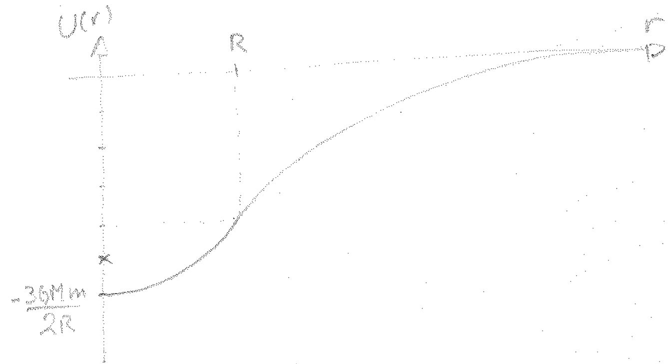
Ej. 1) 1. $\vec{F}(r) = \begin{cases} -\dfrac{GMmr}{R^3}\,\vec{e_r} & 0 \leq r < R \\ -\dfrac{GMm}{r^2}\,\vec{e_r} & R \leq r \end{cases}$

$$U(r) = \begin{cases} \dfrac{GMm}{2R^3} r^2 + H & 0 \leq r < R \\ -\dfrac{GMm}{r} & R \leq r \end{cases}$$

Continuidad para $U(r)\big|_{r=R} \Rightarrow \quad \dfrac{GMm}{2R^3} R^2 + H = -\dfrac{GMm}{R}$

$$H = -\frac{3GMm}{2R}$$

$$U(r) = \begin{cases} \dfrac{GMm}{2R^3} r^2 - \dfrac{3GMm}{2R} & 0 \leq r < R \\ -\dfrac{GMm}{r} & R \leq r \end{cases}$$



2. $\ell = m\sqrt{\dfrac{GMR}{32}}$; $E = -\dfrac{5GMm}{4R}$

$$\ell = mr^2\dot{\varphi} \rightarrow \dot{\varphi} = \frac{\ell}{mr^2} \rightarrow \dot{\varphi}^2 = \frac{\ell^2}{m^2 r^4}$$

$$-\frac{5GMm}{4R} = U(r) + \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\varphi}^2\right) = U(r) + \frac{m\dot{r}^2}{2} + \frac{\ell^2}{2mr^2}$$

$\dot{r} = 0 \Rightarrow \quad -\dfrac{5GMm}{4R} = \dfrac{m}{2r^2}\cdot\dfrac{GMR}{32} + U(r)$

Si $r < R$

$$-\frac{5GMm}{4R} = \frac{GMmR}{64r^2} + \frac{GMm\,r^2}{2R^3} - \frac{3GMm}{2R}$$

$$\left(\div \frac{GMm}{4R}\right) \qquad -5 = \frac{R^2}{16r^2} + \frac{2r^2}{R^2} - 6$$

$$\frac{2r^4}{R^2} - r^2 + \frac{R^2}{16} = 0$$

$$\frac{1 \pm \sqrt{1 - 4\frac{2}{R^2}\cdot\frac{R^2}{16}}}{4/R^2} = \frac{R^2}{4}\left(1 \pm \sqrt{1-1/2}\right) \begin{cases} \frac{R^2}{4}\left(1+\frac{1}{\sqrt{2}}\right) \\ \frac{R^2}{4}\left(1-\frac{1}{\sqrt{2}}\right)\end{cases}$$

$$\boxed{\begin{aligned} r_{min} &= \frac{R}{2}\sqrt{1-\frac{1}{\sqrt{2}}} \\ r_{max} &= \frac{R}{2}\sqrt{1+\frac{1}{\sqrt{2}}}\end{aligned}}$$

3. $\dot{r} = \frac{dr}{d\varphi}\dot{\varphi} = r'\dot{\varphi} = r'\frac{\ell}{mr^2}$

$$-\frac{5GMm}{4R} = \frac{GMmR}{64r^2} + \frac{GMm\,r^2}{2R^3} - \frac{3GMm}{2R} + \frac{m\dot{r}^2}{2}$$

$$\dot{r}^2 = r'^2\frac{\ell^2}{m^2r^4} = r'^2\frac{GMR}{32r^4}$$

$$-\frac{5GMm}{4R} = \frac{GMmR}{64r^2} + \frac{GMm\,r^2}{2R^3} - \frac{3GMm}{2R} + \frac{GMmR}{64r^4}r'^2$$

