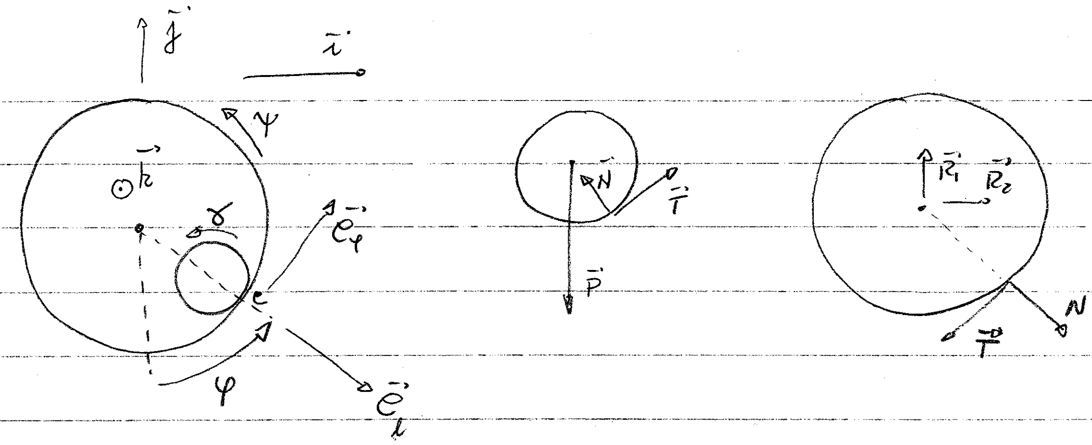
2^{do} parcial 2011

9-VII-2011

Problema ①



ⓐ

2da card. cilindro = $\frac{mr^2}{2}\ddot{\gamma} = Tr$ (I)

1era card. cilindro : $m\left(l\ddot{\varphi}\vec{e}_\varphi - l\dot{\varphi}^2\vec{e}_l\right) = -mg\vec{j} + T\vec{e}_\varphi - N\vec{e}_l$

$l = R - r$

$$\begin{cases} ml\ddot{\varphi} = T - mg\,sen\varphi & (II) \\ -ml\dot{\varphi}^2 = -N + mg\cos\varphi & (III) \end{cases}$$

2da card. tubo = $mR^2\ddot{\psi} = -TR$ (IV)

(I) + (IV) $\Rightarrow$ $r\ddot{\gamma} = -2R\ddot{\psi}$ (V)

Rodadura sin deslizamiento = $\vec{v}_c = R\dot{\psi}\vec{e}_\varphi$
$\vec{v}_c\,' = (l\dot{\varphi} + r\dot{\gamma})\vec{e}_\varphi$ $\Big\}$ $\vec{v}_c = \vec{v}_c\,' \Rightarrow$

$\Rightarrow R\dot{\psi} = l\dot{\varphi} + r\dot{\gamma}$

$\downarrow$

$R\ddot{\psi} = l\ddot{\varphi} + r\ddot{\gamma}$ (VI)

(V+VI) $R\ddot{\psi} = l\ddot{\varphi} - 2R\ddot{\psi} \Rightarrow 3R\ddot{\psi} = l\ddot{\varphi}$

(II+IV) $ml\ddot{\varphi} + mg\,sen\varphi = -mR\ddot{\psi} = -\frac{ml\ddot{\varphi}}{3}$

$3g\,sen\varphi = -4l\ddot{\varphi} \longrightarrow \boxed{\ddot{\varphi} = -\frac{3g}{4l}sen\varphi}$ (VII)

(b) $\int \dot{\varphi}\ddot{\varphi}\,dt = -\frac{3g}{4l}\int sen\varphi\,\dot{\varphi}\,dt \rightarrow \int_{\dot{\varphi}_0}^{\dot{\varphi}} \dot{\varphi}\,d\dot{\varphi} = -\frac{3g}{4l}\int_{\varphi_0}^{\varphi} sen\varphi\,d\varphi$

$$\frac{\dot{\varphi}^2}{2} - \frac{\dot{\varphi}_0^2}{2} = +\frac{3g}{4l}(\cos\varphi - \cos\varphi_0)$$

$$\dot{\varphi} = \sqrt{\frac{3g}{2l}(\cos\varphi - \cos\varphi_0)} \Rightarrow \boxed{2\int_{\varphi_0}^{-\varphi_0} \frac{d\varphi}{\sqrt{\frac{3g}{2l}(\cos\varphi - \cos\varphi_0)}} = T}$$

(c) (II) $T = mg\,sen\varphi + ml\ddot{\varphi} \underset{(VII)}{=} mg\,sen\varphi - \frac{3gm\,sen\varphi}{4} = \frac{mg\,sen\varphi}{4}$.

