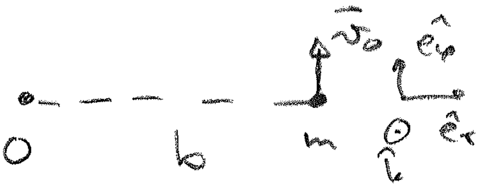
$\boxed{1}$



$$\bar{F} = -k\,\hat{e}_r$$
$$\bar{v}_0 = v_0\,\hat{e}_\varphi$$

$$U = kr$$

$$E = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\varphi}^2 + kr$$

$\bar{L}_0 = cte$ $\qquad \bar{L}_0 = \bar{r}\wedge m\bar{v} = r\hat{e}_r \wedge m(\dot{r}\hat{e}_r + r\dot{\varphi}\hat{e}_\varphi)$

$\bar{L}_0 = mr^2\dot{\varphi}\,\hat{k} \rightarrow l = mr^2\dot{\varphi} = cte$

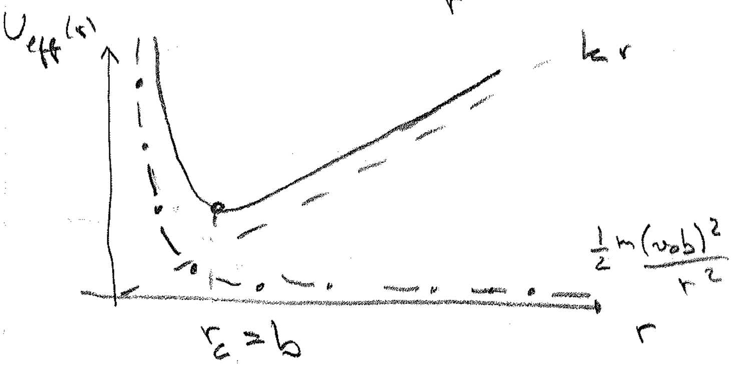
$l = mb^2\dot{\varphi}_0$

$$\dot{\varphi}_0 = \frac{v_0}{b} \rightarrow l = mbv_0$$

$$\dot{\varphi} = \frac{mbv_0}{mr^2} \quad\downarrow\quad \dot{\varphi} = \frac{bv_0}{r^2}$$

$$\Rightarrow U_{eff}(r) = \frac{1}{2}mr^2\left(\frac{v_0 b}{r^2}\right)^2 + kr$$

$$U_{eff}(r) = \frac{1}{2}m\frac{(v_0 b)^2}{r^2} + kr$$



radio de la orbita circular:

$$\left.\frac{dU_{eff}}{dr}\right|_{r_c} = 0 \rightarrow \frac{m(v_0 b)^2}{r^3} = k$$

$$\frac{mv_0^2}{kb} = 1$$

b) ptos de mínima distancia y máxima distancia.

r. t.q $E_0 - U_{eff} = 0$

$$E_0 = \frac{1}{2}mv_0^2 + kb \rightarrow \frac{1}{2}mv_0^2 + kb = \frac{1}{2}m\frac{(v_0b)^2}{r^2} + kr$$

$$\frac{1}{2}mv_0^2\left(1 - \frac{b^2}{r^2}\right) + k(b-r) = 0 \rightarrow \frac{1}{2}mv_0^2\left(\frac{r^2-b^2}{r^2}\right) + k(b-r) = 0$$

c) $\frac{1}{2} m v_0^2 (-b+r) \frac{(r+b)}{r^2} + k(b-r) = 0 \rightarrow r = b$

$-\frac{1}{2} m v_0^2 \frac{(r+b)}{r^2} + k = 0$

$\rightarrow k r^2 - \frac{1}{2} m v_0^2 (r+b) = 0$

$kb = \frac{3}{8} m v_0^2 \rightarrow \frac{1}{2} m v_0^2 = \frac{4}{3} kb$

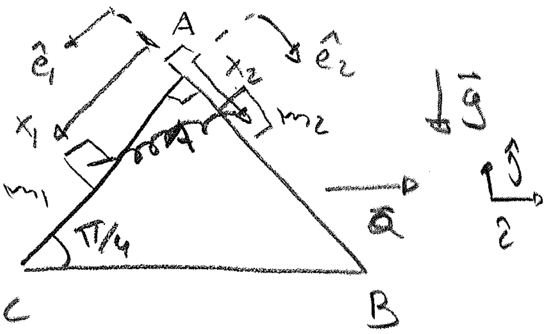
$k r^2 - \frac{4}{3} kb (r+b) = 0 \rightarrow 3r^2 - 4br - 4b^2 = 0$

