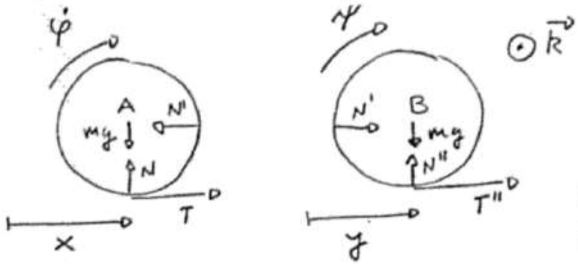
Julio 04

Ejercicio 1

a)



$$\begin{cases} m\ddot{x} = T - N' \\ 0 = N - mg \end{cases} \qquad \begin{cases} m\ddot{y} = T'' + N' \\ 0 = N'' - mg \end{cases}$$

Deslizamiento $\Rightarrow$ $T = \mu N$

$$m\ddot{x} = \mu mg - N'$$

$$m\ddot{x} + \frac{3}{2}m\ddot{y} = \mu mg$$

Hipótesis: $\Delta x = \Delta y$ (discos en contacto)

$$\frac{5}{2}\ddot{y} = \mu g$$

$$\boxed{\ddot{y} = \frac{2\mu g}{5}}$$

$$-I_{B,k}\ddot{\psi}\vec{k} = T''r\vec{k}$$

$$-\frac{mr^2}{2}\ddot{\psi} = T''r$$

$$m\ddot{y} = -\frac{mr\ddot{\psi}}{2} + N'$$

R.S.D. $\Rightarrow$ $\ddot{y} = r\ddot{\psi}$

$$\frac{3}{2}m\ddot{y} = N'$$

$$N' = \frac{3}{5}\mu mg > 0$$

Consistente con la hipótesis realizada.

b)

$$-I_{A,k}\ddot{\varphi}\vec{k} = Tr\vec{k}$$

$$-\frac{mr^2}{2}\ddot{\varphi} = \mu mgr$$

$$r\ddot{\varphi} = -2\mu g$$

$$(r\dot{\varphi} - r\omega_0) = -2\mu gt$$

$$\dot{x} = \frac{2\mu g}{5}t$$

R.S.D $\Leftrightarrow$ $\dot{x} = r\dot{\varphi}$

$$\frac{2\mu gt}{5} = r\omega_0 - 2\mu gt$$

$$\frac{12\mu gt}{5} = r\omega_0$$

$$\boxed{t_{RSD} = \frac{5r\omega_0}{12\mu g}}$$

c) $t \geq t_{RSD}$

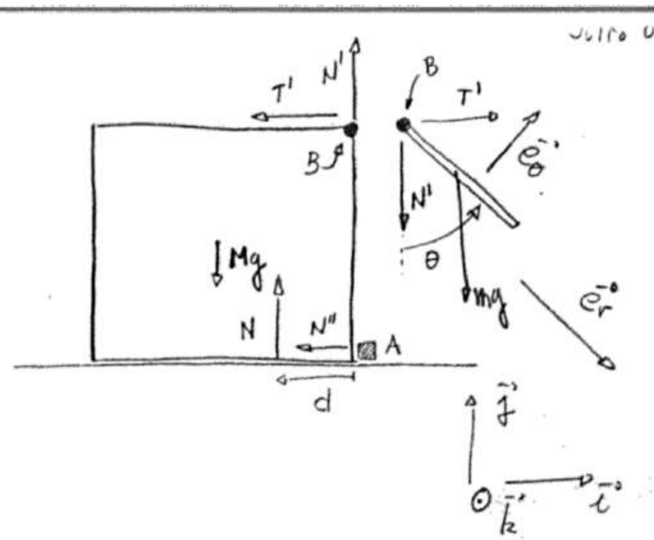
$$-\frac{mr\ddot{\varphi}}{2} = T \Rightarrow T = -\frac{m\ddot{x}}{2} \Rightarrow \frac{3}{2}m\ddot{x} = -N' \Rightarrow \ddot{x} = -\ddot{y}$$

$$\left.\begin{matrix} t \leq t_{RSD} & \ddot{x} = \ddot{y} \\ t \geq t_{RSD} & \ddot{x} = -\ddot{y} \end{matrix}\right\} \Rightarrow \underset{t \geq t_{RSD}}{\ddot{x} = \ddot{y} = 0} \Rightarrow \dot{x} = \dot{y} = \frac{2\mu g}{5}\left(\frac{5r\omega_0}{12\mu g}\right) = \frac{r\omega_0}{6}$$

$$\boxed{\dot{x} = \dot{y} = \frac{r\omega_0}{6}} \quad \forall t \geq t_{RSD}$$

Julio 07

Ejercicio (2)