(III) $N = mg\cos\varphi + ml\left(\frac{3g}{2l}\left(\cos\varphi - \frac{\sqrt{2}}{2}\right)\right) = \frac{5mg}{2}\cos\varphi - \frac{3mg\sqrt{2}}{4}$

R.S.D. $\Rightarrow |T| \leq \mu |N|$

$$\frac{mg|sen\varphi|}{4} \leq \mu \frac{mg}{4} |10\cos\varphi - 3\sqrt{2}|$$

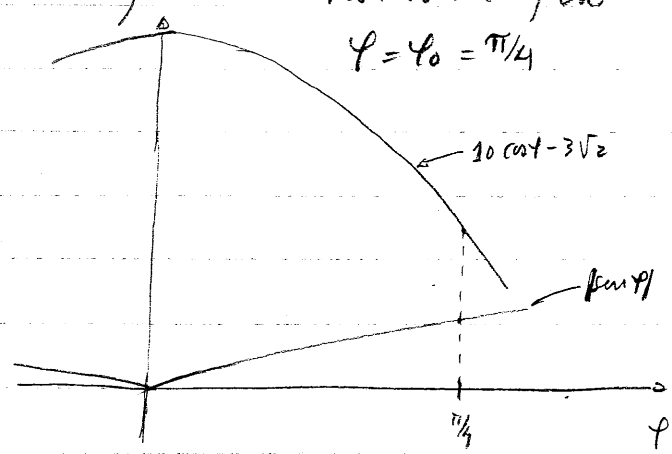
$0 \leq \varphi \leq \pi/4$ $\quad sen\varphi \leq \mu(10\cos\varphi - 3\sqrt{2})$

Condición más restrictiva para $\varphi = \varphi_0 = \pi/4$

$$\frac{\sqrt{2}}{2} \leq \mu(5\sqrt{2} - 3\sqrt{2})$$

$$\boxed{\frac{1}{4} \leq \mu}$$

(Movimiento simétrico para $\varphi \leq 0$)



(d) $(II) \to \quad T = mg\,\text{sen}\,\varphi_0$

$(III) \to \quad N = mg\cos\varphi_0$

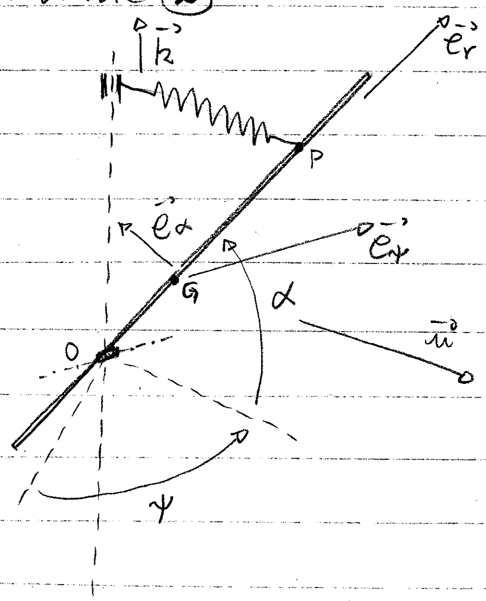
$$\left.\begin{array}{ll}(I+II) \to & \frac{mr}{2}\ddot{\gamma} = mg\,\text{sen}\,\varphi_0 \\ (VI) \to & R\ddot{\psi} = r\ddot{\gamma}\end{array}\right\} \to \frac{mR}{2}\ddot{\psi} = mg\,\text{sen}\,\varphi_0$$

$$\boxed{\ddot{\psi} = \frac{2g}{R}\,\text{sen}\,\varphi_0}$$

$|T| \leq \mu|N| \Rightarrow \text{sen}\,\varphi_0 \leq \mu\cos\varphi_0$

$\text{tg}\,\varphi_0 \leq \mu \qquad \boxed{\varphi_{0max} = A\text{tg}(\mu)}$

Probleme (2)



