PHYSICAL REVIEW E, VOLUME 63, 037102

Unified optimization criterion for energy converters

A. Calvo Hernández,[1] A. Medina,[1,2] J. M. M. Roco,[1] J. A. White,[1] and S. Velasco[1]
[1]*Departamento de Física Aplicada, Universidad de Salamanca, 37008 Salamanca, Spain*
[2]*ETSII de Béjar, Universidad de Salamanca, 37700 Béjar, Salamanca, Spain*
(Received 31 August 2000; published 21 February 2001)

We propose a unified optimization criterion for energy converters. It represents the best compromise between energy benefits and losses for a specific job and neither an explicit evaluation of entropies nor the consideration of environmental parameters are required. For all considered systems the criterion predicts a performance regime laying between those of maximum efficiency and maximum useful energy. Such regime has been invoked as optimum not only in macroscopic heat engines but also in some molecular motors.

DOI: 10.1103/PhysRevE.63.037102 PACS number(s): 05.70.−a, 87.10.+e

The subject of optimization of real devices has received continued attention in thermodynamics, engineering and, recently, in biochemistry [1–6]. The main goal in optimization is to find the pathway that yields optimum performance in a process operating at nonzero rates. To achieve this goal, an objective function that depends on parameters of the problem must be optimized. In principle one has the freedom of choice of such objective function. It has been pointed out [6,7], however, that a *thermodynamic* criterion devoted to analyze the optimum regime of operation in a real process should meet the following requirements: (i) its dependence on the parameters of the process should be a guidance in order to improve the performance of that process; (ii) it should not depend on parameters of the environment; and (iii) it should take into account the unavoidable dissipation of energy provoked by the process. In this letter we address the problem of finding an optimization criterion which, satisfying the above requirements, can be applied to any energy converter.

The two methods most widely used in the optimization of traditional thermodynamic heat devices are the entropy generation minimization and exergy analysis. Both methods are based on the Gouy–Stodola theorem [8], which quantifies the lost available work (or exergy destruction), $W_{\rm lost}=T_0 S_{\rm gen}$, for any system operating under irreversible (finite-time) conditions in terms of the corresponding entropy generation, $S_{\rm gen}$, and the environment temperature, T_0. The application of this theorem to a particular design requires the evaluation of $S_{\rm gen}$ through a model linking the thermodynamic nonideality of the design to the physical characteristics of the system. However, deriving expressions for $S_{\rm gen}$ is a subtle and, sometimes, difficult task (as it happens for situations where the system is far from the equilibrium). Exergetic methods additionally depend on the parameters of the environment which can be unknown or far from the average values [6,7]. A number of different optimization criteria have also been proposed, but they suffer from a lack of generality since they apply to particular heat devices, either heat engines, refrigerators, or heat-pump cycles [5].

An important feature of the proposed criterion is that it gives an optimized efficiency that lies between the maximum efficiency and the efficiency under maximum power conditions. Such operation regime was invoked as optimum in traditional heat engines [9] and agrees with recent observations that some molecular motors seem to be optimized both from the velocity and the efficiency standpoints [10,11]. Although conceptual differences exist between microscopic and macroscopic engines [3,10,11], this fact suggests that the proposed optimization could be used as a unified framework for dealing with molecular and macroscopic engines.

Let us consider an energy converter whose task is to produce a useful energy $E_u(x;\{\alpha\})$ by the conversion of an input energy $E_i(x;\{\alpha\})$ along a given (nonideal) process. Here x denotes an independent variable while $\{\alpha\}$ denotes a set of parameters which can be considered as controls. The conventional efficiency of this converter, defined as the ratio between the useful and input energy $z(x;\{\alpha\})=E_u(x;\{\alpha\})/E_i(x;\{\alpha\})$, satisfies the relation $z_{\min}(\{\alpha\})\leq z(x;\{\alpha\})\leq z_{\max}(\{\alpha\})$, where $z_{\min}(\{\alpha\})$ and $z_{\max}(\{\alpha\})$ are, respectively, the minimum and maximum values of $z(x;\{\alpha\})$ in the allowed range of values of x for given α's [we note that in some energy converters $z_{\min}\{\alpha\}\neq 0$ (see below for an example)]. Then, for a given input energy, one has $z_{\min}(\{\alpha\})E_i(x;\{\alpha\})\leq E_u(x;\{\alpha\})\leq z_{\max}(\{\alpha\})E_i(x;\{\alpha\})$. These limits suggest to define an effective useful energy as $E_{u,\rm eff}(x;\{\alpha\})=E_u(x;\{\alpha\})-z_{\min}(\{\alpha\})E_i(x;\{\alpha\})$ and a lost useful energy as $E_{u,L}(x;\{\alpha\})=z_{\max}(\{\alpha\})E_i(x;\{\alpha\})-E_u(x;\{\alpha\})$. To evaluate the best compromise between useful energy and lost useful energy we introduce the Ω function as the difference between these quantities:

