

$$\prod_{i=2}^n \left( \frac{1-1}{i^2} \right) = \frac{n+1}{2 \cdot n} \quad \text{PROPOSICIÓN P}$$

DEM por inducción.

PASO BASE  $n=2$

$$\prod_{i=2}^2 \left( \frac{1-1}{i^2} \right) = \frac{3}{4} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} P(2) \text{ se cumple}$$

$$\frac{n+1}{2 \cdot n} \xrightarrow{\text{SUSTITUCIÓN } n=2} \frac{2+1}{2 \times 2} = \frac{3}{4}$$

PASO INDUCTIVO

$$\textcircled{H} P(n) = \prod_{i=2}^n \left( \frac{1-1}{i^2} \right) = \frac{n+1}{2 \cdot n}$$

$$\textcircled{T} P(n+1) = \prod_{i=2}^{n+1} \left( \frac{1-1}{i^2} \right) = \frac{(n+1)+1}{2 \times (n+1)} = \frac{n+2}{2n+2}$$

Dem

$$\prod_{i=2}^{n+1} \left( \frac{1-1}{i^2} \right) = \left[ \prod_{i=2}^n \left( \frac{1-1}{i^2} \right) \right] \times \left( \frac{1-1}{(n+1)^2} \right)$$

$\checkmark \textcircled{H}$

$$\left(\frac{n+1}{2n}\right) \left(1 - \frac{1}{(n+1)^2}\right) = \left(\frac{n+1}{2n}\right) \times \left(\frac{n^2+2n}{n^2+2n+1}\right)$$

(\*)

ans

$$1 - \frac{1}{(n+1)^2} = \frac{(n+1)^2 - 1}{(n+1)^2} = \frac{n^2+2n+1-1}{n^2+2n+1} = \frac{n^2+2n}{n^2+2n+1}$$

$$(*) \frac{n^3 + 2n^2 + n^2 + 2n}{2n^3 + 4n^2 + 2n} = \frac{n(n^2 + 3n + 2)}{2n(n^2 + 2n + 1)} = \frac{(n+2)(n+1)}{2(n+1)(n+1)}$$

$$= \frac{n+2}{2(n+1)} = \frac{n+2}{2n+2}$$

↓  
distributive

ans

$$n^2 + 3n + 2 = (n+2)(n+1)$$

$$n^2 + 2n + 1 = (n+1)(n+1)$$