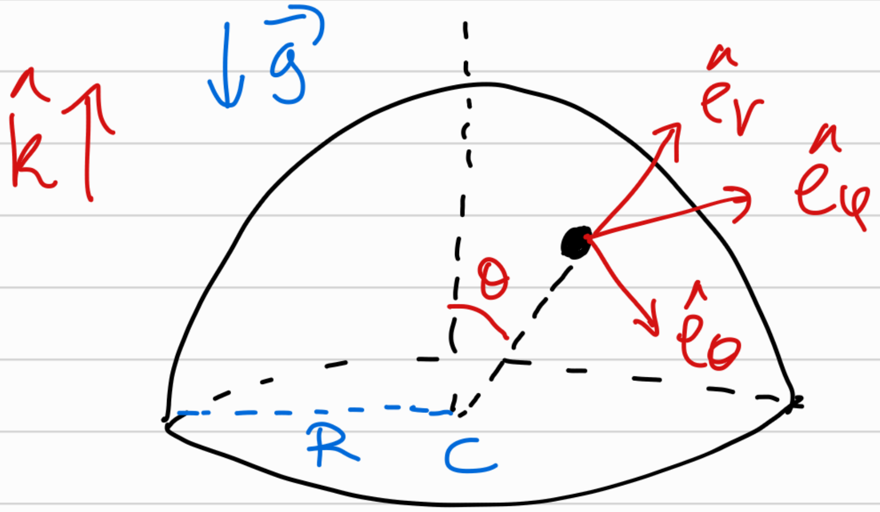
Ej 1



→ $\vec r = R\,\hat e_r$

$\vec v = R\dot\theta\,\hat e_\theta + R\sin\theta\,\dot\varphi\,\hat e_\varphi$

a) $\vec L_c = m\,\vec r\times\vec v$

→ $\hat k$ cte $\Longrightarrow$ $\dfrac{d}{dt}\left(\vec L_c\cdot\hat k\right) = \left(\dfrac{d\vec L_c}{dt}\right)\cdot\hat k$

$$\rightarrow \frac{d\vec L_c}{dt} = \underbrace{m\underbrace{\frac{d\vec r}{dt}}_{\vec v}\times\vec v}_{0} + m\,\vec r\times\underbrace{\frac{d\vec v}{dt}}_{\vec a}$$

Usando $\vec F_{Neta} = m\vec a$

$$\Longrightarrow \frac{d\vec L_c}{dt} = \vec r\times\vec F_{Neta}$$

$$(\vec A\times\vec B)\cdot\vec C = (\vec C\times\vec A)\cdot\vec B$$

$$\Longrightarrow \frac{d}{dt}\left(\vec L_c\cdot\hat k\right) = \left(\vec r\times\vec F_{Neta}\right)\cdot\hat k =$$

$$= \underbrace{\left(\hat k\times\vec r\right)}_{R\sin\theta\,\hat e_\varphi}\cdot\vec F_{Neta}$$

$$\Longrightarrow \frac{d}{dt}(\vec{L_c}\cdot\hat{k}) = R\sin\theta\,(\vec{F}_{neta}\cdot\hat{e}_\varphi)$$

$$\longrightarrow \vec{F}_{Neta} = N\hat{e}_r - mg(\cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta)$$

(contacto liso)

<u>Obs</u> $\vec{F}_{neta}\cdot\hat{e}_\varphi = 0$

$$\Longrightarrow \boxed{\frac{d}{dt}(\vec{L_c}\cdot\hat{k}) = 0}$$

b) $\ell \equiv \vec{L_c}\cdot\hat{k} = m(\vec{r}\times\vec{v})\cdot\hat{k} = mR^2\sin^2\theta\,\dot{\varphi}$

cant cons del movimiento

<u>Obs</u> $P_N = \vec{N}\cdot\vec{v} = 0$
$m\vec{g}$ cons $\Big\} \Rightarrow E$ se cons

$$E = \frac{m}{2}|\vec{v}|^2 + U_g = \frac{m}{2}R^2(\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2) + mgR\cos\theta$$

→ Conds iniciales * $R\cos\theta_{inic} = \frac{R}{2}$

* $|\vec{v}|_{inic} = v_i^2$

* $R\sin\theta\,\dot{\varphi} = v_i$ * $R\sin\theta_{inic} = \sqrt{R^2 - \left(\frac{R}{2}\right)^2} = R\sqrt{\frac{3}{4}}$

$$\Rightarrow \begin{cases} l = m\, v_i\, R \sqrt{\frac{3}{4}} \\ E = \frac{m v_i^2}{2} + mg\frac{R}{2} \end{cases}$$