$$\Omega(x;\{\alpha\})=E_{u,\rm eff}(x;\{\alpha\})-E_{u,L}(x;\{\alpha\}) =\frac{2z(x;\{\alpha\})-z_{\min}(\{\alpha\})-z_{\max}(\{\alpha\})}{z(x;\{\alpha\})}E_u(x;\{\alpha\}), \tag{1}$$

which is our proposal as objective function to analyze the operation mode of any energy converter giving the best compromise between energy benefits and losses.

We first apply the criterion to macroscopic heat devices used in thermodynamics, distinguishing among heat engines (HE), refrigerators (RE), and heat pumps (HP). In a HE the useful energy is the work delivered $|W|$ and the input energy is the heat supply $|Q_H|$; a RE extracts a refrigeration load $|Q_L|$ from a cold space at the cost of an expenditure of work $|W|$; and a HP delivers a heating load $|Q_H|$ to a warm space while a given work $|W|$ is supplied. The efficiencies of these systems are well known: $z_{HE}\equiv\eta=|W|/|Q_H|$, $z_{RE}\equiv\epsilon=|Q_L|/|W|$ is the coefficient of performance (COP) of the RE and $z_{HP}\equiv\nu=|Q_H|/|W|$ is the COP of the HP. Note that η and ϵ can attain the value zero, while $\nu=\epsilon+1$. As a consequence, in a HP ν is never below unity and the effective useful heating load is $|Q_H|-|W|$. The HP is an explicit example where $z_{min}\neq 0$. From the above considerations and using Eq. (1) we obtain for these heat devices the following expressions for Ω:

$$\Omega_{HE}=2|W|-|W|_{max}=(2\eta-\eta_{max})|Q_H|=(2\eta-\eta_{max})|W|/\eta, \quad (2)$$

$$\Omega_{RE}=2|Q_L|-|Q_L|_{max}=(2\epsilon-\epsilon_{max})|W|=(2\epsilon-\epsilon_{max})|Q_L|/\epsilon, \quad (3)$$

$$\Omega_{HP}=2|Q_H|-|W|-|Q_H|_{max}=(2\nu-1-\nu_{max})|W|=(2\nu-1-\nu_{max})|Q_H|/\nu, \quad (4)$$

which can be considered, respectively, as the best compromise between maximum work performed and minimum lost work in a HE, between maximum cooling load and minimum lost cooling load in a RE, and between the maximum heating load and minimum lost heating load in a HP.

In order to obtain concrete results we focus on the so-called irreversible Carnot-type models. They are widely used in finite-time thermodynamics [5] because, in spite of their relative analytical simplicity, are able to account for the main irreversibilities that usually arise in real heat devices: finite-rate heat transfer between the working fluid and the external heat sources, internal dissipation of the working fluid, and heat leak between reservoirs. For an irreversible Carnot-type model of a HE, the power $\dot{W}$ and efficiency η are [12]

$$\dot{W}(a_h;\tau,I,\sigma_{hc},\sigma_{ih})\propto\frac{I(a_h-1)-\sigma_{hc}(a_h-1)^2-\tau(a_h^2-a_h)}{a_h(I+\sigma_{hc})-\sigma_{hc}a_h^2}, \quad (5)$$

$$\eta(a_h;\tau,I,\sigma_{hc},\sigma_{ih})=\left[1-\frac{a_h\tau}{I-\sigma_{hc}(a_h-1)}\right]\times\left[\frac{a_h-1}{a_h-1+\sigma_{ih}a_h(1-\tau)}\right], \quad (6)$$

where $a_h\geqslant 1$ is the ratio of the hot reservoir temperature to the working fluid temperature in the upper isothermal process (our independent variable x) and τ, I, σ_{hc}, and σ_{ih} are the set $\{\alpha\}$ of controls accounting for, respectively, the ratio of the cold reservoir to the hot reservoir temperature, the internal dissipations of the working fluid, the ratio of the external hot-end to cold-end conductances and the ratio of the internal heat conductance to the external hot-end conductance. For this model $\eta_{min}=0$ and the rate-dependent version of Eq. (2) becomes

