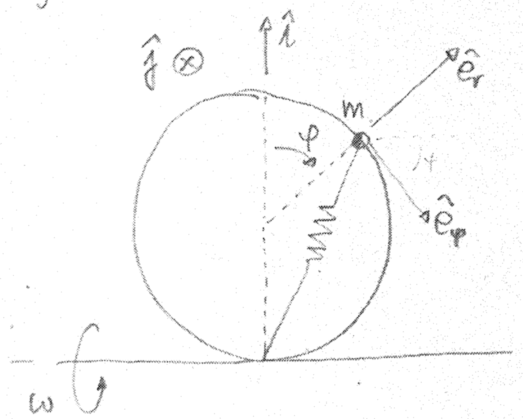
Ejercicio (1)



(a)

$$\vec{v} = r\dot{\varphi}\,\hat{e}_\varphi + (r + r\cos\varphi)\,\omega\,\hat{j}$$

$$\vec{a} = r\ddot{\varphi}\,\hat{e}_\varphi - r\,\text{sen}\varphi\,\omega\dot{\varphi}\,\hat{j} + r\dot{\varphi}\,\dot{\hat{e}}_\varphi + (r + r\cos\varphi)\,\omega\,\dot{\hat{j}}$$

$$\dot{\hat{e}}_\varphi = \vec{\Omega}\times\hat{e}_\varphi = (\dot{\varphi}\,\hat{j} + \omega\,\hat{k})\times\hat{e}_\varphi = -\dot{\varphi}\,\hat{e}_r - \omega\,\text{sen}\varphi\,\hat{j}$$

$$\dot{\hat{j}} = \omega\hat{k}\times\hat{j} = -\omega\,\hat{\imath}$$

$$\vec{a} = r\ddot{\varphi}\,\hat{e}_\varphi - r\,\text{sen}\varphi\,\omega\dot{\varphi}\,\hat{j} + r\dot{\varphi}(-\dot{\varphi}\,\hat{e}_r - \omega\,\text{sen}\varphi\,\hat{j}) - r(1+\cos\varphi)\,\omega^2\,\hat{\imath}$$

b)

$$m\vec{a} = N\hat{e}_r + N'\hat{j} - k(r\hat{\imath} + r\hat{e}_r) \rightarrow \underline{\text{Ec. mov}} \qquad m\vec{a}\cdot\hat{e}_\varphi = -k\,r\,\hat{\imath}\cdot\hat{e}_\varphi$$

$$m\vec{a}\cdot\hat{e}_\varphi = m\left(r\ddot{\varphi} + r(1+\cos\varphi)\,\omega^2\,\text{sen}\varphi\right) = +k\,r\,\text{sen}\varphi$$

$$\boxed{\ddot{\varphi} + (1+\cos\varphi)\,\text{sen}\varphi\,\omega^2 - \frac{k}{m}\,\text{sen}\varphi = 0}$$

Pos. equilibrio $\Rightarrow$ $\left[(1+\cos\varphi)\,\omega^2 - \frac{k}{m}\right]\text{sen}\varphi = 0$ $\rightarrow$ $\text{sen}\,\varphi_{eq} = 0, \pi$

$\hookrightarrow$ $\cos\varphi_{eq} = \left(\frac{k/m}{\omega^2} - 1\right)$

$$\cos\varphi_{eq} = \pm\pi/2 \Leftrightarrow \frac{k/m}{\omega^2} = 1 \Rightarrow \boxed{\omega_r = \sqrt{k/m}}$$

(c)

$$\ddot{\varphi} = \omega_r^2\,\text{sen}\varphi - (1+\cos\varphi)\,\text{sen}\varphi\,\omega^2$$

$$\dot{\varphi}\ddot{\varphi} = \left((\omega_r^2 - \omega^2)\,\text{sen}\varphi - \cos\varphi\,\text{sen}\varphi\,\omega^2\right)\dot{\varphi}$$

$$\frac{\dot{\varphi}^2}{2} - \frac{\dot{\varphi}_0^2}{2} = -(\omega_r^2 - \omega^2)(\cos\varphi - \cos\varphi_0) - \frac{\omega^2}{2}(\text{sen}^2\varphi - \text{sen}^2\varphi_0)$$

$$\dot{\varphi}_0 = 0 \;;\; \varphi_0 = 0 \Rightarrow \quad \dot{\varphi}^2 = 2(\omega_r^2 - \omega^2)(1 - \cos\varphi) - \omega^2\,\text{sen}^2\varphi$$

$$\dot{\varphi}^2 = 0 \Rightarrow 2(\omega_r^2 - \omega^2)(1 - \cos\varphi) - \omega^2\,\text{sen}^2\varphi = 0$$