→ Cons de l implica: $\boxed{\dot{\varphi} = \frac{v_i}{R \sin^2\theta}\sqrt{\frac{3}{4}}}$

→ Sustituyendo en E se tiene

$$\boxed{\begin{aligned} &\frac{mR^2}{2}\dot{\theta}^2 + \frac{m v_i^2}{2}\frac{3}{4}\frac{1}{\sin^2\theta} + mgR\cos\theta \\ &= \frac{m v_i^2}{2} + mg\frac{R}{2} \end{aligned}}$$

c) Para que no se desprenda en $t=0$ se necesita

$$\vec{N}\cdot\hat{e}_r\Big|_{t=0} > 0$$

→ Ec de Newton: $m\vec{a} = \vec{N} + m\vec{g}$

$$\Rightarrow \vec{N}\cdot\hat{e}_r = m\vec{a}\cdot\hat{e}_r \underbrace{- m\vec{g}\cdot\hat{e}_r}_{mg\cos\theta \,=\, \frac{U_g}{R}}$$

Obs $\vec{a}\cdot\hat{e}_r = \frac{d\vec{v}}{dt}\cdot\hat{e}_r = \frac{d}{dt}(\vec{v}\cdot\hat{e}_r) - \vec{v}\cdot\frac{d\hat{e}_r}{dt}$

Usando que $\vec{r} = R\,\hat{e}_r \Rightarrow |\vec{r}|^2 = R^2 = cte$

$$\Rightarrow \begin{cases} \vec{v}\cdot\vec{r} = 0 \Rightarrow \vec{v}\cdot\hat{e}_r = 0 \\ \frac{d\hat{e}_r}{dt} = \frac{\vec{v}}{R} \end{cases}$$

$$\Rightarrow \boxed{\vec{a}\cdot\hat{e}_r = -\frac{|\vec{v}|^2}{R}}$$

$$\rightarrow E = \frac{m|\vec{v}|^2}{2} + U_g \Rightarrow \boxed{m\vec{a}\cdot\hat{e}_r = (U_g - E)\frac{2}{R}}$$

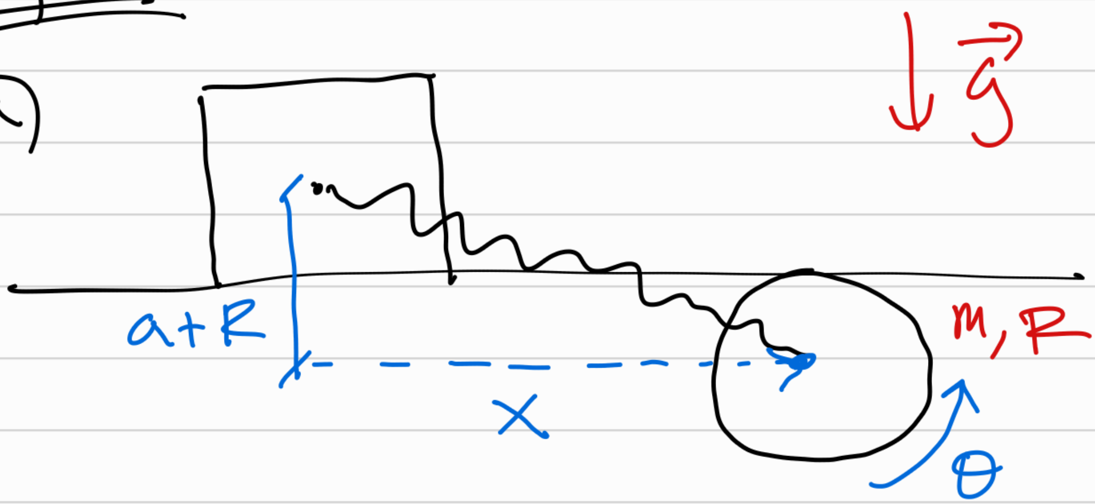
$$\Rightarrow \vec{N}\cdot\hat{e}_r = \frac{(3U_g - 2E)}{R}$$

$$\Rightarrow \vec{N}\cdot\hat{e}_r\Big|_{t=0} = \frac{1}{R}\left[\frac{3mgR}{2} - mv_i^2 - mgR\right]$$

$$\Rightarrow \boxed{\vec{N}\cdot\hat{e}_r\Big|_{t=0} > 0 \Leftrightarrow v_i^2 < \frac{gR}{2}}$$

Ej 2

a)