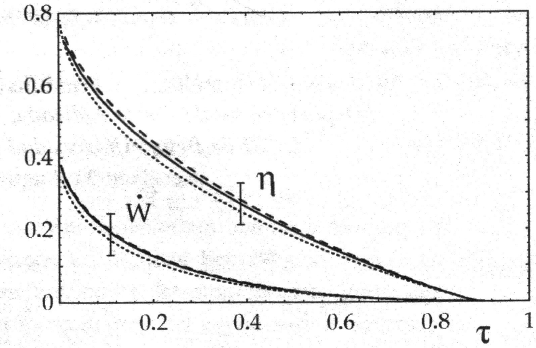

FIG. 1. Efficiency, η, and (dimensionless) power, $\dot{W}$, for an irreversible Carnot-type HE model ($I=0.9$, $\sigma_{hc}=1$, $\sigma_{ih}=0.1$) versus τ. Upper part: maximum η (dashed line) and η under conditions of maximum $\dot{\Omega}_{HE}$ (solid line) and maximum $\dot{W}$ (dotted line). Lower part: maximum $\dot{W}$ (dashed line) and $\dot{W}$ under conditions of maximum $\dot{\Omega}_{HE}$ (solid line) and maximum η (dotted line).

$$\dot{\Omega}_{HE}(a_h;\tau,I,\sigma_{hc},\sigma_{ih})=[2\eta(a_h;\tau,I,\sigma_{hc},\sigma_{ih})-\eta_{max}(\tau,I,\sigma_{hc},\sigma_{ih})]\times\dot{W}(a_h;\tau,I,\sigma_{hc},\sigma_{ih})/\eta(a_h;\tau,I,\sigma_{hc},\sigma_{ih}).$$

For given values of controls, the functions η, $\dot{W}$, and $\dot{\Omega}_{HE}$ always present a maximum for some $a_h\geqslant 1$. The maximum efficiency and the efficiencies under conditions of maximum $\dot{W}$ and maximum $\dot{\Omega}_{HE}$ are plotted versus τ in the upper part of Fig. 1 for a set of realistic values of controls, while the lower part shows the maximum power and the power under conditions of maximum η and maximum $\dot{\Omega}_{HE}$. As it can be seen, the $\dot{\Omega}_{HE}$ regime gives efficiencies and powers whose values are between those obtained from the maximum efficiency and maximum power regimes. We have checked that this happens for any allowed value of the controls.

For an irreversible Carnot-type RE the cooling rate, $\dot{Q}_L$, and the COP, ϵ, are [13]

$$|\dot{Q}_L|(a_h;\tau,I,\sigma_{hc},\sigma_{ih})\propto\frac{\alpha a_h-\beta}{\gamma a_h-(\gamma-1)}, \quad (7)$$

$$\epsilon(a_h;\tau,I,\sigma_{hc},\sigma_{ih})=\frac{\alpha a_h-\beta}{(a_h-1)(\gamma a_h-\delta)}, \quad (8)$$

with $\alpha=I\tau-\sigma_{ih}(1+I\sigma_{hc})(1-\tau)$, $\beta=\alpha+\sigma_{ih}(1-\tau)$, $\gamma=1+I\sigma_{hc}$, and $\delta=I(\sigma_{hc}+\tau)$. Now $a_h\geqslant 1$ denotes the ratio of the temperature of the refrigerant in the upper isothermal process to the temperature of the external hot reservoir,

