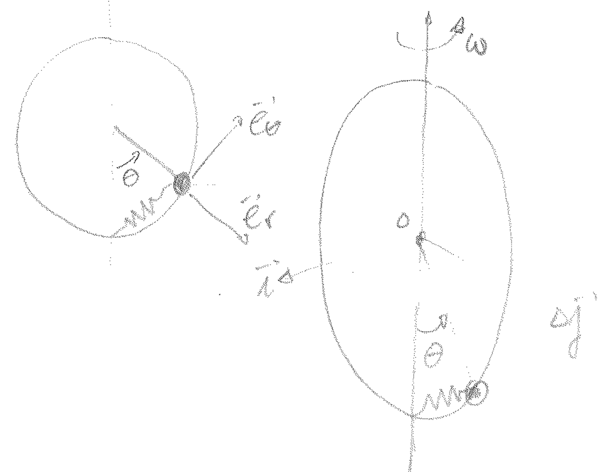
Problema 1 (a)




$$\vec{v}' = \vec{v}_0 + \vec{\omega}\times\vec{r}' + \vec{v}_r$$

$$\vec{v}_r = r\dot{\theta}\,\vec{e}_\theta$$

$$\omega\vec{k}\times r\vec{e}_r = -r\omega\,\mathrm{sen}\,\theta\,\vec{i}$$

$$\vec{v}_0 = 0$$

$$\vec{v}' = -r\omega\,\mathrm{sen}\,\theta\,\vec{i} + r\dot{\theta}\,\vec{e}_\theta$$

$$\vec{a}' = \vec{a}_0 + \frac{d\vec{\omega}}{dt}\times\vec{r}' + \vec{\omega}\times(\vec{\omega}\times\vec{r}') + 2\vec{\omega}\times\vec{v}' + \vec{a}_r$$

$$\vec{a}_0 = 0 \quad ; \quad \frac{d\vec{\omega}}{dt} = 0$$

$$\omega\vec{k}\times(\omega\vec{k}\times r\vec{e}_r) = \omega\vec{k}\times(-r\omega\,\mathrm{sen}\,\theta\,\vec{i}) = -r\omega^2\,\mathrm{sen}\,\theta\,\vec{j}$$

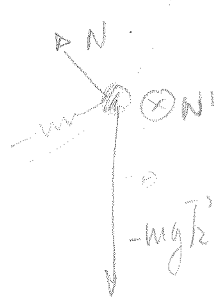
$$2\omega\vec{k}\times(r\dot{\theta}\,\vec{e}_\theta) = -2r\omega\dot{\theta}\cos\theta\,\vec{i}$$

$$\vec{a}_r = r\ddot{\theta}\,\vec{e}_\theta - r\dot{\theta}^2\,\vec{e}_r$$

$$\vec{a}' = \underbrace{-r\omega^2\,\mathrm{sen}\,\theta\,\vec{j}}_{\vec{a}_T} \underset{\vec{a}_C}{- 2r\omega\dot{\theta}\cos\theta\,\vec{i}} + \underset{\vec{a}_r}{r\ddot{\theta}\,\vec{e}_\theta - r\dot{\theta}^2\,\vec{e}_r}$$

$$\vec{F}_{res} = -kr\vec{k} - kr\vec{e}_r$$
$(kr = mg)$

(b)




$$m\vec{a}\cdot\vec{e}_\theta = -mg\,\mathrm{sen}\,\theta - kr\,\mathrm{sen}\,\theta$$

$$mr\ddot{\theta} - mr\omega^2\,\mathrm{sen}\,\theta\cos\theta + 2mg\,\mathrm{sen}\,\theta = 0$$

(c)

$$mr\dot{\theta}\ddot{\theta} - mr\omega^2\,\mathrm{sen}\,\theta\cos\theta\,\dot{\theta} + 2mg\,\mathrm{sen}\,\theta\,\dot{\theta} = 0$$

integral en el tiempo

$$mr\frac{\dot{\theta}^2}{2} - mr\omega^2\frac{\mathrm{sen}^2\theta}{2} - 2mg\cos\theta = cte$$

$$\underbrace{\frac{m}{2}(r\dot{\theta})^2}_{T_{rel}} \underbrace{- mr^2\omega^2 \frac{\operatorname{sen}^2\theta}{2} - 2mgr\cos\theta}_{U_{ef}} = c^{te}$$

$$U_{ef} = -\left(2mgr\cos\theta + mr^2\omega^2 \frac{\operatorname{sen}^2\theta}{2}\right)$$

Ptos. eq:

$$\frac{\partial U_{ef}}{\partial\theta} = 0 \Leftrightarrow \sum \vec{F}\cdot\vec{e}_\theta = 0 \Longrightarrow 2g\operatorname{sen}\theta - r\omega^2\operatorname{sen}\theta\cos\theta = 0$$

$$\longrightarrow \operatorname{sen}\theta = 0 \Leftrightarrow \theta = 0, \ \theta = \pi$$

$$2g - r\omega^2\cos\theta = 0$$

$$\longrightarrow \cos\theta = \frac{2g}{r\omega^2}$$

$$\exists \Leftrightarrow 2g < r\omega^2$$

$$\frac{\partial^2 U_{ef}}{\partial\theta^2} = 2mgr\cos\theta - mr^2\omega^2\cos^2\theta + mr^2\omega^2\operatorname{sen}^2\theta =$$

$$= 2mgr\cos\theta - 2mr^2\omega^2\cos^2\theta + mr^2\omega^2$$

$$\left.\frac{\partial^2 U_{ef}}{\partial\theta^2}\right|_{\theta=0} = 2mgr - mr^2\omega^2 > 0 \Leftarrow 2g/r > \omega^2 \Rightarrow \theta = 0 \text{ estable} \Updownarrow \omega^2 < 2g/r$$

$$\left.\frac{\partial^2 U_{ef}}{\partial\theta^2}\right|_{\theta=\pi} = -2mgr - mr^2\omega^2 < 0 \text{ siempre} \Rightarrow \theta = \pi \text{ inestable siempre.}$$