(a)

$$0 = -N'' - T' \quad \text{(hipótesis de equilibrio)}$$

$$m(l\ddot{\theta}\,\vec{e}_\theta - l\dot{\theta}^2\vec{e}_r) = (-N' - mg)\vec{j} + T'\vec{i}$$

$$I_{B,k}\,\ddot{\theta}\,\vec{k} = -mg\,l\,\text{sen}\,\theta\,\vec{k}$$

$$\left(I_{B,k} = \frac{ml^2}{3} + ml^2 = \frac{4}{3}ml^2\right)$$

$$\frac{4}{3}ml^2\ddot{\theta} = -mgl\,\text{sen}\,\theta$$

$$l\ddot{\theta} = -\frac{3}{4}g\,\text{sen}\,\theta$$

$$l\dot{\theta}\ddot{\theta} = -\frac{3}{4}g\,\text{sen}\,\theta\,\dot{\theta}$$

$$l\frac{\dot{\theta}^2}{2} = \frac{3}{4}g\cos\theta$$

$(\dot{\theta}_0 = 0;\ \theta_0 = \pi/2)$

$$m(l\ddot{\theta}\cos\theta - l\dot{\theta}^2\,\text{sen}\,\theta) = T'$$

$$m\left(-\frac{3}{4}g\,\text{sen}\,\theta\cos\theta - \frac{3}{2}g\,\text{sen}\,\theta\cos\theta\right) = T'$$

$$-\frac{9}{4}mg\,\text{sen}\,\theta\cos\theta = T'$$

$$\Downarrow$$

$$N'' = \frac{9}{4}mg\,\text{sen}\,\theta\cos\theta = \frac{9}{8}mg\,\text{sen}(2\theta)$$

$$N'' \leq F \ \forall\, \theta \in (0, \pi/2)$$

$$N''_{max} = N''(\theta = \pi/4) = 9/8\,mg$$

$$\boxed{m \leq \frac{8F}{9g}}$$

(b)

$$\vec{M}_A = -Nd + Mg\frac{a}{2} + T'a = 0$$

$$(0 = N + N' - Mg)$$

$$m(l\ddot{\theta}\,\text{sen}\,\theta + l\dot{\theta}^2\cos\theta) = -mg - N'$$

$$-\frac{3}{4}mg\,\text{sen}^2\theta + \frac{3}{2}mg\cos^2\theta + mg = -N'$$

$$d = \frac{a}{2}\cdot\frac{Mg - 9/2\,mg\,\text{sen}\,\theta\cos\theta}{\left(-\frac{3}{4}\text{sen}^2\theta + \frac{3}{2}\cos^2\theta + 1\right)mg + Mg}$$

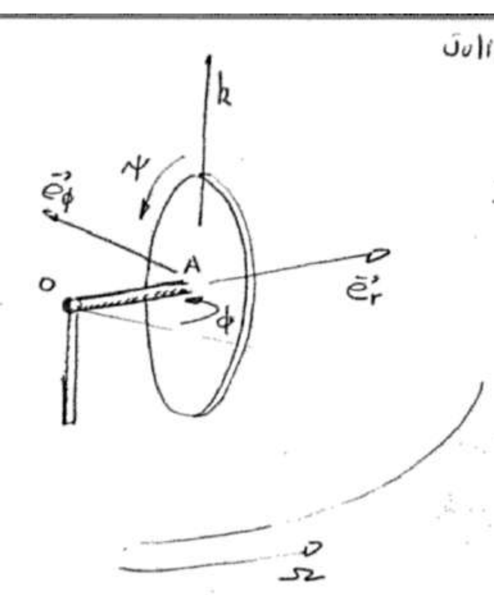
$$d = \frac{a}{2}\cdot\frac{2M - 9m\,\text{sen}\,\theta\cos\theta}{\left(-\frac{3}{2}\text{sen}^2\theta + 3\cos^2\theta + 2\right)m + 2M} \geq 0 \quad \text{para impedir vuelco en torno a A.}$$

$$\frac{\frac{2M}{m} - 9m\,\text{sen}\,\theta\cos\theta}{\underbrace{\frac{2M}{m} + \left(\frac{1}{2} + \frac{9}{2}\cos^2\theta\right)}_{\text{siempre positivo}}} \geq 0 \iff \frac{M}{m} \geq \frac{9}{2}\text{sen}\,\theta\cos\theta = \frac{9}{4}\text{sen}(2\theta)$$

$$\boxed{\left(\frac{M}{m}\right)_{min} = 9/4}$$

Julio '04

Ejercicio 3



(a)

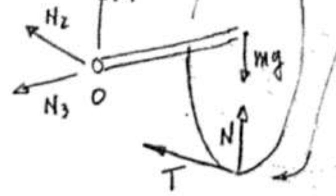
$$\vec{\omega} = -\dot{\psi}\,\vec{e}_r + \dot{\phi}\,\vec{k}$$

$$\mathbb{I}_A = \frac{ma^2}{4}\begin{bmatrix} 2 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

$$\underset{(A=G)}{\mathbb{I}_O = \mathbb{I}_A} + ma^2\begin{bmatrix} 0 & & \\ & 1 & \\ & & 1 \end{bmatrix} = \frac{ma^2}{4}\begin{bmatrix} 2 & & \\ & 5 & \\ & & 5 \end{bmatrix}$$

$$\vec{L}_O = \mathbb{I}_O\,\vec{\omega} = \frac{ma^2}{4}\left(-2\dot{\psi}\,\vec{e}_r + 5\dot{\phi}\,\vec{k}\right)$$