→ RSD $\boxed{\dot{x} = R\dot{\theta}}$

→ Obs RSD ⇒ $P_{\vec{F}roz} = 0$

$P_{\vec{N}} = 0$, demás fuerzas cons

⇒ mov del disco cons E

→ $E = \dfrac{m|\vec{v}_{CM}|^2}{2} + \dfrac{I_{disco\,CM}}{2}\dot{\theta}^2 + U_{elástica}$

($U_g = cte$)

* $|\vec{v}_{CM}|^2 = \dot{x}^2$
* $\dot{\theta}^2 = \left(\dfrac{\dot{x}}{R}\right)^2$ * $I_{disco\,CM} = \dfrac{mR^2}{2}$

* $U_{elástica} = \dfrac{kx^2}{2}$ (ignoramos cte $\dfrac{k(a+R)^2}{2}$)

(long nat nula)

$$\Rightarrow E = \frac{m}{2}\frac{3}{2}\dot{x}^2 + \frac{kx^2}{2}$$

$$\frac{d}{dt}E = \boxed{0 = m\frac{3}{2}\ddot{x} + kx}$$ Ec de mov

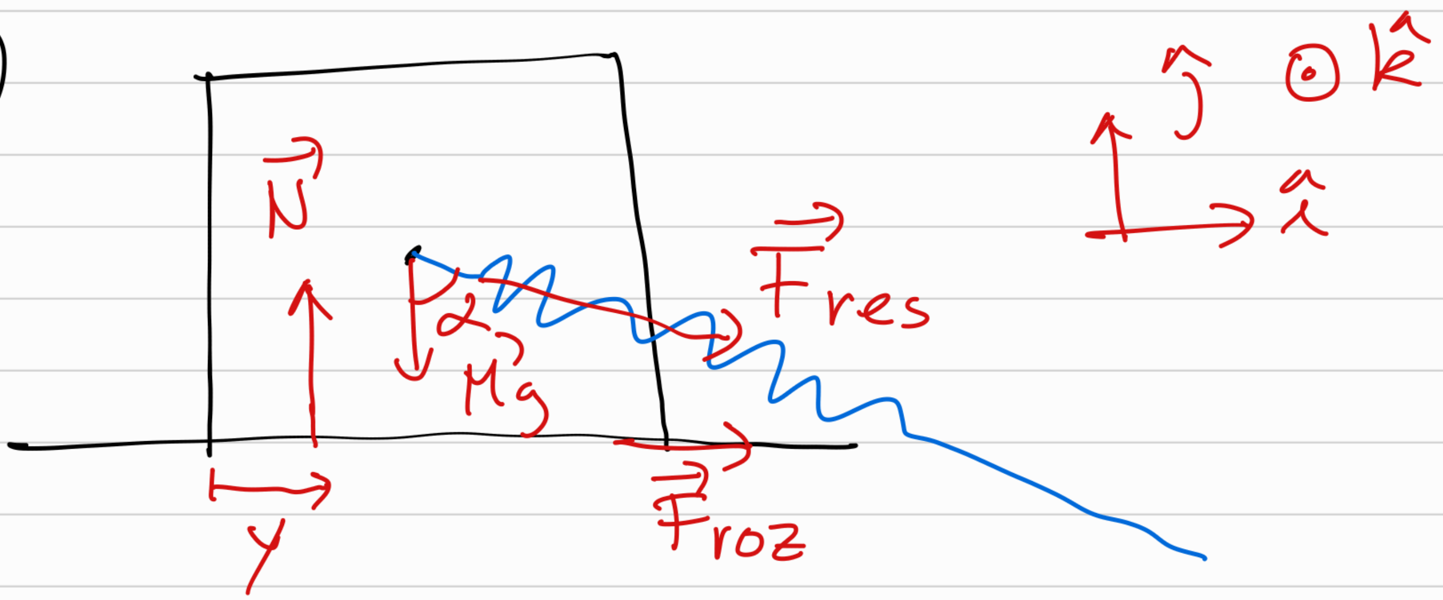
Obs Por cond inic $\begin{cases} x|_{t=0} = 0 \\ \dot{x}|_{t=0} = v_0 \end{cases}$

$$\Rightarrow E = \frac{m}{2}\frac{3}{2}v_0^2$$

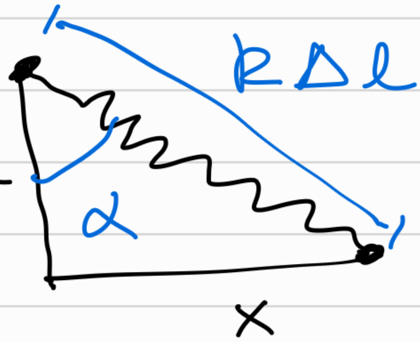
Obs cómo $\dot{x}^2 \geq 0$ y $E = cte \Rightarrow x$ máximo sucede cuando $\dot{x} = 0$