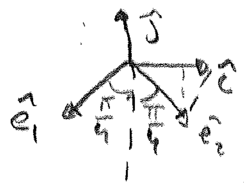
$\rightarrow r = 2b$

Distancia máxima $2b$

mínima b

$\boxed{2}$





$$\overline{AC} = \overline{AB} = L$$

$$\hat{\imath} = \cos\frac{\pi}{4}\,\hat{e}_2 - \sin\frac{\pi}{4}\,\hat{e}_1$$

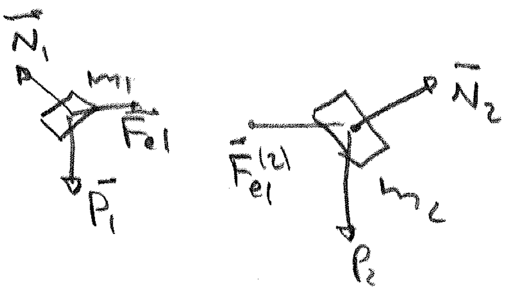
$$\hat{\imath} = \frac{1}{\sqrt{2}}\hat{e}_2 - \frac{1}{\sqrt{2}}\hat{e}_1$$

$$\hat{j} = -\frac{1}{\sqrt{2}}\hat{e}_1 - \frac{1}{\sqrt{2}}\hat{e}_2$$

a) $\vec{a}_1 = a\hat{\imath} + \ddot{x}_1\hat{e}_1$

$$\vec{a}_1 = \left(\ddot{x}_1 - \frac{a}{\sqrt{2}}\right)\hat{e}_1 + \frac{a}{\sqrt{2}}\hat{e}_2$$

$$\vec{a}_2 = \left(\ddot{x}_2 + \frac{a}{\sqrt{2}}\right)\hat{e}_2 - \frac{a}{\sqrt{2}}\hat{e}_1$$



$$\vec{F}_{el}^{(1)} = -k\left(x_1\hat{e}_1 - x_2\hat{e}_2\right)$$

$$\vec{F}_{el}^{(2)} = k\left(x_1\hat{e}_1 - x_2\hat{e}_2\right)$$

$$\vec{N}_1 = -N_1\hat{e}_2$$

$$\vec{N}_2 = -N_2\hat{e}_1$$

$$\vec{P}_1 = -m_1 g\hat{j} = m_1 g\left(\frac{1}{\sqrt{2}}\hat{e}_1 + \frac{1}{\sqrt{2}}\hat{e}_2\right)$$

$$\vec{P}_2 = -m_2 g\hat{j} = m_2 g\left(\frac{1}{\sqrt{2}}\hat{e}_1 + \frac{1}{\sqrt{2}}\hat{e}_2\right)$$

$$m_1\left(\ddot{x}_1 - \frac{a}{\sqrt{2}}\right)\hat{e}_1 + \frac{a}{\sqrt{2}}\hat{e}_2 = -kx_1\hat{e}_1 + kx_2\hat{e}_2 - N_1\hat{e}_2 + m_1 g\frac{1}{\sqrt{2}}\hat{e}_1 + \frac{m_1 g}{\sqrt{2}}\hat{e}_2$$

$$m_2\left(\ddot{x}_2 + \frac{a}{\sqrt{2}}\right)\hat{e}_2 - m_2\frac{a}{\sqrt{2}}\hat{e}_1 = kx_1\hat{e}_1 - kx_2\hat{e}_2 - N_2\hat{e}_1 + m_2 g\frac{1}{\sqrt{2}}\hat{e}_1 + \frac{m_2 g}{\sqrt{2}}\hat{e}_2$$

$$\hookrightarrow\ m_1\left(\ddot{x}_1 - \frac{a}{\sqrt{2}}\right) = -kx_1 + m_1 g\frac{1}{\sqrt{2}}$$

$$m_2\left(\ddot{x}_2 + \frac{a}{\sqrt{2}}\right) = -kx_2 + m_2 g\frac{1}{\sqrt{2}}$$

ii_ pos eq relativas $\ddot{x}_1 = 0$ y $\ddot{x}_2 = 0$

$$x_1^e = \frac{m_1}{\sqrt{2}\,k}(a+g)$$

$$x_2^e = \frac{m_2}{\sqrt{2}\,k}(g-a)$$

ambos existen si se cumple: $\frac{m_1}{\sqrt{2}\,k}(a+g) < L$

$\frac{m_2}{\sqrt{2}\,k}(g-a) < L$

b) Si $|\vec{a}| < |\vec{g}|$

i) a partir de las ec. de mov. $x_1(0) = \frac{L}{2}$ $\dot{x}_1(0) = 0$

$x_2(0) = \frac{L}{2}$ $\dot{x}_2(0) = 0$

$$\ddot{x}_1 = \frac{1}{\sqrt{2}}(a+g) - \frac{k}{m_1}x_1 \Rightarrow x_1(t) = A\cos(\omega_1 t) + x_1^e \,;\ \omega_1 = \sqrt{\frac{k}{m_1}}$$

$$\ddot{x}_2 = \frac{1}{\sqrt{2}}(g-a) - \frac{k}{m_2}x_2 \Rightarrow x_2(t) = B\cos(\omega_1 t) + x_2^e \,;\ \omega_2 = \sqrt{\frac{k}{m_2}}$$

