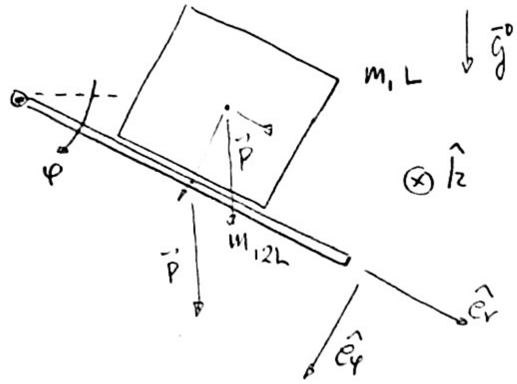
2do Parcial 2022/1

Ejercicio 1

(a)



Barra =

$$I_G^{|} = \frac{mL^2}{3} \xrightarrow{\text{Steiner}} I_O^{|} = \frac{mL^2}{3} + mL^2 = \frac{4}{3} mL^2$$

Placa =

$$I_G^{\square} = \frac{mL^2}{6} \xrightarrow{\text{Steiner}} I_O^{\square} = \frac{mL^2}{6} + m\left(L^2 + \frac{L^2}{4}\right) =$$

$$= mL^2\left(\frac{1}{6} + \frac{5}{4}\right) = \frac{17}{12} mL^2$$

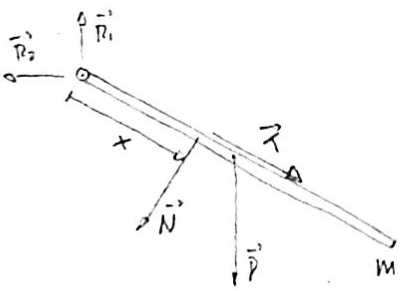
(H) No deslizamiento $\Rightarrow$ conjunto rígido

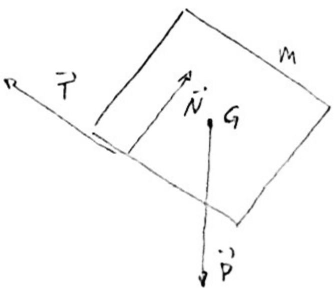
$$I_O = I_O^{\square} + I_O^{|} = \left(\frac{4}{3} + \frac{17}{12}\right) mL^2 = \frac{11}{4} mL^2$$

$$I_O \ddot{\varphi} \hat{k} = \vec{M}^{ext} \cdot \hat{k} \longrightarrow \frac{11}{4} mL^2 \ddot{\varphi} = mgL\cos\varphi + mgL\cos\varphi + mg\frac{L}{2}\operatorname{sen}\varphi$$

$$\boxed{\ddot{\varphi} = \frac{8}{11} g/L \cos\varphi + \frac{2}{11} g/L \operatorname{sen}\varphi}$$

(b)





Placa:

$$\vec{r}_G = L\hat{e}_r - L/2\,\hat{e}_\varphi$$

$$\vec{v}_G = L\dot{\varphi}\hat{e}_\varphi + L/2\,\dot{\varphi}\hat{e}_r$$

$$\vec{a}_G = \left(L\ddot{\varphi} + L/2\,\dot{\varphi}^2\right)\hat{e}_\varphi + \left(\frac{L}{2}\ddot{\varphi} - L\dot{\varphi}^2\right)\hat{e}_r$$

$$m\vec{a}_G = -N\hat{e}_\varphi - T\hat{e}_r + mg\cos\varphi\,\hat{e}_\varphi + $$
$$+ mg\operatorname{sen}\varphi\,\hat{e}_r$$

Barra:

$$I_O^{|}\ddot{\varphi} = M_O^{ext} = Nx + mgL\cos\varphi$$

$$\frac{4}{3} mL^2\ddot{\varphi} = Nx + mgL\cos\varphi$$

$(\hat{e}_r)$ $\quad m\left(\frac{L}{2}\ddot{\varphi} - L\dot{\varphi}^2\right) = mg\operatorname{sen}\varphi - T$

$(\hat{e}_\varphi)$ $\quad m\left(L\ddot{\varphi} + L/2\,\dot{\varphi}^2\right) = mg\cos\varphi - N$