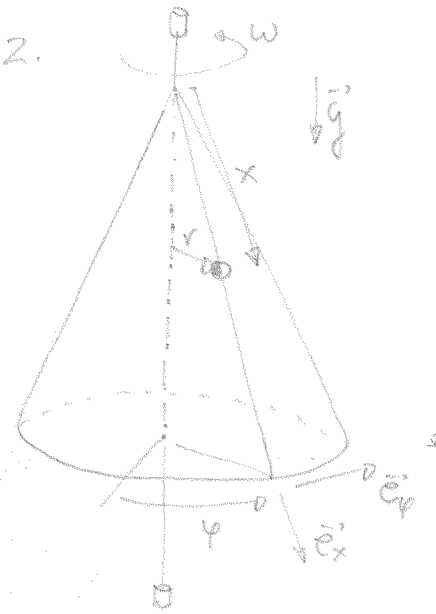
$$\underbrace{-\frac{5}{4R} - \frac{R}{64r^2}}\ - \frac{r^2}{2R^3} + \underbrace{\frac{3}{2R}}_{\frac{1}{4R}} = \frac{R}{64r^4}r'^2$$

$$\frac{16r^4}{R^2} - r^2 - \frac{32r^6}{R^4} = r'^2$$

$$\boxed{\frac{dr}{d\varphi} = r\sqrt{16\left(\frac{r}{R}\right)^2 - 1 - 32\left(\frac{r}{R}\right)^4}}$$

Ej. 2.

1.



$$\vec{v} = \dot{x}\,\vec{e}_x + r\dot{\varphi}\,\vec{e}_\varphi \quad ; \quad \dot{\varphi} = \omega(t)$$

$$\vec{L}\cdot\vec{k} = c^{te}$$

$$\vec{L}\cdot\vec{k} = I\omega + mr^2\omega$$

$$I\omega_0 = I\omega_f + mR^2\omega_f$$

$$\boxed{\omega_f = \left(\frac{I}{I + mR^2}\right)\omega_0 = \left[\frac{1}{1 + \frac{mR^2}{I}}\right]\omega_0}$$

$E = c^{te}$

$$\frac{1}{2} I\omega_0^2 = \frac{1}{2} I\omega_f^2 - mgh + \frac{1}{2} m\left(\dot{x}_f^2 + R^2\omega_f^2\right)$$

$$I\omega_0^2 = \frac{I\omega_0^2}{\left(1 + mR^2/I\right)^2} - 2mgh + m\dot{x}_f^2 + \frac{mR^2\omega_0^2}{\left(1 + \frac{mR^2}{I}\right)^2}$$

$$I\omega_0^2 = \frac{I\omega_0^2}{\left(1 + \frac{mR^2}{I}\right)^2}\left(1 + \frac{mR^2}{I}\right) - 2mgh + m\dot{x}_f^2$$

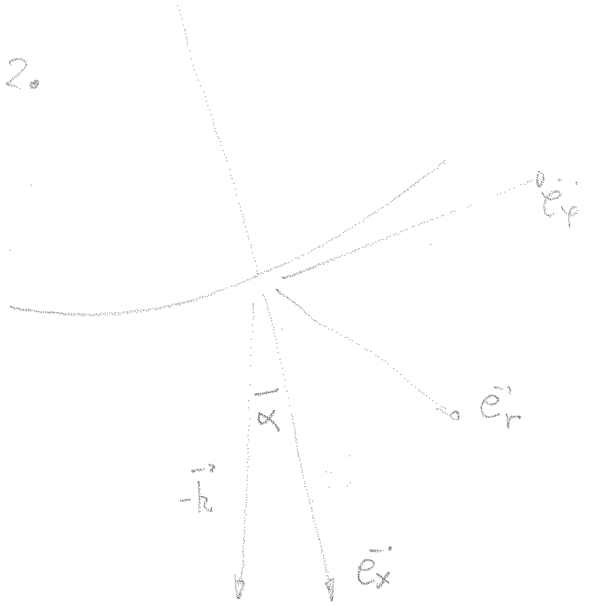
$$I\omega_0^2\left(1 - \frac{1}{1 + \frac{mR^2}{I}}\right) + 2mgh = m\dot{x}_f^2$$

$$\cancel{I}\omega_0^2\left(\frac{\cancel{m}R^2/\cancel{I}}{1 + mR^2/I}\right) + 2\cancel{m}gh = \cancel{m}\dot{x}_f^2$$

$$\boxed{\sqrt{2gh + \frac{R^2\omega_0^2}{1 + mR^2/I}} = \dot{x}_f}$$

$$\vec{v}_f = \dot{x}_f\,\vec{e}_x + R\omega_f\,\vec{e}_\varphi$$

2.



