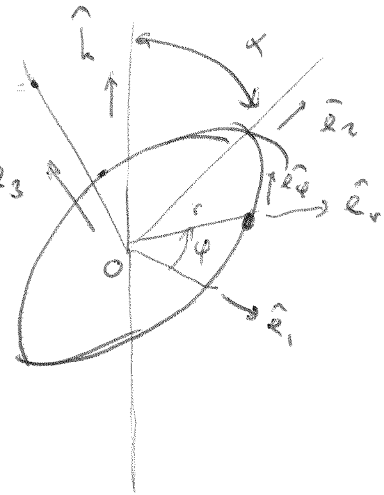
1.-



$$\vec{a}_R \cdot \hat{e}_\varphi = r\ddot{\varphi}$$

$$\vec{a}_T \cdot \hat{e}_\varphi = \{\vec{\omega} \wedge [\vec{\omega} \wedge (P-O)]\} \cdot \hat{e}_\varphi =$$

$$= [\vec{\omega} \cdot (P-O)]\, \vec{\omega} \cdot \hat{e}_\varphi - \omega^2 \underbrace{(P-O) \cdot \hat{e}_\varphi}_{=0}$$

$$= r\omega^2 (\hat{k} \cdot \hat{e}_r)(\hat{k} \cdot \hat{e}_\varphi)$$

$\hat{k} = \cos\alpha\, \hat{e}_2 + \operatorname{sen}\alpha\, \hat{e}_3 \quad \therefore \quad \hat{k} \cdot \hat{e}_\varphi = \cos\alpha\, \hat{e}_2 \cdot \hat{e}_\varphi = \cos\alpha \cos\varphi$

$\hat{k} \cdot \hat{e}_r = \cos\alpha\, \hat{e}_2 \cdot \hat{e}_r = \cos\alpha \operatorname{sen}\varphi$

$$\vec{a}_C \cdot \hat{e}_\varphi = 0$$

$$\vec{F} \cdot \hat{e}_\varphi = -mg\, \hat{k} \cdot \hat{e}_\varphi + \underbrace{\hat{R} \cdot \hat{e}_\varphi}_{=0} = -mg \cos\alpha \cos\varphi$$

Newton: $\boxed{\ddot{\varphi} + \cos\alpha \cos\varphi \left(\omega^2 \cos\alpha \operatorname{sen}\varphi + \frac{g}{r} \right) = 0}$

2.- $\underline{\alpha = \frac{\pi}{2}}$ — todas las posiciones son de equilibrio

$0 \leq \alpha < \frac{\pi}{2}$

$\varphi_1 = \frac{\pi}{2}$

$\varphi_2 = -\frac{\pi}{2}$

$\varphi_3 = \operatorname{Arcsen}\left(-\frac{g}{r\omega^2 \cos\alpha} \right)$; existe y es $\neq \varphi_2$ si $\omega^2 > \frac{g}{r\cos\alpha}$

Si $\alpha = \frac{\pi}{2}$, todas las posiciones son de equilibrio inestable.-

Si $0 \leq \alpha < \frac{\pi}{2}$ $\quad g(\varphi) = \cos\alpha \cos\varphi \left(\omega^2 \cos\alpha \operatorname{sen}\varphi + \frac{g}{r} \right)$

$$g'(\varphi) = -\cos\alpha \,\text{sen}\,\varphi \left(\omega^2 \cos\alpha \,\text{sen}\,\varphi + \frac{g}{r}\right) +$$

$$+ \omega^2 \cos^2\alpha \cos^2\varphi$$

$$g'(\varphi_1) = -\cos\alpha \left(\omega^2\cos\alpha + \frac{g}{r}\right) < 0$$

$\Rightarrow$ $\boxed{\varphi_1 \text{ es inestable}}$

$$g'(\varphi_2) = \cos\alpha \left(\frac{g}{r} - \omega^2\cos\alpha\right)$$

si $\frac{g}{r} = \omega^2\cos\alpha$, $\quad g'(\varphi) = -\omega^2\cos^2\alpha \,\text{sen}\,\varphi\,(1+\text{sen}\,\varphi)$

$$g''(\varphi) = -\omega^2\cos^2\alpha \cos\varphi\,(1+2\,\text{sen}\,\varphi)$$

$$g''(\varphi_2) = 0$$

$$g'''(\varphi) = -\omega^2\cos^2\alpha \left[2\cos^2\varphi - \text{sen}\,\varphi(1+2\,\text{sen}\,\varphi)\right]$$

$$g'''(\varphi_2) = \omega^2\cos^2\alpha > 0$$

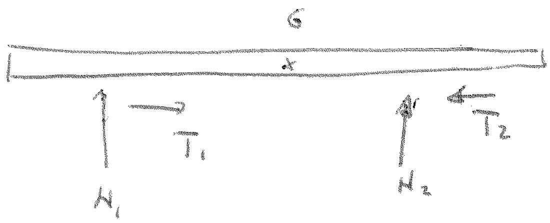
$\Rightarrow$

φ_2 es estable	si	$\omega^2 \leq \dfrac{g}{r\cos\alpha}$
" inestable	si	$\omega^2 > \dfrac{g}{r\cos\alpha}$