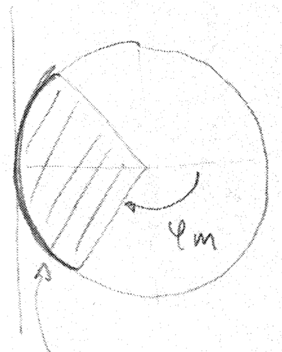
$$2\left(\omega r^2 - \frac{3}{4}\omega r^2\right)(1-\cos\varphi) - \frac{3}{4}\omega r^2 \operatorname{sen}^2\varphi = 0$$

$$\frac{1}{2}(1-\cos\varphi) - \frac{3}{4}(1-\cos^2\varphi) = 0$$

$$2(1-\cos\varphi) - 3(1-\cos^2\varphi) = 0$$

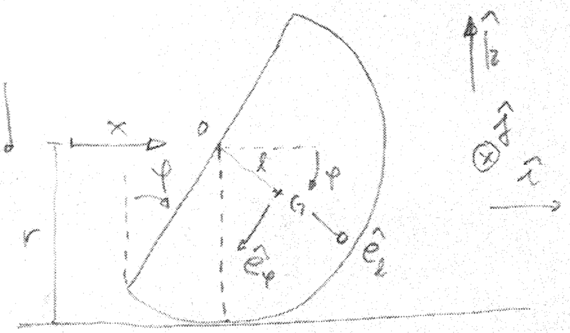
$$3\cos^2\varphi - 2\cos\varphi - 1 = 0$$

- $\cos\varphi_0 = 1 \Rightarrow \varphi_0 = 0$ ✓
- $\cos\varphi_m = -1/3 \Rightarrow \varphi_m \cong 70{,}5°$



$$\dot\varphi^2 \geq 0 \Leftrightarrow \varphi_0 \leq \varphi \leq \varphi_m$$

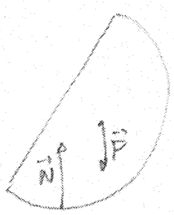
2)



$$\vec r_G = x\hat\imath + l\hat e_l$$

$$\vec v_G = \dot x\hat\imath + l\dot\varphi\hat e_\varphi$$

Diagrama de cuerpo libre ⇒



$$m\vec a_G = N\hat k - P\hat k \Rightarrow m\vec a_G\cdot\hat\imath = 0 \Rightarrow \vec v_G\cdot\hat\imath = c^{te}$$

$$\vec v_G\cdot\hat\imath = \dot x - l\dot\varphi\operatorname{sen}\varphi = c^{te} = 0$$

← $\dot\varphi(0)=0,\ \dot x(0)=0$.

Sistema conservativo ⇒ $K + U = E_{mec} = c^{te}$ $\qquad U = -mgl\operatorname{sen}\varphi$

$$K = \frac{1}{2}m v_G^2 + \frac{1}{2}\dot\varphi^2 I_{G,\hat\jmath}$$

$$\vec v_G\cdot\vec v_G = \dot x^2 + l^2\dot\varphi^2 - 2l\dot x\dot\varphi\operatorname{sen}\varphi \quad ; \quad \frac{mr^2}{2} = I_{G,\hat\jmath} + ml^2 \Rightarrow I_{G,\hat\jmath} = \frac{mr^2}{2} - ml^2$$

$$E_{mec} = \frac{m}{2}\left(\dot x^2 + l^2\dot\varphi^2 - 2l\dot x\dot\varphi\operatorname{sen}\varphi\right) + \frac{1}{2}\left(\frac{mr^2}{2} - ml^2\right)\dot\varphi^2 - mgl\operatorname{sen}\varphi = 0$$