Condiciones de movimiento en $t = 0$ $\longrightarrow$ $\varphi(0) = 0$, $\dot{\varphi}(0) = 0$

$$\left.\begin{aligned} &\ddot{\varphi} = \frac{8}{11} g/L \quad ; \quad mL\ddot{\varphi} = mg - N \\ &\frac{mL}{2}\ddot{\varphi} = -T \quad ; \quad \frac{4}{3} mL^2\ddot{\varphi} = Nx + mgL \end{aligned}\right\} \rightarrow \begin{aligned} &T = -\frac{4}{11} mg \\ &N = mg - \frac{8}{11} mg = \frac{3}{11} mg \end{aligned}$$

$$\left(\frac{4}{3}\cdot\frac{8}{11}\right)mg = \frac{3}{11}mg\left(\frac{x}{L}\right) + mg \longrightarrow \frac{32}{33} - 1 = \frac{3}{11}\left(\frac{x}{L}\right) \longrightarrow \left(\frac{x}{L}\right) = -\frac{1}{9}$$

Condiciones para mantener el vinculo de reposo relativo entre placa y barra:

$N \geq 0 \Rightarrow \frac{3}{11}mg \geq 0$ verificado $\Rightarrow$ se mantiene el vinculo de apoyo.

$|T| \leq \mu_s N \Rightarrow \frac{4}{11}mg \leq \frac{5}{3}\cdot\frac{3}{11}mg$

$4 \leq 5$ verificado $\Rightarrow$ se mantiene el vinculo de NO-deslizamiento

$\frac{L}{2} \leq x \leq \frac{3L}{2}$ reacción normal $\vec{N}$ dentro de la base de apoyo, No se verifica ($x < 0$).

La placa vuelca, rotando en sentido anti-horario.

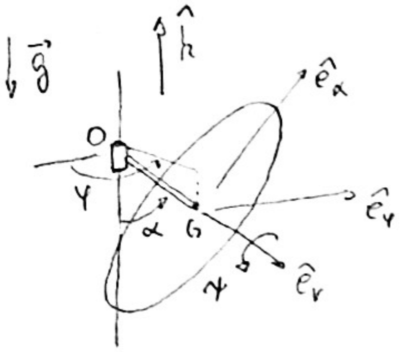
(2) (a)

$$\vec{M}^{r}_{o} = -k\varphi\hat{k}$$

$$P = \frac{dW}{dt} = -k\varphi\hat{k}\cdot\dot{\varphi}\hat{k} = -k\varphi\dot{\varphi} = -\frac{d}{dt}\left(\frac{k\varphi^2}{2}\right) \Rightarrow \Delta W = -\Delta\left(\frac{k\varphi^2}{2}\right)$$

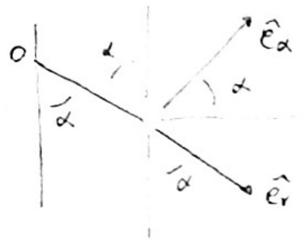
$$\Delta W = -\Delta U \Rightarrow \boxed{U = \frac{k\varphi^2}{2}}$$

(b)



$$\underset{(\hat{e}_r,\hat{e}_\varphi,\hat{e}_\alpha)}{\mathbb{I}_G} = \frac{mr^2}{2}\begin{bmatrix} 2 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

$$\vec{\omega}_{cro} = \dot{\psi}\,\hat{e}_r + \dot{\varphi}\,\hat{k} = \dot{\psi}\,\hat{e}_r + \dot{\varphi}(-\cos\alpha\,\hat{e}_r + \operatorname{sen}\alpha\,\hat{e}_\alpha)$$



$$\vec{L}_G = \mathbb{I}_G\,\vec{\omega} = \frac{mr^2}{2}\begin{bmatrix} 2 & & \\ & 1 & \\ & & 1 \end{bmatrix}\begin{bmatrix} \dot{\psi} - \dot{\varphi}\cos\alpha \\ 0 \\ \dot{\varphi}\operatorname{sen}\alpha \end{bmatrix}$$

$$\vec{L}_G = mr^2(\dot{\psi} - \dot{\varphi}\cos\alpha)\,\hat{e}_r + \frac{mr^2}{2}\dot{\varphi}\operatorname{sen}\alpha\,\hat{e}_\alpha$$

$$\vec{L}_o = \vec{L}_G + m\vec{v}_G \times \left(-\frac{r}{\sqrt{2}}\hat{e}_r\right) = \vec{L}_G + \frac{mr^2}{2}\operatorname{sen}\alpha\,\dot{\varphi}\,\hat{e}_\alpha$$

$$\left(\vec{v}_G = \frac{r}{\sqrt{2}}\operatorname{sen}\alpha\,\dot{\varphi}\,\hat{e}_\varphi\right)$$

$$\vec{L}_o = mr^2(\dot{\psi} - \dot{\varphi}\cos\alpha)\,\hat{e}_r + mr^2\operatorname{sen}\alpha\,\dot{\varphi}\,\hat{e}_\alpha$$