(b)



(Froz se opone a la velocidad relativa de los puntos de contacto.)

$$\begin{cases} \dot{\vec{L}}_O = \vec{M}^{ext}_O \\ \dot{\vec{L}}_O = \dfrac{ma^2}{4}\left(-2\ddot{\psi}\,\vec{e}_r - 2\dot{\psi}\dot{\phi}\,\vec{e}_\phi + 5\ddot{\phi}\,\vec{k}\right) \\ \vec{M}^{ext}_O = mga\,\vec{e}_\phi - Na\,\vec{e}_\phi + Ta\,\vec{k} - Ta\,\vec{e}_r \end{cases}$$

$$\Rightarrow\quad -\frac{ma^2}{2}\ddot{\psi} = +Ta \;;\quad \frac{5ma^2}{4}\ddot{\phi} = +Ta \;;\quad -\frac{ma^2}{2}\dot{\psi}\dot{\phi} = (mg-N)a$$

$$\boxed{-\ddot{\psi} = \frac{5}{2}\ddot{\phi}} \Rightarrow -\dot{\psi} = 5/2\,\dot{\phi} \quad \left(\dot{\psi}(0) = \dot{\phi}(0) = \phi\right)$$

Deslizamiento $\Rightarrow$ $|T| = fN$ $\Longrightarrow$

$$\frac{5ma}{4}\ddot{\phi} = T = fmg + \frac{fma}{2}\dot{\psi}\dot{\phi}$$

$$\frac{5a}{4}\ddot{\phi} = fg - \frac{fa}{2}\dot{\phi}\cdot\left(\frac{5}{2}\dot{\phi}\right)$$

$$\boxed{\ddot{\phi} + f\dot{\phi}^2 = \frac{4fg}{5a}}$$

(c)

$$\left.\begin{aligned} u(\phi) &= \dot{\phi}^2 \\ u'(\phi) &= 2\dot{\phi}\cdot\frac{d\dot{\phi}}{d\phi} = 2\dot{\phi}\,\ddot{\phi}\,\frac{1}{\dot{\phi}} = 2\ddot{\phi} \end{aligned}\right\} \Rightarrow \quad \frac{u'}{2} + fu = \frac{4fg}{5a}$$

$$\boxed{u' + 2fu = \frac{8fg}{5a}}$$

$$u' + 2fu = \frac{8fg}{5a}$$

<u>Sol. homogénea:</u> $u' = -2fu$

$$u = u_0\, e^{-2f\phi}$$

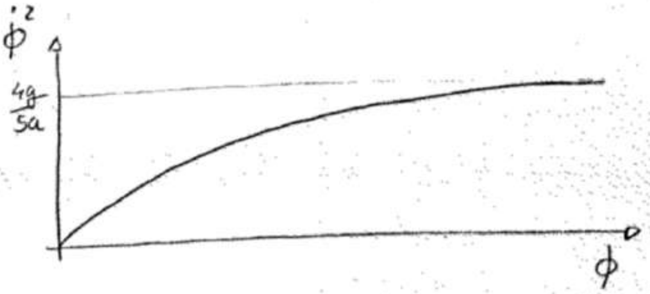
<u>Sol. particular:</u> $u = \frac{4g}{5a}$

<u>Sol</u>:

$$u = \frac{4g}{5a} + u_0 e^{-2f\phi}$$

$$u(\phi=0) = 0 \Rightarrow u_0 = -\frac{4g}{5a} \Rightarrow \boxed{u(\phi) = \frac{4g}{5a}\left(1 - e^{-2f\phi}\right)}$$

$(\dot\phi = 0)$



(d)

Sea C: p^{to} contacto.

$$C \in \text{disco} \Rightarrow \vec{v}_C = \vec{v}_A + \vec{\omega}_D \times -a\vec{k} = a\dot\phi\,\vec{e}_\phi + (-\dot\psi\,\vec{e}_r + \dot\phi\,\vec{k}) \times -a\vec{k}$$

$$\vec{v}_C = a\dot\phi\,\vec{e}_\phi - a\dot\psi\,\vec{e}_\phi = a(\dot\phi - \dot\psi)\,\vec{e}_\phi = \frac{7}{2}a\dot\phi\,\vec{e}_\phi$$

$$C \in \text{plano} \Rightarrow \vec{v}_C = a\Omega\,\vec{e}_\phi$$

Siempre hay deslizamiento $\Leftrightarrow$ $a\Omega\,\vec{e}_\phi > \frac{7}{2}a\dot\phi\,\vec{e}_\phi \quad \forall t$

$$\underbrace{\Omega > \frac{7}{2}\dot\phi \quad \forall t}$$

$$\Downarrow$$

$$\Omega > \frac{7}{2}(\dot\phi_{max}) = \frac{7}{2}\left(2\sqrt{\frac{g}{5a}}\right)$$

$$\boxed{\Omega > 7\sqrt{g/5a}}$$