← $\dot\varphi(0)=0$, $\varphi(0)=0$

$\dot{x} = l\dot{\varphi}\,\text{sen}\,\varphi$

$$E_{mec} = \frac{m}{2}\left(l^2\dot{\varphi}^2\text{sen}^2\varphi + l^2\dot{\varphi}^2 - 2l^2\dot{\varphi}^2\text{sen}^2\varphi\right) + \frac{m}{2}\left(\frac{r^2}{2} - l^2\right)\dot{\varphi}^2 - mgl\,\text{sen}\,\varphi = \emptyset$$

$$\dot{\varphi}^2\left(\frac{r^2}{2l^2} - \text{sen}^2\varphi\right) - \frac{2g}{l}\,\text{sen}\,\varphi = \emptyset$$

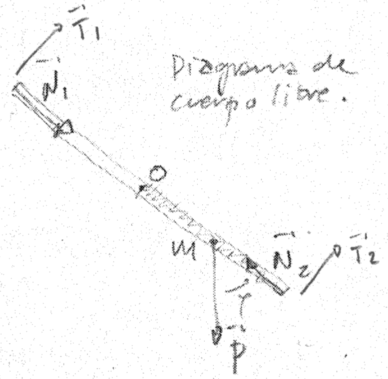
$$\dot{\varphi}^2 = \frac{2g/l\,\text{sen}\,\varphi}{\frac{r^2}{2l^2} - \text{sen}^2\varphi}$$

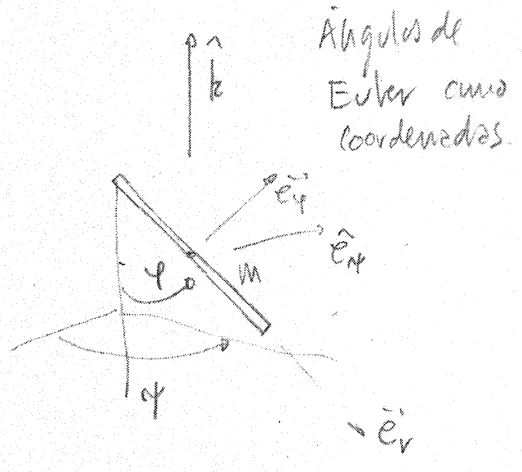
$$\frac{\dot{\varphi}}{\sqrt{\dfrac{2g/l\,\text{sen}\,\varphi}{\frac{r^2}{2l^2} - \text{sen}^2\varphi}}} = 1$$

$$\boxed{\int_0^{\pi} \frac{d\varphi}{\sqrt{\dfrac{2g/l\,\text{sen}\,\varphi}{\frac{r^2}{2l^2} - \text{sen}^2\varphi}}} = T}$$

③ ⓐ

El p^{to} O es fijo





$\vec{M}_O^{ext} = mg\frac{r}{2}\,\text{sen}\,\varphi\,\hat{e}_\psi$

$\vec{M}_O^{ext}\cdot\hat{k} = 0 \rightarrow \boxed{\vec{L}_O\cdot\hat{k} = c^{te}}$

$\vec{L}_O = \mathbb{I}_O\vec{\omega}$

$$\underset{(\hat{e}_r,\hat{e}_\psi,\hat{e}_\varphi)}{\mathbb{I}_O} = \frac{mr^2}{12}\begin{bmatrix}0 & & \\ & 1 & \\ & & 1\end{bmatrix} + \frac{mr^2}{4}\begin{bmatrix}0 & & \\ & 1 & \\ & & 1\end{bmatrix} = \frac{mr^2}{3}\begin{bmatrix}0 & & \\ & 1 & \\ & & 1\end{bmatrix}$$

$$\vec{\omega} = -\dot{\varphi}\,\hat{e}_\psi + \dot{\psi}\,\hat{k} = -\dot{\varphi}\,\hat{e}_\psi + \dot{\psi}\left(-\cos\varphi\,\hat{e}_r + \text{sen}\,\varphi\,\hat{e}_\varphi\right)$$

③ (cont.)