(a) $\vec{\Omega} = \omega\vec{k} - \dot{\alpha}\,\vec{e_\psi}$

$\vec{r_G} = r\,\vec{e_r} \Rightarrow \vec{v_G} = r\,\dot{\vec{e_r}} = r\omega\cos\alpha\,\vec{e_\psi} + r\dot{\alpha}\,\vec{e_\alpha}$

$\left(\dot{\vec{e_r}} = \vec{\Omega}\times\vec{e_r} = \omega\cos\alpha\,\vec{e_\psi} + \dot{\alpha}\,\vec{e_\alpha}\right)$

$\vec{a_G} = -r\omega\,\text{sen}\,\alpha\,\dot{\alpha}\,\vec{e_\psi} + r\omega\cos\alpha\,\dot{\vec{e_\psi}} + r\ddot{\alpha}\,\vec{e_\alpha}$
$\quad + r\dot{\alpha}\,\dot{\vec{e_\alpha}}$

$$\left(\begin{array}{l}\dot{\vec{e_\psi}} = \vec{\Omega}\times\vec{e_\psi} = -\omega\vec{u} = -\omega(\cos\alpha\,\vec{e_r} - \text{sen}\,\alpha\,\vec{e_\alpha}) \\ \dot{\vec{e_\alpha}} = \vec{\Omega}\times\vec{e_\alpha} = -\omega\,\text{sen}\,\alpha\,\vec{e_\psi} - \dot{\alpha}\,\vec{e_r}\end{array}\right)$$

$\vec{a_G} = -(r\dot{\alpha}^2 + r\omega^2\cos^2\alpha)\,\vec{e_r} - 2r\omega\dot{\alpha}\,\text{sen}\,\alpha\,\vec{e_\psi} + (r\ddot{\alpha} + r\omega^2\,\text{sen}\,\alpha\cos\alpha)\,\vec{e_\alpha}$

(b) 2^{da} cardinal barra en O $\quad \dot{\vec{J}}_0 = \vec{M}_0^{ext}$

$$\mathbb{I}_G \underset{(\vec{e}_r, \vec{e}_\varphi, \vec{e}_\alpha)}{=} \begin{bmatrix} 0 & & \\ & I_G & \\ & & I_G \end{bmatrix} \underset{(\text{steiner})}{\Rightarrow} \quad \mathbb{I}_0 = \mathbb{I}_G + \begin{bmatrix} 0 & & \\ & mr^2 & \\ & & mr^2 \end{bmatrix} \Rightarrow \mathbb{I}_0 = \begin{bmatrix} 0 & & \\ & m(L^2/3 + r^2) & \\ & & m(L^2/3+r^2) \end{bmatrix}$$

$$I_G = \frac{mL^2}{3} \qquad\qquad = \begin{bmatrix} 0 & & \\ & I_0 & \\ & & I_0 \end{bmatrix}$$

$$\vec{J}_0 = \mathbb{I}_0 \vec{\Omega} = -I_0 \dot{\alpha}\, \vec{e}_\varphi + I_0 \omega \cos\alpha\, \vec{e}_\alpha$$

$$\vec{\Omega} = \omega \operatorname{sen}\alpha\, \vec{e}_r - \dot{\alpha}\, \vec{e}_\varphi + \omega \cos\alpha\, \vec{e}_\alpha$$

$$\dot{\vec{J}}_0 = I_0 \left(-\ddot{\alpha}\, \vec{e}_\varphi - \dot{\alpha}\, \dot{\vec{e}}_\varphi - \omega \operatorname{sen}\alpha \dot{\alpha}\, \vec{e}_\alpha + \omega \cos\alpha\, \dot{\vec{e}}_\alpha \right) =$$
$$= I_0 \left(-\ddot{\alpha}\, \vec{e}_\varphi - \dot{\alpha}\left(-\omega \cancel{\cos}\alpha\, \vec{e}_r + \omega \operatorname{sen}\alpha\, \vec{e}_\alpha\right) - \omega \operatorname{sen}\alpha\, \dot{\alpha}\, \vec{e}_\alpha \right.$$
$$\left. + \omega \cos\alpha \left(-\omega \operatorname{sen}\alpha\, \vec{e}_\varphi - \cancel{\dot{\alpha}\, \vec{e}_r}\right)\right) =$$
$$= I_0 \left(-\ddot{\alpha} - \omega^2 \cos\alpha \operatorname{sen}\alpha \right) \vec{e}_\varphi + I_0 \left(-2\omega\dot{\alpha} \operatorname{sen}\alpha \right) \vec{e}_\alpha$$