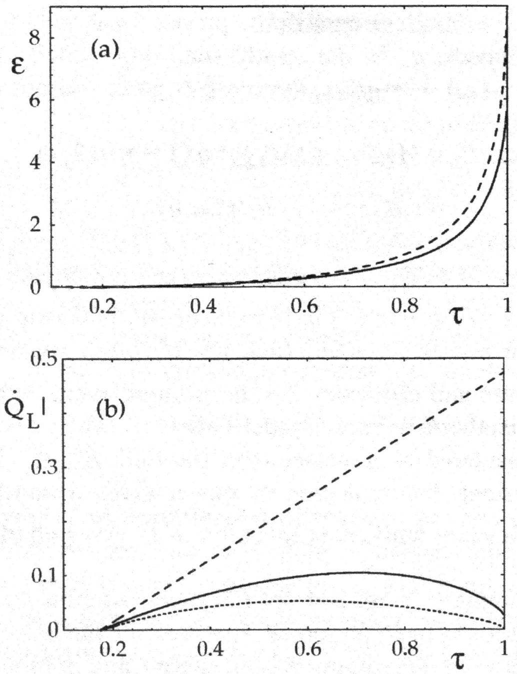

FIG. 2. COP, ϵ, and (dimensionless) cooling power, $|\dot{Q}_L|$, for an irreversible Carnot-type RE model ($I=0.9$, $\sigma_{hc}=1$, $\sigma_{ih}=0.1$) versus τ. (a) Maximum ϵ (dashed line) and ϵ under conditions of maximum $\dot{\Omega}_{\mathrm{HE}}$ (solid line) and maximum $|\dot{Q}_L|$ (dotted line); (b) maximum $|\dot{Q}_L|$ (dashed line) and $|\dot{Q}_L|$ under conditions of maximum $\dot{\Omega}_{\mathrm{HE}}$ (solid line) and maximum ϵ (dotted line).

$\epsilon_{\min}=0$ and $\dot{\Omega}_{\mathrm{RE}}(a_h;\tau,I,\sigma_{hc},\sigma_{ih})=[2\epsilon(a_h;\tau,I,\sigma_{hc},\sigma_{ih})-\epsilon_{\max}(\tau,I,\sigma_{hc},\sigma_{ih})]|\dot{Q}_L|(a_h;\tau,I,\sigma_{hc},\sigma_{ih})/\epsilon(a_h;\tau,I,\sigma_{hc},\sigma_{ih})$. Figure 2(a) shows the maximum COP and the COP under condition of maximum $\dot{\Omega}_{\mathrm{RE}}$ (the COP under condition of maximum $|\dot{Q}_L|$ is zero) and Fig. 2(b) shows the maximum $|\dot{Q}_L|$ and $|\dot{Q}_L|$ under conditions of maximum ϵ and maximum $\dot{\Omega}_{\mathrm{RE}}$. Note again that the proposed criterion gives a COP below that corresponding to the maximum COP regime and a cooling power lying between the maximum one and that obtained under maximum COP. Results for the Carnot-type irreversible HP are straightforward and they are not shown.