$$\dot{\vec{L}}_o\cdot\hat{k} = \vec{M}^{ext}_{O}\cdot\hat{k} = -k\varphi$$

$$\dot{\vec{L}}_o = mr^2(\ddot{\psi} - \ddot{\varphi}\cos\alpha)\,\hat{e}_r + mr^2\operatorname{sen}\alpha\,\ddot{\varphi}\,\hat{e}_\alpha + mr^2(\dot{\psi} - \dot{\varphi}\cos\alpha)\operatorname{sen}\alpha\,\dot{\varphi}\,\hat{e}_\varphi + mr^2\operatorname{sen}\alpha\,\dot{\varphi}\cos\alpha\,\dot{\varphi}\,\hat{e}_\varphi$$

$$-mr^2(\ddot{\psi} - \ddot{\varphi}\cos\alpha)\cos\alpha + mr^2\ddot{\varphi}\operatorname{sen}^2\alpha = -k\varphi$$

$$\boxed{-\ddot{\psi}\cos\alpha + \ddot{\varphi} + \frac{k}{mr^2}\varphi = 0} \quad (\mathrm{I})$$

$$E_{mec} = K + U_M = cte \quad ; \quad K = \frac{1}{2}\vec{\omega}\,\mathbb{I}_o\,\vec{\omega} = \frac{1}{2}\vec{\omega}\cdot\vec{L}_o =$$

$$= \frac{1}{2}\left(mr^2(\dot{\psi} - \dot{\varphi}\cos\alpha)^2 + mr^2\dot{\varphi}^2\operatorname{sen}^2\alpha\right) =$$

$$= \frac{mr^2}{2}\left(\dot{\psi}^2 + \dot{\varphi}^2 - 2\dot{\psi}\dot{\varphi}\cos\alpha\right)$$

Condiciones iniciales de movimiento $\dot{\psi}(0) = 0,\ \dot{\varphi}(0) = 0,\ \varphi = \varphi_0$

$$\boxed{\frac{mr^2}{2}\left(\dot{\psi}^2 + \dot{\varphi}^2 - 2\dot{\psi}\dot{\varphi}\cos\alpha\right) + \frac{k\varphi^2}{2} = \frac{k\varphi_0^2}{2}} \quad (\mathrm{II})$$

c) ¿ $\exists$ solución $\varphi_s(t)$ / $\dot{\psi}_s = \cos\alpha\, \dot{\varphi}_s$?

Si $\ddot{\psi}_s = \cos\alpha\, \ddot{\varphi}_s$

ecuación (I)

$$\longrightarrow - \cos^2\alpha\, \ddot{\varphi}_s + \ddot{\varphi}_s + \frac{k}{mr^2}\varphi = 0$$

$$\ddot{\varphi}_s + \left(\frac{k}{\operatorname{sen}^2\alpha\, mr^2}\right)\varphi_s = 0$$

condiciones iniciales
$\varphi(0) = \varphi_0$
$\dot{\varphi}(0) = 0$

$$\longrightarrow \boxed{\varphi_s = \varphi_0 \cos\Omega t \qquad \Omega = \sqrt{\frac{k}{mr^2 \operatorname{sen}^2\alpha}}}$$

ecuación (II)

$$\frac{mr^2}{2}\left(\cos^2\alpha\, \dot{\varphi}_s^2 + \dot{\varphi}_s^2 - 2\dot{\varphi}_s^2\cos^2\alpha\right) + \frac{k}{2}\varphi^2 = \frac{k}{2}\varphi_0^2$$

$$\frac{mr^2}{2}\dot{\varphi}_s^2 \operatorname{sen}^2\alpha + \frac{k}{2}\varphi^2 = \frac{k}{2}\varphi_0^2$$

$$\dot{\varphi}_s = -\varphi_0\Omega \operatorname{sen}\Omega t$$

$$\longrightarrow \frac{mr^2}{2}\left(\varphi_0^2\Omega^2\operatorname{sen}^2\Omega t\right)\operatorname{sen}^2\alpha + \frac{k}{2}\varphi^2 =$$

$$\left(\Omega^2 = \frac{k}{mr^2\operatorname{sen}^2\alpha}\right)$$

$$= \frac{k}{2}\left(\varphi_0^2\operatorname{sen}^2\Omega t + \varphi_0^2\cos^2\Omega t\right) = \frac{k}{2}\varphi_0^2$$

verifico $E_{mec} = c^{te}$