$$\vec{M}_0^{ext} = -k(L\cos\alpha) L \operatorname{sen}\alpha\, \vec{e}_\varphi + mgr\cos\alpha\, \vec{e}_\varphi + M_1^{react}\, \vec{e}_\alpha + M_2^{react}\, \vec{e}_r$$

$$-I_0 \left(\ddot{\alpha} + \omega^2 \cos\alpha \operatorname{sen}\alpha \right) = -mgr\cos\alpha - kL^2 \operatorname{sen}\alpha \cos\alpha$$

$$\boxed{\ddot{\alpha} + \left(\omega^2 - \frac{kL^2}{I_0} \right) \operatorname{sen}\alpha \cos\alpha + \frac{mgr}{I_0} \cos\alpha = 0}$$

(c) $\vec{M}_0^{ext} \cdot \vec{e}_\alpha = \dot{\vec{J}}_0 \cdot \vec{e}_\alpha \Rightarrow \quad M_1^{react} = -2\omega\dot{\alpha} I_0 \operatorname{sen}\alpha$

$\vec{M}_0^{ext} \cdot \vec{e}_r = \dot{\vec{J}}_0 \cdot \vec{e}_r \Rightarrow \quad M_2^{react} = 0$

$$\boxed{\vec{M}_0^{react} = -2\omega\dot{\alpha}\, m\left(\frac{L^2}{3} + r^2 \right) \operatorname{sen}\alpha\, \vec{e}_\alpha}$$

$$m\vec{a}_G = -mg\vec{k} - kL\cos\alpha\,\vec{u} + \vec{R}$$

$$\vec{R} = \left(-mr\dot{\alpha}^2 - mr\omega^2\cos^2\alpha + mg\,\text{sen}\,\alpha + kL\cos^2\alpha\right)\vec{e}_r$$
$$+ \left(-2mr\omega\dot{\alpha}\,\text{sen}\,\alpha\right)\vec{e}_\varphi +$$
$$+ \left(mr\ddot{\alpha} + mr\omega^2\,\text{sen}\,\alpha\cos\alpha + mg\cos\alpha - kL\cos\alpha\,\text{sen}\,\alpha\right)\vec{e}_\alpha$$

d) $P = \vec{\mu}_O^{\,react.}\cdot\vec{\Omega} = -2\omega^2\dot{\alpha}\,m\left(\frac{L^2}{3}+r^2\right)\text{sen}\,\alpha\cos\alpha$

e) Pos. eq. relat. $\Rightarrow$ $\left.\begin{array}{l}\alpha(t_0)=\alpha_{eq}\\ \dot{\alpha}(t_0)=0\end{array}\right\} \Rightarrow \dot{\alpha}(t)=0\ \forall t \Rightarrow \ddot{\alpha}(t)=0\ \forall t$

$$\left(\omega^2-\frac{kL^2}{I_0}\right)\text{sen}\,\alpha_{eq}\cos\alpha_{eq} + \frac{mgr}{I_0}\cos\alpha_{eq} = 0. \qquad \text{(de (b))}$$

$$\left[\left(\omega^2-\frac{kL^2}{I_0}\right)\text{sen}\,\alpha_{eq} + \frac{mgr}{I_0}\right]\cos\alpha_{eq} = 0$$

$\cos\alpha_{eq} = 0 \Rightarrow$ $\boxed{\alpha_{eq} = \pi/2,\ -\pi/2}$

$\text{sen}\,\alpha_{eq} = \dfrac{mgr}{(kL^2 - I_0\omega^2)}$ $\qquad \boxed{\alpha_{eq} = \text{Asen}\left(\dfrac{mgr}{kL^2-I_0\omega^2}\right)}$

$\exists \Leftrightarrow mgr < |kL^2 - I_0\omega^2|$