\cdot\vec e_\theta$

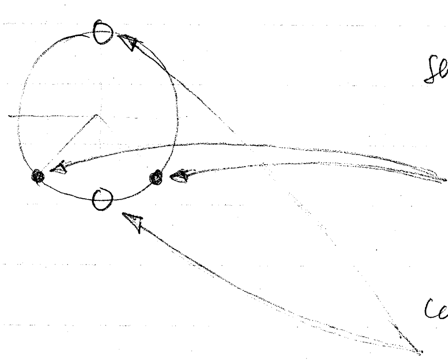
$$m\left(-\cancel{d\ddot\varphi\,\text{sen}\,\theta} + r\ddot\theta + r\dot\varphi^2\cos\theta\,\text{sen}\,\theta\right) = -mg\cos\theta$$

$$r\ddot\theta + r\dot\varphi^2\cos\theta\,\text{sen}\,\theta - \cancel{d\ddot\varphi\,\text{sen}\,\theta} = -g\cos\theta$$

$\ddot\varphi = \dot\omega = 0$

$$\boxed{\ddot\theta + \omega^2\,\text{sen}\,\theta\cos\theta + \frac{g}{r}\cos\theta = 0}$$

(b) Pos. eq. ⇒ $\left(\omega^2\,\text{sen}\,\theta_0 + g/r\right)\cos\theta_0 = 0$.



$\text{sen}\,\theta_0 = -\dfrac{g}{r\omega^2}$ $\exists \Leftrightarrow$ $\boxed{r\omega^2 > g}$

$\theta_0 = \text{Arcsen}(-g/r\omega^2)$ (2 raíces)

$\cos\theta_0 = 0$ (2 raíces)

$\theta_0 = \pm\pi/2$

$$f(\theta) = \omega^2 \operatorname{sen}\theta \cos\theta + g/r \cos\theta$$

$$\frac{\partial f(\theta)}{\partial \theta} = \omega^2 \cos^2\theta - \omega^2 \operatorname{sen}^2\theta - g/r \operatorname{sen}\theta = \omega^2 - 2\omega^2 \operatorname{sen}^2\theta - g/r \operatorname{sen}\theta$$

$$\left.\frac{\partial f(\theta)}{\partial \theta}\right|_{\theta_0 = \operatorname{Arcsen}(-g/r\omega^2)} = \omega^2 - 2\omega^2 \cdot \frac{g^2}{r^2\omega^4} + g/r \cdot g/r\omega^2 = \omega^2 - \frac{g^2}{r^2\omega^2}$$

$$\theta_0 = \operatorname{Arcsen}(-g/r\omega^2) \text{ estable} \Leftrightarrow \omega^2 > \frac{g^2}{r^2\omega^2}$$

$$r^2\omega^4 > g^2$$

$$\boxed{\theta_0 = \operatorname{Arcsen}(-g/r\omega^2) \text{ es siempre estable (ya que de otro modo } \nexists)} \leftarrow \boxed{r\omega^2 > g}$$

$$\left.\frac{\partial f(\theta)}{\partial \theta}\right|_{\theta_0 = +\pi/2} = \omega^2 - 2\omega^2 - g/r = -\omega^2 - g/r < 0 \quad \forall \omega$$

$$\boxed{\theta_0 = +\pi/2 \quad \text{inestable siempre}}$$

$$\left.\frac{\partial f(\theta)}{\partial \theta}\right|_{\theta_0 = -\pi/2} = \omega^2 - 2\omega^2 + g/r = -\omega^2 + g/r > 0$$

$$\Updownarrow$$

$$g/r > \omega^2$$

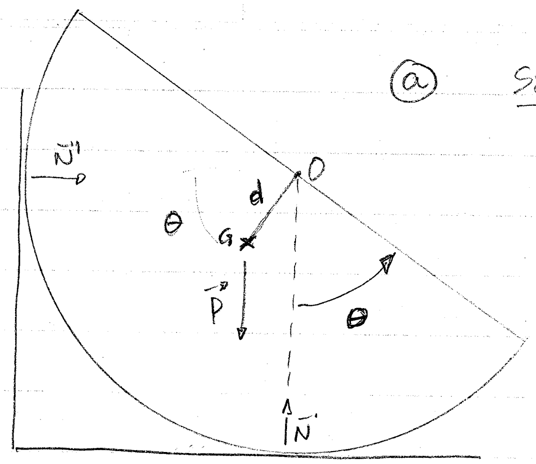
$$\boxed{\theta_0 = -\pi/2 \text{ es estable} \Leftrightarrow \omega^2 < g/r}$$