$$\operatorname{sen}\alpha = \frac{R}{\sqrt{R^2+h^2}}$$

$$\vec{v}_f = \dot{x}_f \operatorname{sen}\alpha \, \vec{e}_r - \dot{x}_f \cos\alpha \, \vec{k} + R\omega_f \vec{e}_\varphi$$

$$h = g\frac{t_v^2}{2} + \dot{x}_f \cos\alpha \, t_v$$

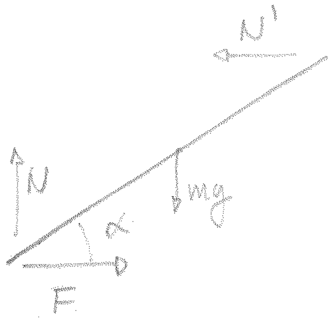
$$\frac{g}{2} t_v^2 + \dot{x}_f \cos\alpha \, t_v - h = 0$$

$$t_v = \frac{-\dot{x}_f \cos\alpha}{g} + \sqrt{\frac{\dot{x}_f^2 \cos^2\alpha + 4 \frac{g}{2} h}{g^2}}$$

$$t_v = \sqrt{\frac{\dot{x}_f^2 \cos^2\alpha}{g^2} + \frac{2h}{g}} - \frac{\dot{x}_f \cos\alpha}{g}$$

$$\boxed{\vec{v}_{piso} = \dot{x}_f \operatorname{sen}\alpha \, \vec{e}_r + R\omega_f \vec{e}_\varphi - \vec{k} \cdot \left(\dot{x}_f \cos\alpha + g \, t_v\right)}$$

Ej. 3.



$$\begin{cases} N - mg = 0 \\ F - N' = 0 \end{cases}$$

$$F \leq fN$$

$$N' \leq f mg$$

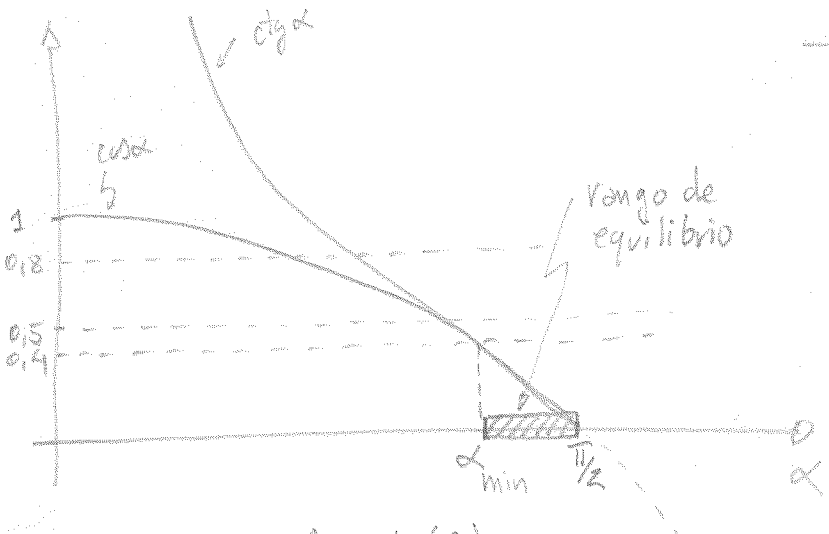
$$0 = N' L \operatorname{sen}\alpha - mg \frac{L}{2} \cos\alpha$$

$$N' \operatorname{sen}\alpha = \frac{mg\cos\alpha}{2}$$

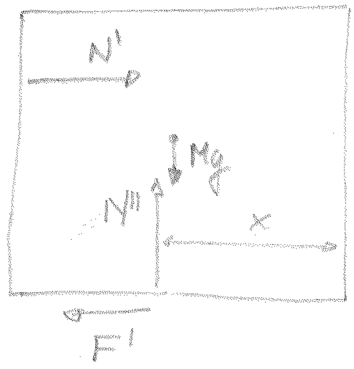
$$N' = \frac{mg}{2} \operatorname{ctg}\alpha$$

