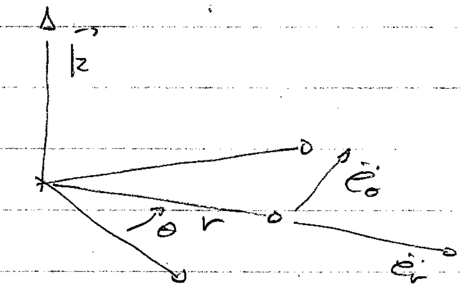
① $U(r) = -C/r^4 \; ; \quad C > 0$

$C = cte$



$$E_{mec} = T + U = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) - \frac{C}{r^4}$$

Mov. central $\Rightarrow$ $\vec{L} = \vec{r} \times \vec{p} = cte$

$$\vec{L} = r \vec{e}_r \times m(\dot{r} \vec{e}_r + r\dot{\theta} \vec{e}_\theta) = m r^2 \dot{\theta} \vec{k}$$

$$\ell = m r^2 \dot{\theta} = cte \qquad \ell^2 = m^2 r^4 \dot{\theta}^2$$

Mov. central $\Rightarrow$ mov. plano

$\vec{k} \perp$ plano

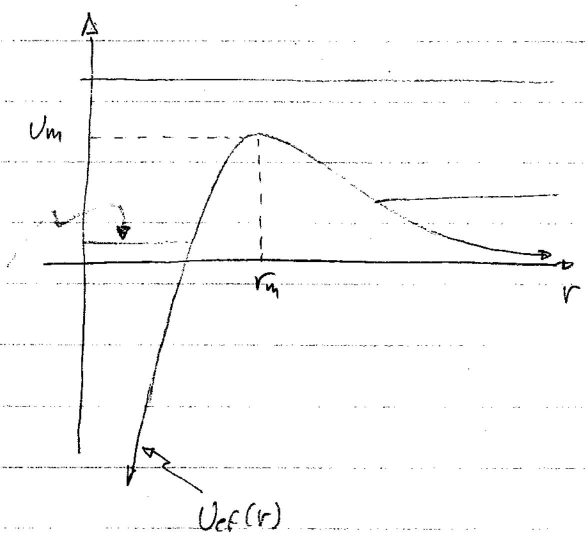
$$\vec{r} = r \vec{e}_r$$

$$\vec{v} = \dot{r} \vec{e}_r + r\dot{\theta} \vec{e}_\theta$$

$$v^2 = \vec{v} \cdot \vec{v} = \dot{r}^2 + r^2 \dot{\theta}^2$$

$$E_{mec} = \frac{1}{2} m \dot{r}^2 + \underbrace{\frac{\ell^2}{2mr^2} - \frac{C}{r^4}}_{U_{ef}}$$

$$U_{ef} = \frac{\ell^2}{2mr^2} - \frac{C}{r^4}$$



$$\frac{\partial U_{ef}}{\partial r} = -\frac{\ell^2}{m r_m^3} + \frac{4C}{r_m^5} = 0$$

$$-\frac{\ell^2}{m} r_m^2 + 4C = 0$$

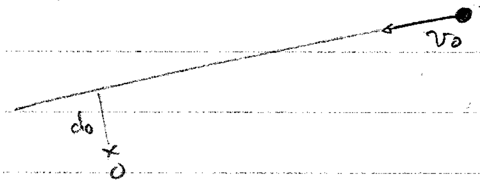
$$\boxed{r_m = \frac{2\sqrt{Cm}}{\ell}} \Rightarrow U_m = \frac{\ell^2}{2m\left(\frac{4Cm}{\ell^2}\right)} - \frac{C}{\frac{16 C^2 m^2}{\ell^4}} =$$

$$= \frac{\ell^4}{8Cm^2} - \frac{\ell^4}{16Cm^2} = \boxed{\frac{\ell^4}{16Cm^2}}$$

ⓐ Trayectoria no acotada $\Leftrightarrow$ $\boxed{E_{mec} > \frac{\ell^4}{16Cm^2}}$