As a second application to heat devices, we consider the so-called *endoreversible* models [1,14]. These models, subject to criticisms during the last years [15] (see however [16]), assume an internally reversible Carnot engine coupled to two external heat reservoirs through linear finite-rate heat transfer laws. They emerge from the irreversible Carnot-type models if $I=1$ and $\sigma_{ih}=0$. Now $\eta_{\max}=1-\tau\equiv\eta_C$, $\epsilon_{\max}=\tau/(1-\tau)\equiv\epsilon_C$, $\nu_{\max}=1/(1-\tau)\equiv\nu_C$ and the values of involved functions under maximum $\dot{\Omega}$ condition can be worked out analytically. In particular, the results for the efficiency and the COP's are $\eta_{\max\dot{\Omega}_{\mathrm{HE}}}=1-\sqrt{\tau(\tau+1)/2}$, $\epsilon_{\max\dot{\Omega}_{\mathrm{RE}}}=\tau/(\sqrt{2-\tau}-\tau)$, and $\nu_{\max\dot{\Omega}_{\mathrm{HP}}}=\epsilon_{\max\dot{\Omega}_{\mathrm{RE}}}+1$. It is also found that $\eta_{\mathrm{CA}}\leqslant\eta_{\max\dot{\Omega}_{\mathrm{HE}}}\leqslant\eta_C$, where $\eta_{\mathrm{CA}}\equiv1-\sqrt{\tau}$, is the (Curzon–Ahlborn [14]) efficiency under maximum power conditions. Two of the above τ-dependent values have been reported previously. Angulo-Brown [17] first derived $\eta_{\max\dot{\Omega}_{\mathrm{HE}}}$ by applying the so-called ecological criterion (the best compromise between power production and the product of entropy production and the cold reservoir temperature T_C: $E_{\mathrm{HE}}=\dot{W}-T_C\dot{S}_{\mathrm{gen}}$) to the optimization of an endoreversible Carnot HE. Later, this criterion was reinterpreted by Yan [18] in exergetic terms as $E_{\mathrm{HE}}=\dot{W}-T_0\dot{S}_{\mathrm{gen}}$ with T_0 denoting the environment temperature. The $\epsilon_{\max\dot{\Omega}_{\mathrm{RE}}}$ result was first reported by Yan and Chen [19] in the optimization of an endoreversible Carnot RE under the ecological criterion $E_{\mathrm{RE}}=|\dot{Q}_L|-\epsilon_C T_0\dot{S}_{\mathrm{gen}}$ (the best compromise between the maximum rate of refrigeration and the minimum rate of exergy loss) when T_0 takes the value of the hot reservoir T_H. The optimized $\nu_{\max\dot{\Omega}_{\mathrm{HP}}}$ value can be also obtained from the optimization of an endoreversible HP under the criterion $E_{\mathrm{HP}}=(|\dot{Q}_H|-|\dot{W}|)-\nu_C T_0\dot{S}_{\mathrm{gen}}$ with $T_0=T_C$. Accordingly, the Ω criterion is an ecological-like optimization but without requiring environmental parameters and explicit calculations of S_{gen}.

An entirely different energy converter is the isothermal linear model for systems in nonreversible steady states as considered by Stucki [20], Santillán *et al.* [21], and Prost *et al.* [3,10] in the analysis of the efficiency in linear biological motors. For such energy converter power, P, and efficiency, η, are $P=-TJ_1X_1$ and $\eta=-J_1X_1/J_2X_2$, where J_1 and J_2 are the generalized currents and X_1 and X_2 are the generalized forces, with $J_1X_1<0$ and $J_2X_2>0$ denoting, respectively, the driven and driver processes in the steady state. Under a constant driver force X_2, these magnitudes can be expressed in terms of a relevant variable $x=-X_1L_{11}/X_2L_{12}$ $[0\leqslant x\leqslant1]$ and a (control) irreversibility parameter $q=L_{12}/\sqrt{L_{11}L_{22}}$ $[0\leqslant q^2\leqslant1]$ measuring the coupling degree between driver and driven processes through the phenomenological constants L_{ij}, as

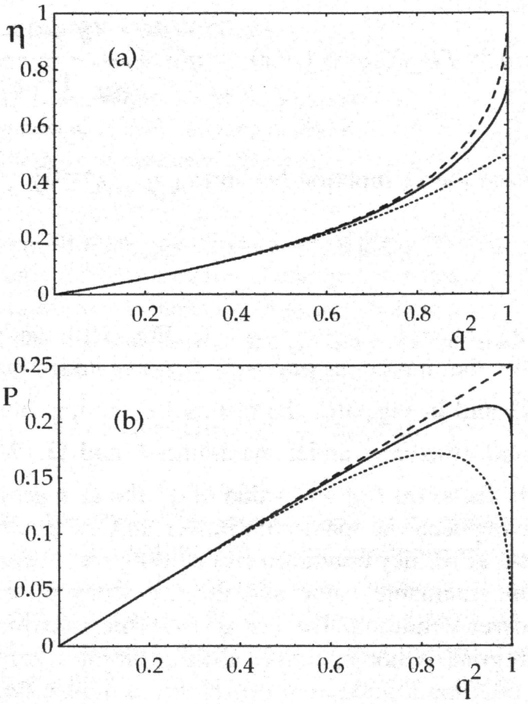

FIG. 3. As in Fig. 1 but for the efficiency, η, and (dimensionless) power, P, of the isothermal linear model versus q^2.

$$P(x;q)=TL_{22}X_2^2q^2x(1-x), \quad \eta(x;q)=\frac{q^2x(1-x)}{1-q^2x}. \tag{9}$$