$$\Longrightarrow \boxed{x_{max} = \sqrt{\frac{m3v_0^2}{2k}}}$$

b)



$$\rightarrow \sum \vec{F}_{\text{sobre placa}} = 0$$



$$\hat{\imath})\ F_{roz} + \underbrace{k\Delta l \sin\alpha}_{x} = 0$$

$$\hat{\jmath})\ N - Mg - \underbrace{k\Delta l \cos\alpha}_{R+a} = 0$$

$\rightarrow$ Para que no deslice en instante t

$$|F_{roz}(t)| \leq f\,|N|$$

$$\Rightarrow k\,|x(t)| \leq f\left(Mg + k(R+a)\right)$$

$\rightarrow$ Para que no deslice en ningún instante basta con imponer que esto suceda cuando $x = x_{max}$

$$\Rightarrow k\,|x_{max}| \leq f\left(Mg + kR + ka\right)$$

$$\boxed{\sqrt{k\, m \tfrac{3}{2} v_0^2} \leq f\left(Mg + kR + ka\right)}$$

$$\rightarrow \sum_{\substack{\text{sobre}\\\text{placa}}} \vec{\tau}_{CM} = 0$$

$$\hat{k})\ F_{Roz}\, a - (a-y)N = 0$$

$$\Rightarrow -Rxa - (a-y)\left(Mg + k(R+a)\right) = 0$$

$$y = \frac{a\left(Mg + k(R+a+x)\right)}{Mg + k(R+a)}$$

→ Para que no se desprenda:

$$0 < y < 2a$$

→ Por la simetría del problema las dos condiciones son equivalentes por lo que basta con imponer una:

$$0 < y \Leftrightarrow 0 < Mg + k(R+a) + kx(t) \;\forall t$$

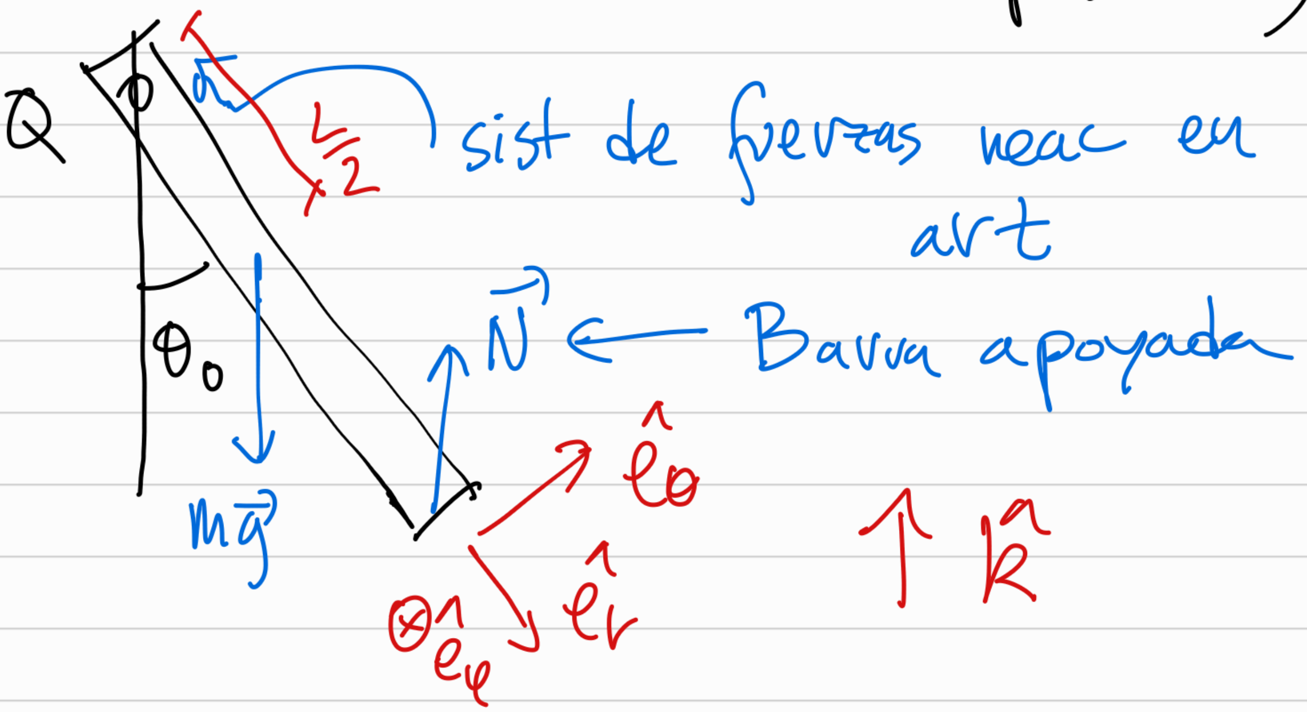
→ Para que se cumpla $\forall t$ basta con imponer que se cumpla cuando $x = -x_{max}$