ⓑ r_{max} en trayectoria acotada $= \frac{2\sqrt{Cm}}{\ell}$

(c)



$$\ell = m d_0 v_0$$

$$E_{mec} = \underbrace{\frac{1}{2} m v_0^2}_{E_0} = \frac{\ell^2}{2 m r_{min}^2} - \frac{C}{r_{min}^4} \qquad (\dot{r}_{min} = 0)$$

$$\frac{m v_0^2}{2} = \frac{m d_0^2 v_0^2}{2\, r_{min}^2} - \frac{C}{r_{min}^4}$$

$$\frac{m v_0^2}{2} r_{min}^4 - \frac{m d_0^2 v_0^2}{2} r_{min}^2 + C = 0$$

$$r_{min}^2 = \frac{+\frac{m d_0^2 v_0^2}{2} \pm \sqrt{\frac{m^2 d_0^4 v_0^4}{4} - 4 \cdot \frac{m v_0^2}{2} \cdot C}}{\cancel{2}\,\frac{m v_0^2}{\cancel{4}}} = \frac{d_0^2}{2} \pm \sqrt{\frac{d_0^4}{4} - \frac{2C}{m v_0^2}}$$

$$U_m = \frac{\ell^4}{16 C m^2} = \frac{m^4 d_0^4 v_0^4}{16 C m^2} > \frac{m v_0^2}{2} = E_0$$

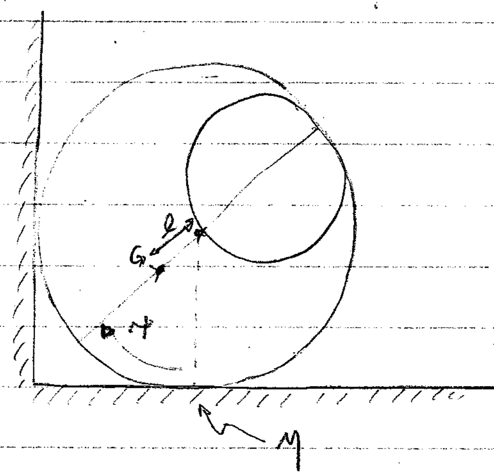
$$\frac{m d_0^4 v_0^2}{8C} > 1 \Rightarrow \frac{d_0^4}{4} > \frac{2C}{m v_0^2} \quad ✓$$

$$r_{min}^2 = \frac{d_0^2}{2} + \sqrt{\frac{d_0^4}{4} - \frac{2C}{m v_0^2}}$$

(2)

(a)

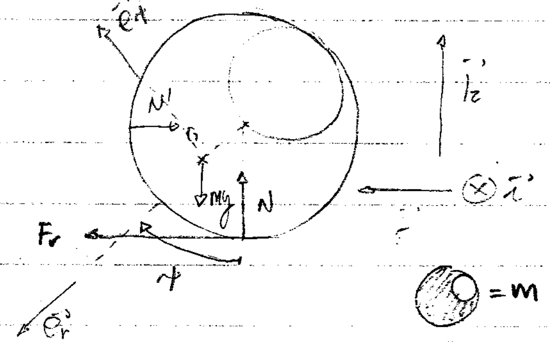
$$\begin{cases} N - mg = 0 \\ -N' + F_r = 0 \\ -mg\dfrac{r}{6}\operatorname{sen}\varphi + F_r r = 0 \end{cases}$$



$$|F_r| \le \mu_s N$$

$$\left.\begin{matrix} F_r = +\dfrac{mg}{6}\operatorname{sen}\varphi \\ N = mg \end{matrix}\right\} \to \left|\frac{\operatorname{sen}\varphi}{6}\right| \le \mu_s$$

$$\boxed{1/6 \le \mu_s}$$



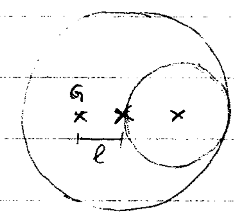
(b) $\mu_s < 1/6$

$\varphi_0 = \pi/2$

= m

$= m'' = m/3$

$= m' = 4m/3$



$$I_0\ddot{\varphi} = -mg\frac{r}{6}\operatorname{sen}\varphi + F_r r$$

$$\begin{cases} m\left(\dfrac{r}{6}\ddot{\varphi}\,\vec{e}_\varphi - \dfrac{r}{6}\dot{\varphi}^2\,\vec{e}_r\right) = \\ = -mg\,\vec{k} + F_r\,\vec{j} + N\,\vec{k} - N'\,\vec{j} \end{cases}$$

$$-\left(\rho r^2\pi L - \rho\frac{r^2}{4}\pi L\right)\ell + \left(\rho\frac{r^2}{4}\pi L\right)\frac{r}{2} = 0.$$

$$\rho\frac{r^2}{4}\pi L\left(-3\ell + \frac{r}{2}\right) = 0 \Rightarrow \ell = r/6$$