(c) Partícula en equilibrio relativo $\Rightarrow$ $\dot\theta = 0$; $\ddot\theta = 0$

$$m(-d\omega^2 \vec{e_d} - r\omega^2 \cancel{\cos\theta_0}\, \vec{e_d} - r\omega^2 \cos\theta_0\, \vec{e_\varphi}) = -mg\, \vec{i_z} + \vec{R}$$

$$\boxed{\vec{R} = mg\, \vec{i_z} - mr\omega^2 \cos\theta_0\, \vec{e_\varphi} - m(d + r\cancel{\cos\theta_0})\omega^2\, \vec{e_d}}$$

(2)



(a) Sistema conservativo. $E_{mec.} = cte$

$$U_g = -mg\, d \,\text{sen}\,\theta$$

$$T = \frac{1}{2} I_O \dot{\theta}^2 \qquad O: p^{to} \text{ fijo}$$

$$I_O = \frac{mr^2}{2}$$

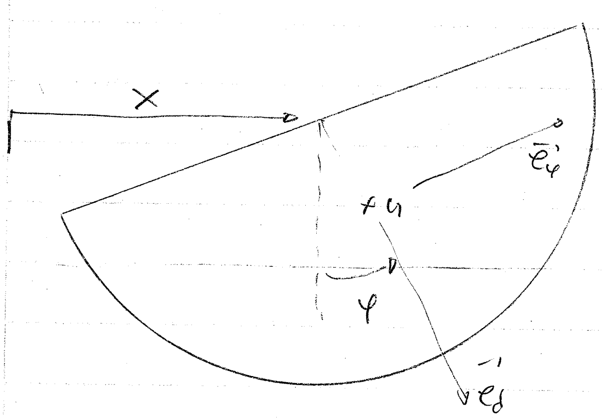
$$T = \frac{mr^2}{4}\dot{\theta}^2 \Rightarrow E_{mec} = \frac{mr^2}{4}\dot{\theta}^2 - mg\,d\,\text{sen}\,\theta$$

$$E_{mec}(t=0) = 0 \Rightarrow \frac{mr^2}{4}\dot{\theta}^2 - mgd\,\text{sen}\,\theta = 0$$

$$\left(\dot{\theta}(\theta=\pi/2) = 4\sqrt{\frac{g}{3\pi r}}\right) \Leftarrow \dot{\theta}(\theta=\pi/2) = \sqrt{\frac{4gd}{r^2}} = \frac{2}{r}\sqrt{gd}$$

(b) Sistema conservativo, → Conservación Emec.
(Única fuerza que trabaja es el peso)

Las fuerzas exteriores tienen proyección nula sobre dirección horizontal $\vec{i}$ → $\vec{P}\cdot\vec{i} = c^{te}$



$$\vec{r}_G = x\vec{i} + d\vec{e}_d$$
$$\vec{v}_G = \dot{x}\vec{i} + d\dot{\varphi}\vec{e}_\varphi$$

$$\vec{P} = m\dot{x}\vec{i} + md\dot{\varphi}\vec{e}_\varphi$$

$$\vec{P}\cdot\vec{i} = m\dot{x} + md\dot{\varphi}\cos\varphi = c^{te}$$

$$\vec{P}\cdot\vec{i}\,(\theta=\pi/2) = m\frac{2}{r}\sqrt{gd}\, d$$

$$\boxed{\dot{x} + d\dot{\varphi}\cos\varphi = \frac{2d}{r}\sqrt{gd}}$$

$$E_{mec} = \overbrace{\left(\frac{1}{2} m v_G^2 + \frac{1}{2} I_G \dot{\varphi}^2\right)}^{T} - mgd\cos\varphi$$

$$I_O = I_G + md^2 = \frac{mr^2}{2} \Rightarrow I_G = \frac{mr^2}{2} - md^2$$

$$E_{mec} = \frac{m}{2}\left(\dot{x}^2 + d^2\dot{\varphi}^2 + 2d\dot{x}\dot{\varphi}\cos\varphi\right) + \frac{1}{2}\left(\frac{mr^2}{2} - md^2\right)\dot{\varphi}^2 - mgd\cos\varphi$$