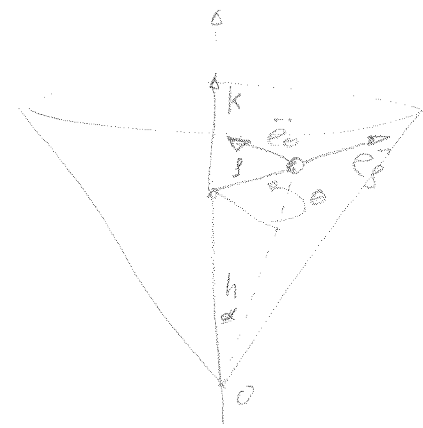
$$\left.\frac{\partial^2 U_{ef}}{\partial\theta^2}\right|_{\theta = A\cos(g/r\omega^2)} = 2mg\cancel{r}\cdot\frac{2g}{\cancel{r}\omega^2} - 2m\cancel{r^2}\cancel{\omega^2}\cdot\frac{4g^2}{\cancel{r^2}\omega^{\cancel{4}2}} + mr^2\omega^2 = -\frac{m4g^2}{\omega^2} + mr^2\omega^2 =$$

$$= \frac{m}{\omega^2}(r^2\omega^4 - 4g^2) > 0 \qquad \theta = A\cos(2g/r\omega^2) \text{ estable}$$

d) $(\vec{i}\,)$ $\quad m\vec{a}\cdot\vec{i} = -m\,2r\omega\dot{\theta}\cos\theta = -N'$

$$P = \vec{N}'\cdot\vec{v} = 2mr^2\omega^2\dot{\theta}\cos\theta\operatorname{sen}\theta$$

Problema (2)




(a)

$$\vec{L} = (h\vec{k} + \rho\vec{e}_\rho) \times m(\dot{h}\vec{k} + \dot{\rho}\vec{e}_\rho + \rho\dot{\theta}\vec{e}_\theta)$$

$$\rho = h\,\text{sen}\,\alpha$$

$$\vec{L}\cdot\vec{k} = m\rho^2\dot{\theta} = m h^2 \text{sen}^2\alpha\,\dot{\theta}$$

$$\vec{L}\cdot\vec{k} = \ell \qquad \dot{\ell} = 2m\rho\dot{\rho}\dot{\theta} + m\rho^2\ddot{\theta}$$

$$\vec{a} = (\ddot{\rho} - \rho\dot{\theta}^2)\vec{e}_\rho + (\rho\ddot{\theta} + 2\dot{\rho}\dot{\theta})\vec{e}_\theta + \ddot{h}\vec{k}$$

$$m\vec{a}\cdot\vec{e}_\theta = 0 \,;\; \sum \vec{F}\cdot\vec{e}_\theta = 0 \Rightarrow \underbrace{m\rho\ddot{\theta} + 2m\dot{\rho}\dot{\theta}}_{\dot{\ell}/\rho} = 0$$

$$\dot{\ell}/\rho = 0 \Rightarrow \underline{\ell = c^{te}}$$

(b)

$$\vec{v}_0 = \rho_0\dot{\theta}_0\vec{e}_\theta = h_0\,tg\alpha\,\dot{\theta}_0\vec{e}_\theta$$

Sistema conservativo $\Rightarrow E_{mec} = c^{te}$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{\rho}^2 + \rho^2\dot{\theta}^2 + \dot{h}^2)$$

$$U = mgh$$

$$E_0 = \frac{1}{2}m\left(\dot{h}^2(1+tg^2\alpha) + h^2\,tg^2\alpha\,\dot{\theta}^2\right) + mgh$$

$$E_0 = \frac{1}{2}m\left(\dot{h}^2(1+tg^2\alpha) + \frac{h_0^2 v_0^2}{h^2}\right) + mgh = \frac{1}{2}mv_0^2 + mgh_0$$

$$\left(h^2\,tg^2\alpha\,\dot{\theta} = h_0\,tg\alpha\,v_0 \qquad \dot{\theta} = \frac{h_0 v_0}{h^2\,tg\alpha}\right)$$

$$\dot{h}^2(1+tg^2\alpha) = 2g(h_0-h) + v_0^2\left(1-\frac{h_0^2}{h^2}\right)$$

$$\dot{h}^2 = \left(2g(h_0-h) + v_0^2\left(\frac{h^2-h_0^2}{h^2}\right)\right)\frac{1}{1+tg^2\alpha}$$

(c)

$$\dot{h}_m = \emptyset \Rightarrow -2g + v_0^2\left(\frac{h_m + h_0}{h_m^2}\right) = \emptyset.$$

$$+2g h_m^2 = v_0^2 (h_m + h_0)$$

$$2g h_m^2 - v_0^2 h_m - v_0^2 h_0 = \emptyset.$$

$$\frac{+v_0^2 \pm \sqrt{v_0^4 + 4.2g.v_0^2 h_0}}{4g} = \frac{v_0^2}{4g}\left(1 \pm \sqrt{1 + 8gh_0/v_0^2}\right) = h_m$$

$$\boxed{h_m = \frac{v_0^2}{4g}\left(1 + \sqrt{1 + 8gh_0/v_0^2}\right)}$$

$$\boxed{h_m' = h_0}$$