$$\to \frac{mr}{6}\left(\ddot{\varphi}\operatorname{sen}\varphi + \dot{\varphi}^2\cos\varphi\right) = N - mg$$

$$|F_r| = \mu_d N = \mu_d\left(mg + \frac{mr}{6}\left(\ddot{\varphi}\operatorname{sen}\varphi + \dot{\varphi}^2\cos\varphi\right)\right)$$

$$\frac{13mr^2}{24}\ddot{\varphi} = -mg\frac{r}{6}\operatorname{sen}\varphi + F_r r$$

$$(\varphi = \pi/2,\ \dot{\varphi} = 0) \Rightarrow \frac{13mr^2}{24}\ddot{\varphi} = -mg\frac{r}{6} + \mu_d mgr + \frac{m\mu_d}{6}r^2\ddot{\varphi}$$

$$I_{0,\text{body}} + I_{0,\text{small}} = I_{0,\text{full}} = \frac{m'r^2}{2}$$

$$I_{0,\text{small}} = \frac{m''r^2}{8} + \frac{m''r^2}{4} = \frac{3m''r^2}{8}$$

$$I_{0,\text{body}} = \frac{m'r^2}{2} - \frac{3m''r^2}{8} =$$

$$= \frac{(4m' - 3m'')r^2}{8} = \frac{16m/3 - m}{8}r^2 =$$

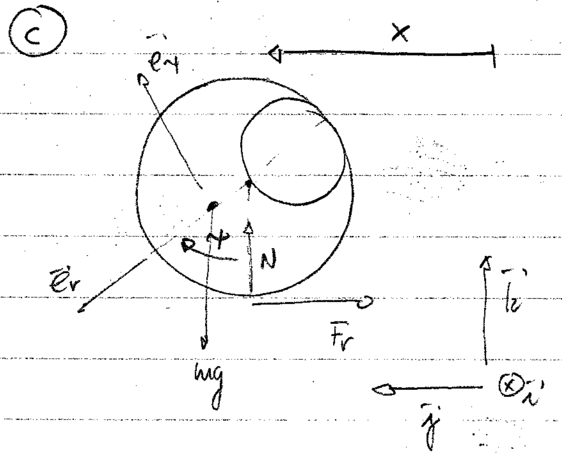
$$= \frac{13mr^2}{24}$$

$$\frac{13\,r\ddot{\psi}}{24} - \frac{\mu_d}{6}\,r\ddot{\psi} = \left(\mu_d g - \frac{1}{6}g\right)$$

$$r\ddot{\psi}\,(13 + 4\mu_d) = 4g\,(6\mu_d - 1)$$

$$\ddot{\psi} = \frac{4g\,(6\mu_d - 1)}{r\,(13 - 4\mu_d)} = -\frac{4g\,(1 - 6\mu_d)}{r\,(13 - 4\mu_d)}$$

c)



$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G\,\omega^2$$

$$U = -mg\,\frac{r}{6}\cos\psi$$

$$\vec{v}_G = \dot{x}\,\vec{j} + \frac{r}{6}\dot{\psi}\,\vec{e}_\psi$$

$$v_G^2 = \dot{x}^2 + \frac{r^2}{36}\dot{\psi}^2 + \frac{2r}{6}\dot{x}\dot{\psi}\cos\psi$$

<u>RSD</u> : $\dot{x} = -r\dot{\psi}$

$$v_G^2 = r^2\dot{\psi}^2 + \frac{r^2}{36}\dot{\psi}^2 - \frac{1}{3}r^2\dot{\psi}^2\cos\psi =$$

$$= \frac{r^2\dot{\psi}^2}{36}\left(37 - 12\cos\psi\right)$$

$$E_{mec} = \frac{1}{2} m \frac{r^2\dot{\psi}^2}{36}(37 - 12\cos\psi) + \frac{1}{2}\left(\frac{13mr^2}{24} - \frac{mr^2}{36}\right)\dot{\psi}^2 - mg\frac{r}{6}\cos\psi = \emptyset$$

$$\frac{r\dot{\psi}^2}{36}\left(37 - 12\cos\psi + \frac{39}{2} - 1\right) = \frac{g}{3}\cos\psi$$

$$r\dot{\psi}^2\left(\frac{111}{2} + 12\cos\psi\right) = 12\,g\cos\psi$$

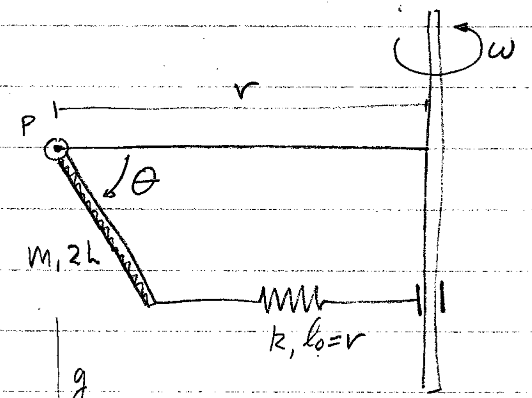
$$\boxed{\dot{\psi}^2 = \frac{g}{r}\,\frac{\cos\psi}{\left(\cos\psi + 111/24\right)}}$$