$$E_{mec}(\theta = \pi/2) = \varnothing$$

$$\frac{m}{2}\dot{x}^2 + md\dot{x}\dot{\varphi}\cos\varphi + \cancel{\frac{md^2\dot{\varphi}^2}{2}} - \cancel{\frac{md^2\dot{\varphi}^2}{2}} + \frac{mr^2}{4}\dot{\varphi}^2 - mgd\cos\varphi = \varnothing$$

$$\frac{m}{2}\left(\frac{2d}{r}\sqrt{gd} - d\dot{\varphi}\cos\varphi\right)^2 + md\dot{\varphi}\cos\varphi\left(\frac{2d}{r}\sqrt{gd} - d\dot{\varphi}\cos\varphi\right) + \frac{mr^2}{4}\dot{\varphi}^2 = mg\underbrace{d\cos\varphi}_{-h}$$

$$\frac{1}{2}\left(\frac{4d^2}{r^2}(gd) + d^2\dot{\varphi}^2\cos^2\varphi - \cancel{\frac{4d^2}{r}\sqrt{gd}\,\dot{\varphi}\cos\varphi}\right) + \cancel{\frac{2d^2}{r}\sqrt{gd}\,\dot{\varphi}\cos\varphi} - d^2\dot{\varphi}^2\cos^2\varphi + \frac{r^2\dot{\varphi}^2}{4} = -gh$$

$$\frac{2d^3g}{r^2} - \frac{d^2\dot{\varphi}^2}{2}\cos^2\varphi + \frac{r^2\dot{\varphi}^2}{4} = -gh$$

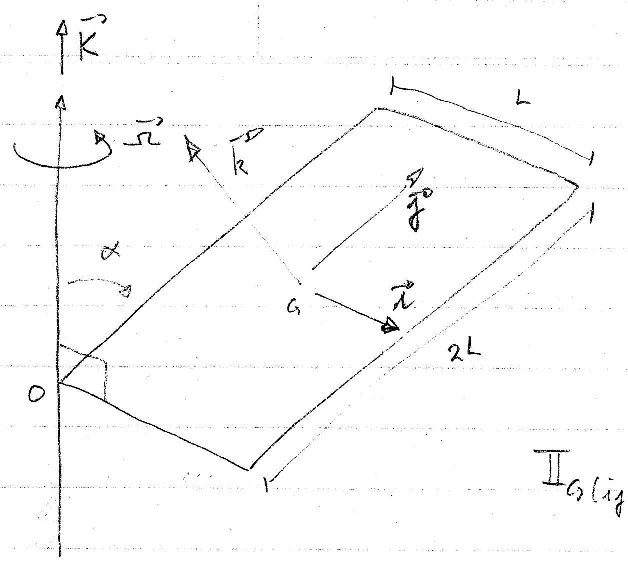
$$h(\varphi, \dot{\varphi}) = -\frac{r^2\dot{\varphi}^2}{4g} + \frac{d^2\dot{\varphi}^2\cos^2\varphi}{2g} - \frac{2d^3}{r^2}$$

Altura máxima $\Rightarrow$ en p^to^ de retroceso $\dot{\varphi} = \varnothing$.

$$h(\varphi, \dot{\varphi} = 0) = -\frac{2d^3}{r^2}$$

$$\boxed{\text{Altura máxima} = r - \frac{2d^3}{r^2}} \qquad \left(h_m = r\left(1 - \frac{128}{27\pi}\right)\right)$$

③



ⓐ

$$\vec{\omega} = \Omega \vec{K} = $$
$$= \Omega \cos\alpha \, \vec{j} + \Omega \operatorname{sen}\alpha \, \vec{k}$$

$$\mathbb{I}_{G(ijk)} = \frac{M}{12}\begin{bmatrix} 4L^2 & & \\ & L^2 & \\ & & 5L^2 \end{bmatrix}$$

$$\mathbb{I}_O = \frac{mL^2}{12}\begin{bmatrix} 4 & & \\ & 1 & \\ & & 5 \end{bmatrix} + m\begin{bmatrix} L^2 & -L^2/2 & \emptyset \\ -L^2/2 & \frac{L^2}{4} & \emptyset \\ \emptyset & \emptyset & \frac{5L^2}{4} \end{bmatrix}$$

$$\mathbb{I}_O = \begin{bmatrix} \frac{mL^2}{3} + mL^2 & -\frac{mL^2}{2} & \emptyset \\ -\frac{mL^2}{2} & \frac{mL^2}{12} + \frac{mL^2}{4} & \emptyset \\ \emptyset & \emptyset & \frac{5mL^2}{3} \end{bmatrix}$$

$$\mathbb{I}_O = mL^2\begin{bmatrix} 4/3 & -1/2 & \emptyset \\ -1/2 & 1/3 & \emptyset \\ \emptyset & \emptyset & 5/3 \end{bmatrix}$$