$$\boxed{\operatorname{ctg}\alpha \leq 2f}$$

no desliz. barra



$$\alpha_{min} = \operatorname{Arc\,ctg}(f)$$



$$N' - F' = 0$$

$$N'' - Mg = 0$$

$$-N' L \operatorname{sen}\alpha + Mg \frac{L}{2} - N'' x = 0$$

$$-\frac{mg}{2} \operatorname{ctg}\alpha \, L \operatorname{sen}\alpha + Mg \frac{L}{2} - Mg x = 0$$

$$-m \cos\alpha \, L + ML - 2Mx = 0$$

$$\boxed{\frac{L}{2}\left(1 - \frac{m}{M}\cos\alpha\right) = x}$$

$$\frac{L}{2}\left(1 - 2\cos\alpha\right) = x \geq 0$$

$$L\left(\frac{1}{2} - \cos\alpha\right) \geq 0$$

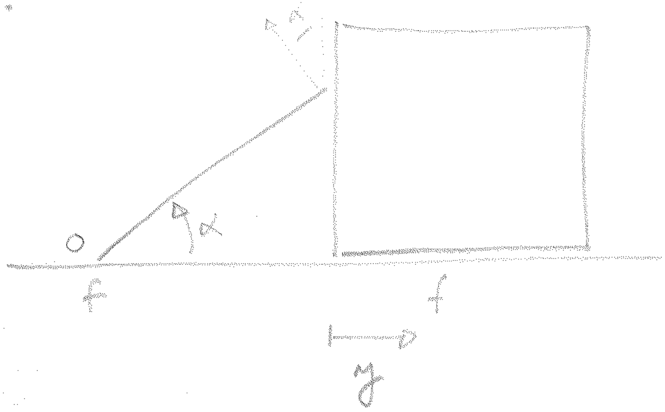
$$\longrightarrow \boxed{\cos\alpha \leq 1/2}$$

$$|F'| \leq f Mg$$

$$\frac{mg\operatorname{ctg}\alpha}{2} \leq f Mg$$

$$\boxed{\operatorname{ctg}\alpha \leq f}$$

2.



$$M\ddot{y} = N' - F'$$

$$F' = fN'' = fMg$$

$$M\ddot{y} = N' - fMg$$

$$I_0\ddot{\alpha} = N' L\,\text{sen}\,\alpha - mg\frac{L}{2}\cos\alpha$$

Vinculo entre α e $y \Rightarrow$ $L\dot{\alpha}\,\text{sen}\,\alpha = -\dot{y}$

$$L\ddot{\alpha}\,\text{sen}\,\alpha + L\dot{\alpha}^2\cos\alpha = -\ddot{y}$$

$$I_0\ddot{\alpha} = -M\left(L\ddot{\alpha}\,\text{sen}\,\alpha + L\dot{\alpha}^2\cos\alpha\right)L\,\text{sen}\,\alpha + fMgL\,\text{sen}\,\alpha - mg\frac{L}{2}\cos\alpha$$

$t = t_0 \Rightarrow \dot{\alpha}(t_0) = 0$

$$I_0\ddot{\alpha}_0 = -ML^2\text{sen}^2\alpha_0\,\ddot{\alpha}_0 + fMgL\,\text{sen}\,\alpha_0 - mg\frac{L}{2}\cos\alpha_0$$

$$\left(I_0 + ML^2\text{sen}^2\alpha_0\right)\ddot{\alpha}_0 = fMgL\,\text{sen}\,\alpha_0 - MgL\cos\alpha_0$$

$$\boxed{\ddot{\alpha}_0 = \frac{MgL\left(f\,\text{sen}\,\alpha_0 - \cos\alpha_0\right)}{\left(I_0 + ML^2\text{sen}^2\alpha_0\right)}}$$

$$I_0 = \frac{mL^2}{3} = \frac{2ML^2}{3}$$

$$\boxed{\ddot{\alpha}_0 = \frac{3g\left(f\,\text{sen}\,\alpha_0 - \cos\alpha_0\right)}{L\left(2 + 3\,\text{sen}^2\alpha_0\right)}}$$