③

ⓐ

$I_G = \frac{mL^2}{3}$

$I_p = \frac{mL^2}{3} + mL^2 = \frac{4mL^2}{3}$

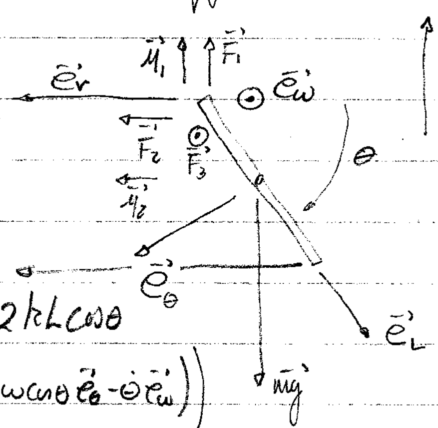


$\vec{v}_p = r\omega \vec{e}_\omega \quad (\neq 0 \,!)$

$\vec{a}_p = -r\omega^2 \vec{e}_r$

$\vec{M}_p^{ext} = \vec{\mu}_1 + \vec{\mu}_2 -$

$- mgL\cos\theta \, \vec{e}_\omega$



$\vec{\omega} = \omega \vec{k} - \dot\theta \vec{e}_\omega =$

$= -\omega \operatorname{sen}\theta \, \vec{e}_L - \omega\cos\theta \, \vec{e}_\theta - \dot\theta \vec{e}_\omega$

$$\frac{d}{dt}\left(\frac{4mL^2}{3}\begin{bmatrix} 0 & & \\ & 1 & \\ & & 1 \end{bmatrix}(-\omega\operatorname{sen}\theta\,\vec{e}_L - \omega\cos\theta\,\vec{e}_\theta - \dot\theta\vec{e}_\omega)\right)$$

$$= \vec{M}_p^{ext} - m(\vec{r}_G - \vec{r}_p) \times \vec{a}_p$$

$mL\vec{e}_L \times -r\omega^2 \vec{e}_r =$

$= mrL\omega^2 \operatorname{sen}\theta \, \vec{e}_\omega$

$$\rightarrow \frac{4mL^2}{3}\frac{d}{dt}\left(-\omega\cos\theta\,\vec{e}_\theta - \dot\theta\vec{e}_\omega\right) =$$

$\dot{\vec{e}}_\theta = (\omega\vec{k} - \dot\theta\vec{e}_\omega) \times \vec{e}_\theta = \omega\operatorname{sen}\theta\,\vec{e}_\omega - \dot\theta\vec{e}_L$

$\dot{\vec{e}}_\omega = \vec{\omega} \times \vec{e}_\omega = (\omega\vec{k} - \dot\theta\vec{e}_\omega) \times \vec{e}_\omega =$

$= -\omega \, \vec{e}_r$

$$= \frac{4mL^2}{3}\Big(\omega\dot\theta\operatorname{sen}\theta\,\vec{e}_\theta - \omega\cos\theta(\omega\operatorname{sen}\theta\,\vec{e}_\omega - \dot\theta\vec{e}_L) - \ddot\theta\vec{e}_\omega + \dot\theta\omega\vec{e}_r\Big) =$$

$$= \frac{4mL^2}{3}\Big(-\ddot\theta\vec{e}_\omega + \dot\theta\omega(-\cos\theta\,\vec{e}_L + \operatorname{sen}\theta\,\vec{e}_\theta) + \omega\dot\theta\operatorname{sen}\theta\,\vec{e}_\theta + \omega\dot\theta\cos\theta\,\vec{e}_L - \omega^2\operatorname{sen}\theta\cos\theta\,\vec{e}_\omega\Big) =$$

$$= \frac{4mL^2}{3}\Big(-(\ddot\theta + \omega^2\operatorname{sen}\theta\cos\theta)\,\vec{e}_\omega + 2\omega\dot\theta\operatorname{sen}\theta\,\vec{e}_\theta\Big) = \vec{\mu}_p - mgL\cos\theta\,\vec{e}_\omega -$$

$$- 2kL\cos\theta\, 2L\operatorname{sen}\theta\,\vec{e}_\omega - mrL\omega^2\operatorname{sen}\theta\,\vec{e}_\omega$$