$$\vec{L}_0 = \frac{mr^2}{3}\left(-\dot\varphi\,\hat e_\psi + \dot\psi \,\text{sen}\varphi\,\hat e_\varphi\right) \Rightarrow \vec{L}_0\cdot\hat k = \frac{mr^2}{3}\dot\psi\,\text{sen}^2\varphi = c^{te}$$

$$\boxed{\dot\psi\,\text{sen}^2\varphi = c^{te}}$$

$$E_{mec} = c^{te} \Rightarrow \left.\begin{array}{l}\frac{1}{2}\vec\omega\,\mathbb{I}_0\,\vec\omega = K\\ -mg\frac{r}{2}\cos\varphi = U\end{array}\right\} \Rightarrow E_m = -mg\frac{r}{2}\cos\varphi + \frac{mr^2}{6}\left(\dot\varphi^2 + \dot\psi^2\text{sen}^2\varphi\right)$$

$$\boxed{\dot\varphi^2 + \dot\psi^2\text{sen}^2\varphi - 3g/r\cos\varphi = c^{te}}$$

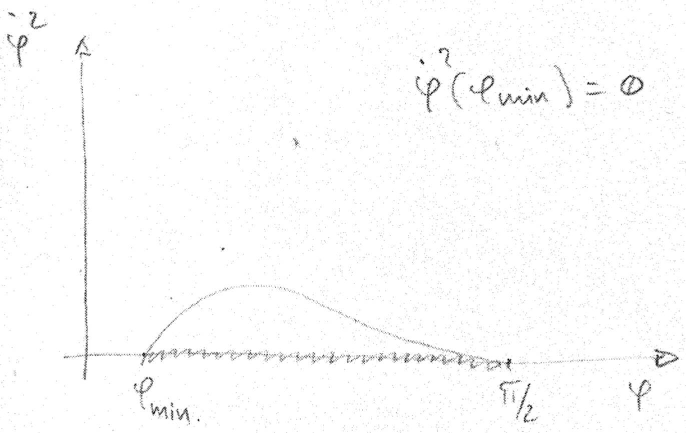
ⓑ

$$\left.\begin{array}{l}\varphi(0) = \pi/2\\ \dot\psi(0) = \dot\psi_0\end{array}\right\} \Rightarrow \left|\begin{array}{l}\dot\psi\,\text{sen}^2\varphi = \dot\psi_0\\ \dot\varphi^2 + \dot\psi^2\text{sen}^2\varphi - 3g/r\cos\varphi = \dot\psi_0^2\end{array}\right.$$

$$\dot\varphi^2 + \frac{\dot\psi_0^2}{\text{sen}^2\varphi} - \frac{3g}{r}\cos\varphi = \dot\psi_0^2$$

$$\dot\varphi^2 = \dot\psi_0^2 - \frac{\dot\psi_0^2}{\text{sen}^2\varphi} + \frac{3g}{r}\cos\varphi$$

Altura mínima del baricentro implica calcular φ_{min} $\left(h_m = -\frac{r}{2}\cos\varphi_m\right)$



$$\dot\varphi^2(\varphi_{min}) = 0 \Rightarrow \dot\psi_0^2\left(1 - \frac{1}{\text{sen}^2\varphi_m}\right) = -\frac{3g}{r}\cos\varphi_m$$

$$\dot\psi_0^2\,\frac{(\text{sen}^2\varphi_m - 1)}{\text{sen}^2\varphi_m} = -\frac{3g}{r}\cos\varphi_m$$

$$\dot\psi_0^2\cos\varphi_m = \frac{3g}{r}\text{sen}^2\varphi_m = \frac{3g}{r}(1-\cos^2\varphi_m)$$

$$\frac{3g}{r} - \frac{3g}{r}\cos^2\varphi_m - \dot\psi_0^2\cos\varphi_m = 0.$$

$$\cos^2\varphi_m + \frac{r\dot\psi_0^2}{3g}\cos\varphi_m - 1 = 0.$$

$$x^2 + \frac{r\dot\psi_0^2}{3g}x - 1 = 0$$ se podría continuar y obtener una expresión de $\cos\varphi_m$ ....