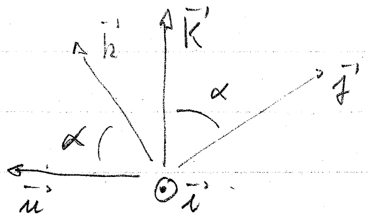
$O =$ fijo $\Rightarrow$ $$\vec{L_O} = \mathbb{I}_O \vec{\omega} = mL^2\Omega\left(-\frac{\cos\alpha}{2}\vec{i} + \frac{\cos\alpha}{3}\vec{j} + \frac{5\operatorname{sen}\alpha}{3}\vec{k}\right)$$

$$\boxed{\vec{L_O} = mL^2\Omega\left(\frac{-\cos\alpha}{2}\vec{i} + \frac{\cos\alpha}{3}\vec{j} + \frac{5}{3}\operatorname{sen}\alpha\,\vec{k}\right)}$$

(b)

$$\dot{\vec{L}}_O = \vec{M}_O = \vec{M}_O^{react} + \vec{M}_O^{peso}$$

$$\vec{M}_O^{peso} = \left(\frac{L}{2}\vec{i}' + L\vec{j}'\right) \times -mg\vec{K}' = -mg\frac{L}{2}\left(\vec{u}\right) - mgL\,\text{sen}\alpha\,\vec{i}' =$$



$$= -mg\frac{L}{2}\left(\cos\alpha\,\vec{k}' - \text{sen}\alpha\,\vec{j}'\right) - mgL\,\text{sen}\alpha\,\vec{i}'$$

$$\dot{\vec{L}}_O = mL^2\Omega\left(-\frac{\cos\alpha}{2}\dot{\vec{i}}' + \frac{\cos\alpha}{3}\dot{\vec{j}}' + \frac{5}{3}\text{sen}\alpha\,\dot{\vec{k}}'\right) =$$

$$\left(\begin{array}{l} \dot{\vec{k}}' = \Omega\left(\cos\alpha\,\vec{j}' + \text{sen}\alpha\,\vec{k}'\right) \times \vec{k}' = \Omega\cos\alpha\,\vec{i}' \\ \dot{\vec{j}}' = \Omega\left(\cos\alpha\,\vec{j}' + \text{sen}\alpha\,\vec{k}'\right) \times \vec{j}' = -\Omega\,\text{sen}\alpha\,\vec{i}' \\ \dot{\vec{i}}' = \Omega\left(\cos\alpha\,\vec{j}' + \text{sen}\alpha\,\vec{k}'\right) \times \vec{i}' = -\Omega\cos\alpha\,\vec{k}' + \Omega\,\text{sen}\alpha\,\vec{j}' \end{array}\right)$$

$$= mL^2\Omega\left(+\frac{\cos^2\alpha}{2}\Omega\,\vec{k}' - \frac{\cos\alpha}{2}\text{sen}\alpha\,\Omega\,\vec{j}' - \frac{\cos\alpha}{3}\text{sen}\alpha\,\Omega\,\vec{i}' + \right.$$

$$\left. + \frac{5}{3}\text{sen}\alpha\cos\alpha\,\Omega\,\vec{i}'\right)$$

$$\vec{M}_O^{react} = -\vec{M}_O^{peso} + \dot{\vec{L}}_O = \left(\frac{4mL^2\Omega^2}{3}\text{sen}\alpha\cos\alpha + mgL\,\text{sen}\alpha\right)\vec{i}'$$

$$+ \left(-\frac{mL^2\Omega^2}{2}\text{sen}\alpha\cos\alpha - \frac{mgL}{2}\text{sen}\alpha\right)\vec{j}'$$

$$+ \left(\frac{mL^2\Omega^2}{2}\cos^2\alpha + \frac{mgL}{2}\cos\alpha\right)\vec{k}'$$