$$g'(\varphi_3) = \omega^2\cos^2\alpha\cos^2\varphi_3 > 0$$

$\therefore$ $\boxed{\varphi_3 \text{ es estable}}$

1.-



$T_1 = f N_1$, $T_2 = f N_2$;

$$\left.\begin{aligned} m\ddot{x} &= T_1 - T_2 \\ N_1 + N_2 &= mg \\ a N_2 &= x m g \end{aligned}\right\} \Rightarrow \begin{aligned} N_1 &= \frac{a-x}{a}\, mg \\ N_2 &= \frac{x}{a}\, mg \\ \ddot{x} + \frac{2fg}{a}x &= fg \end{aligned}$$

$$\Rightarrow \boxed{x(t) = \left(x_0 - \frac{a}{2}\right)\cos\omega t + \frac{a}{2}}^{(1)} \quad ; \quad \omega^2 = \frac{2fg}{a}$$

2.- Cambia los sentidos de T_1 y T_2, lo que equivale a cambiar f por $(-f)$.-

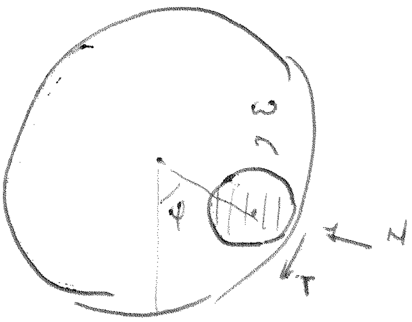
$$\Rightarrow \quad \ddot{x} - \frac{2fg}{a}x = -fg \quad \Rightarrow \quad \boxed{x(t) = \left(x_0 - \frac{a}{2}\right)\operatorname{ch}\omega t + \frac{a}{2}}$$

3.- $N_1 \geq 0$, $N_2 \geq 0 \Rightarrow 0 \leq x \leq a$

En el caso 1) si x_0 verifica $0 \leq x_0 \leq a$ se puede ver de (1) que $0 \leq x(t) \leq a$ para todo t.-

En el caso 2) si $x_0 \neq \frac{a}{2}$ el $\operatorname{ch}\omega t$ tiende a ∞ cuando t aumenta por lo que la barra siempre pierde contacto con los discos.- Si $x_0 = \frac{a}{2}$ la barra está en equilibrio.

1.-



$T = f|N|, \quad N \geq 0$

Ecs. card. $\boxed{3mr\ddot{\varphi} = -T, \quad 3mr\dot{\varphi}^2 = N; \quad \frac{mr^2}{2}\dot{\omega} = -rT}$

$$\Rightarrow 3mr\ddot{\varphi} = \frac{mr\dot{\omega}}{2} \quad \therefore \quad \dot{\omega} = 6\ddot{\varphi} \quad \therefore \quad \omega = 6\dot{\varphi} + C$$

$$C = \underset{=0}{\omega_0} - 6\dot{\varphi}_0 = -6\frac{v_0}{3r} = -\frac{2v_0}{r}$$

$$\boxed{\omega = 6\dot{\varphi} - \frac{2v_0}{r}}$$

$$N = 3mr\dot{\varphi}^2 \geq 0$$

2.- $3mr\ddot{\varphi} = -f\,3mr\dot{\varphi}^2 \Rightarrow \boxed{\ddot{\varphi} = -f\dot{\varphi}^2}$

3.- Las ecuaciones anteriores son válidas mientras $3r\dot{\varphi} + r\omega > 0$

$$u = \dot{\varphi} \Rightarrow \dot{u} = -fu^2 \quad \therefore \quad \frac{du}{dt} = fu^2; \quad \frac{du}{u^2} = -f\,dt; \quad -\frac{1}{u} = -ft + K$$

$$K = -\frac{3r}{v_0} \Rightarrow \dot{\varphi} = \frac{v_0/3r}{1 + \frac{v_0}{3r}ft}$$

$$3r\dot{\varphi} + r\omega = 9r\dot{\varphi} - 2v_0 > 0 \quad \therefore \quad \boxed{t < \frac{3}{2}\frac{r}{v_0 f}}$$

4.- $\varphi(t) = \frac{1}{f}\ln\left|1 + \frac{v_0}{3r}ft\right|; \quad \varphi(t_0) = 2\pi = \frac{1}{f}\ln\left(1 + \frac{v_0 f}{3r}t_0\right)$

$$\rightarrow \boxed{t_0 = \frac{e^{2\pi f} - 1}{\frac{v_0}{3r}f}}$$

$$t_0 < \frac{3}{2}\frac{r}{v_0 f} \rightarrow \boxed{f < \frac{1}{2\pi}\ln\frac{3}{2}}$$