Momento en la articulación $= \frac{8mL^2}{3}\omega\dot\theta\operatorname{sen}\theta\,\vec{e}_\theta$

$(\vec{\mu}_p \cdot \vec{e}_\omega \equiv 0)$

art. cilíndrica lisa

(b)

$$\vec{r}_G = r\vec{e}_r + L\vec{e}_L \qquad \dot{\vec{e}}_L = \vec{\omega}\times\vec{e}_L = (\omega\vec{k}-\dot{\theta}\vec{e}_\omega)\times\vec{e}_L =$$

$$\vec{v}_G = r\omega\vec{e}_\omega + L\dot{\vec{e}}_L = \qquad = -\omega\cos\theta\,\vec{e}_\omega + \dot{\theta}\vec{e}_\theta$$

$$= r\omega\vec{e}_\omega + L\dot{\theta}\vec{e}_\theta - L\omega\cos\theta\,\vec{e}_\omega$$

$$\vec{a}_G = r\omega\dot{\vec{e}}_\omega + L\ddot{\theta}\vec{e}_\theta + L\dot{\theta}\dot{\vec{e}}_\theta + L\omega\,\mathrm{sen}\theta\,\dot{\theta}\vec{e}_\omega - L\omega\cos\theta\,\dot{\vec{e}}_\omega =$$

$$= r\omega(-\omega\vec{e}_r) + L\ddot{\theta}\vec{e}_\theta + L\dot{\theta}(\omega\,\mathrm{sen}\theta\,\vec{e}_\omega - \dot{\theta}\vec{e}_L) + L\omega\,\mathrm{sen}\theta\,\dot{\theta}\vec{e}_\omega -$$

$$- L\omega\cos\theta(-\omega\vec{e}_r) =$$

$$\dot{\vec{e}}_\theta = (\omega\vec{k} - \dot{\theta}\vec{e}_\omega)\times\vec{e}_\theta =$$

$$= \omega\,\mathrm{sen}\theta\,\vec{e}_\omega - \dot{\theta}\vec{e}_L$$

$$= (-r\omega^2 + L\omega^2\cos\theta)\vec{e}_r + L\ddot{\theta}\vec{e}_\theta + 2L\dot{\theta}\omega\,\mathrm{sen}\theta\,\vec{e}_\omega - L\dot{\theta}^2\vec{e}_L$$

$$m\vec{a}_G = -mg\vec{k} + \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + 2kL\cos\theta\,\vec{e}_r$$

$$m\vec{a}_G\cdot\vec{e}_L = +mg\,\mathrm{sen}\theta - 2kL\cos^2\theta + (\vec{F}_1+\vec{F}_2)\cdot\vec{e}_L$$

$$m\left((r\omega^2 - L\omega^2\cos\theta)\cos\theta - L\dot{\theta}^2\right) = mg\,\mathrm{sen}\theta - 2kL\cos^2\theta + \vec{F}\cdot\vec{e}_L$$

$$\vec{F}\cdot\vec{e}_L = 2kL\cos^2\theta - mg\,\mathrm{sen}\theta + m(r - L\cos\theta)\omega^2\cos\theta - mL\dot{\theta}^2$$

Reacción de la barra sobre la articulación $= -(\vec{F}\cdot\vec{e}_L)$

(c) De (a) $\rightarrow \frac{4mL^2}{3}(\ddot{\theta} + \omega^2\,\mathrm{sen}\theta\cos\theta) = mgL\cos\theta + mrL\omega^2\,\mathrm{sen}\theta + 4kL^2\,\mathrm{sen}\theta\cos\theta$

Equilibrio $\theta = 3\pi/4 \Rightarrow -\frac{2}{3}mL^2\omega^2 = -mgL\frac{\sqrt{2}}{2} + mrL\omega^2\frac{\sqrt{2}}{2} - 2kL^2$

$$-\omega^2 m\left(r\sqrt{2}/2 + 2/3\,L\right) = -mg\sqrt{2}/2 - 2kL \qquad \boxed{\omega^2 = \frac{mg\sqrt{2}/2 + 2kL}{(\sqrt{2}/2\,r + 2/3\,L)m}}$$