$$\Rightarrow kx_{max} < k(R+a) + Mg$$

$$\Rightarrow \boxed{\sqrt{km\frac{3}{2}v_0^2} < k(R+a) + Mg}$$

Ej 3

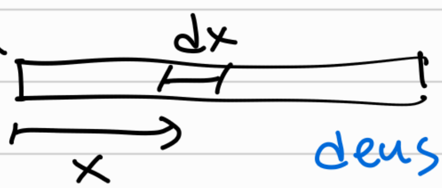
a) $\theta = \theta_0 = cte$ (barra apoyada)



$\rightarrow \vec{L}_Q = \mathbb{I}_Q(\vec{\omega})$

$\rightarrow \vec{\omega} = \omega\left(-\cos\theta_0 \hat{e}_r + \sin\theta_0 \hat{e}_\theta\right)$ barra apoyada

$\rightarrow$



dens lin $\frac{m}{L}$

$$I = \int_0^L x^2 \frac{dx\, m}{L} = \frac{mL^2}{3}$$

$$\Longrightarrow \mathbb{I}_Q^{\{\hat{e}_r, \hat{e}_\varphi, \hat{e}_\theta\}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$$

$$\Longrightarrow \boxed{\vec{L}_Q = \omega \sin\theta_0 I \hat{e}_\theta}$$

b) Para que la barra se mantenga apoyada $\vec{N} \cdot \hat{R} > 0 \quad \forall t$

$\rightarrow$ 2da Card: (Q fijo)

$$\frac{d\vec{L_Q}}{dt} = L\,\hat{e}_r \times \vec{N} + \frac{L}{2}\hat{e}_r \times m\vec{g} + \vec{\tau}_{art}$$

$$\rightarrow \frac{d\vec{L_Q}}{dt} = \omega \sin\theta_0 \frac{mL^2}{3} \frac{d\hat{e}_\theta}{dt} \qquad \omega \cos\theta_0 \hat{e}_\varphi$$

$\rightarrow$ Usando que $\vec{\tau}_{art} \cdot \hat{e}_\varphi = 0$ (art cil lisa) proyectando la 2da card en $\hat{e}_\varphi$ tenemos:

$$L \sin\theta_0 (\vec{N} \cdot \hat{R}) = \frac{L}{2}\sin\theta_0\, mg - \omega^2 \sin\theta_0 \cos\theta_0 \frac{mL^2}{3}$$

$$\Rightarrow \vec{N} \cdot \hat{R} > 0 \iff \boxed{\omega^2 < \frac{g}{L}\frac{3}{2\cos\theta_0}}$$

c) Ahora
$$\begin{cases} \vec{\omega} = \omega(-\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta) - \dot{\theta}\,\hat{e}_\varphi \\ \vec{N} = 0 \end{cases}$$

$$\rightarrow \vec{L_Q} = \omega \sin\theta \frac{mL^2}{3}\hat{e}_\theta - \dot{\theta}\frac{mL^2}{3}\hat{e}_\varphi$$

$\rightarrow$ 2da Card

$$\frac{d\vec{L}_Q}{dt} = \frac{L}{2}\sin\theta\, mg\,\hat{e}_\varphi + \vec{\tau}_{art}$$

$\rightarrow$ **Obs** $\left(\dfrac{d\,\dot{\theta}\,\hat{e}_\varphi}{dt}\right)\cdot\hat{e}_\varphi = \ddot{\theta}\,\hat{e}_\varphi$

$$\left(\frac{d\hat{e}_\theta}{dt}\right)\cdot\hat{e}_\varphi = \omega\cos\theta$$

$\Rightarrow$ Proyectando la 2^{da} Card en $\hat{e}_\varphi$:

$$\boxed{\frac{mL^2}{3}\left(\omega^2\sin\theta\cos\theta - \ddot{\theta}\right) = \frac{mgL}{2}\sin\theta}$$

Ec de movimiento