$$A + x_1^e = \frac{L}{2} \Rightarrow A = \frac{L}{2} - x_1^e$$

$$B = \frac{L}{2} - x_2^e$$

ii) Para que el movimiento sea posible

$0 \leq x_1(t) \leq L$ $\rightarrow$ $0 \leq A\cos(\omega_1 t) + x_1^e \leq L$

$0 \leq x_2(t) \leq L$ $\Downarrow$

$0 \leq 2x_1^e - \frac{L}{2} \leq L$

$N_1(t) > 0$ $\downarrow$

$N_2(t) > 0$

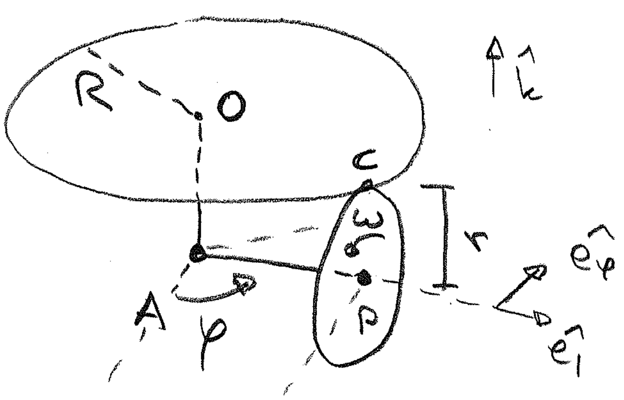
$$\frac{L}{4} \leq \frac{m_1}{\sqrt{2}\,k}(a+g) \leq \frac{3}{4}L$$

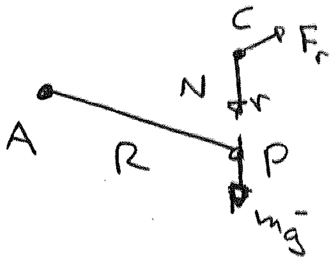
· de igual forma $\frac{L}{4} \leq \frac{m_2}{\sqrt{2}\,k}(g-a) \leq \frac{3}{4}L$

$N_1 > 0 \to$ $\frac{m_1}{\sqrt{2}}(g-a) + k x_2 > 0$ siempre se cumple

$N_2 > 0$ $\frac{m_1}{\sqrt{2}}(a+g) + k x_1 > 0$ siempre se cumple

3





a) $$\mathbb{I}_P^{\{\hat{e}_1,\hat{e}_\varphi,\hat{k}\}} = \frac{m r^2}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$$

$$\mathbb{I}_A^{\{\hat{e}_1,\hat{e}_\varphi,\hat{k}\}} = m\begin{pmatrix} \frac{r^2}{2} & 0 & 0 \\ 0 & \frac{r^2}{4}+R^2 & 0 \\ 0 & 0 & \frac{r^2}{4}+R^2 \end{pmatrix}$$

$$\vec{\omega} = \dot{\varphi}\hat{k} + \omega\hat{e}_1$$

$$\vec{L}_A = m\begin{pmatrix} \frac{r^2}{2} & 0 & 0 \\ 0 & \frac{r^2}{4}+R^2 & 0 \\ 0 & 0 & \frac{r^2}{4}+R^2 \end{pmatrix}\begin{pmatrix} \omega \\ 0 \\ \dot{\varphi} \end{pmatrix}$$

$$\boxed{\vec{L}_A = \frac{m r^2}{2}\omega\,\hat{e}_1 + m\left(\frac{r^2}{4}+R^2\right)\dot{\varphi}\,\hat{k}}$$

$$\dot{\vec{L}}_A = \frac{m r^2}{2}\omega\dot{\varphi}\,\hat{e}_\varphi + m\left(\frac{r^2}{4}+R^2\right)\ddot{\varphi}\,\hat{k}$$

$$\vec{M}_A = R\hat{e}_1 \wedge mg(-\hat{k}) + (R\hat{e}_1 + r\hat{k}) \wedge N(-\hat{k}) + (R\hat{e}_1 + r\hat{k}) \wedge F_r\hat{e}_\varphi$$

$$= mgR\,\hat{e}_\varphi + NR\,\hat{e}_\varphi + F_r R\,\hat{k} - rF_r\,\hat{e}_1$$

- RSD en C $\Rightarrow \vec{v}_C = 0$

$$\vec{v}_C = \vec{v}_A + \vec{\omega} \wedge (\vec{r}_C - \vec{r}_A)$$

$$\vec{v}_C = (\omega \hat{e}_1 + \dot{\varphi} \hat{k}) \wedge (R \hat{e}_1 + r \hat{k}) = -r\omega \hat{e}_\varphi + R\dot{\varphi} \hat{e}_\varphi = 0$$

$$\dot{\varphi} = \frac{r\omega}{R} \Rightarrow \ddot{\varphi} = 0 \Rightarrow F_r = 0$$

$$\Rightarrow \frac{mr^2\omega\dot{\varphi}}{2} = mgR + NR \Rightarrow N = \frac{mr^2\omega\dot{\varphi}}{2R} - mg$$

b) $N > 0 \Rightarrow \frac{mr^2\omega\dot{\varphi}}{2R} - mg > 0$

$$\frac{r^3\omega^2}{2R^2} > g \Rightarrow \omega^2 > \frac{2gR^2}{r^3}$$