In this case the $\dot{\Omega}$ function becomes $[\eta_{min}(q)=0]$

$$\dot{\Omega}(x;q)=TL_{22}X_2^2[2q^2x(1-x)-\eta_{max}(q)(1-q^2x)], \tag{10}$$

where $\eta_{max}(q)=(1-\sqrt{1-q^2})/q^2$. In Fig. 3(b) we plot the results for the maximum power and power under maximum η and $\dot{\Omega}$ and in Fig. 3(a) the results for the maximum efficiency and efficiency under maximum P and $\dot{\Omega}$. We stress two main facts. (a) For any value of q^2 the $\dot{\Omega}$ regime yields a power between the maximum power and the power under maximum efficiency condition and an efficiency between the maximum attainable value and the efficiency under maximum power condition. (b) For $q^2 \to 1$, the maximum efficiency regime is not operative since it implies zero power and the maximum power regime implies a drastic decreasing of the efficiency up to 0.5. Between these two regimes $\dot{\Omega}$ yields an efficiency approaching 3/4 while power remains finite, in agreement with reported results [20,21]. Similar values to those plotted in Fig. 3(a) emerge from an ecological regime [21], E, which can be obtained from $\dot{\Omega}$ by replacing η_{max} by the unity in Eq. (10). A significant difference between them is that the E regime crosses, for some value of q^2, the maximum efficiency and maximum power criteria. Only when $q^2=1$, E and $\dot{\Omega}$ coincide.

Finally, with the aim of showing the wide applicability of the proposed optimization, we analyze a mechanical converter: an Atwood machine [22] with two weights M_2g, the driven force, and M_1g, the driver force (g is the acceleration of gravity), and a friction force proportional to the velocity of the masses, v. In the steady state this velocity is given by $v_{ss}=M_1(1-\eta)g/2\mu$, the (useful) power output is

$$P_u=M_2gv_{ss}=(M_1g)^2\eta(1-\eta)/(2\mu)$$
$$=(M_2g)^2(1-\eta)/(2\mu\eta),$$

where $\eta=M_2/M_1$ is the efficiency, and the objective function is $\dot{\Omega}=(2\eta-1)P_u/\eta$. If we keep M_1 constant, maximization of power gives an efficiency 1/2 while maximization of $\dot{\Omega}$ gives and efficiency 3/4, in full agreement with results for the isothermal linear model (where the driver force was also considered as a constant) in the limit $q^2 \to 1$. Keeping M_2 constant, maximization of power gives a nonoperative zero efficiency while maximization of $\dot{\Omega}$ gives an efficiency 2/3.

In summary, a unified optimization criterion for energy converters has been presented. It represents the best compromise between maximum useful energy and minimum lost useful energy for a specific job, it is independent of any environment parameter, and does not require the explicit derivation of entropy generation. For endoreversible Carnot-type models it recovers in a natural way some temperature-dependent efficiency limits obtained under different ecological-like criteria. For irreversible heat engines it predicts an operation regime lying between those corresponding to maximum efficiency and maximum power. Such regime has been considered as optimum in macroscopic and molecular engines.

We thank F. Angulo-Brown and M. Santillán for stimulating discussions. Financial support from CICYT of Spain (Grant No. PB98-0261) is acknowledged.

[1] A. Bejan, J. Appl. Phys. **79**, 1191 (1996).
[2] S. Sieniutycz and J.S. Shiner, J. Non-Equilib. Thermodyn. **19**, 303 (1994).
[3] F. Julicher, A. Ajdary, and J. Prost, Rev. Mod. Phys. **69**, 1269 (1997).
[4] *Thermodynamic Optimization of Complex Energy Systems*, edited by A. Bejan and E. Mamut (Kluwer Academic, Dordrecht, 1999).
[5] *Recent Advances in Finite Time Thermodynamics*, edited by C. Wu, L. Chen, and J. Chen (Nova Science, New York, 1999).
[6] R. S. Berry, V. A. Kazakov, S. Sieniutycz, Z. Szwast, and A. M. Tsirling, *Thermodynamic Optimization of Finite-Time Processes* (Wiley, Chichester, 2000).
[7] V.A. Mironova, A.M. Tsirling, V.A. Kazakov, and R.S. Berry, J. Appl. Phys. **7**, 629 (1994).
[8] A. Bejan, *Advanced Engineering Thermodynamics*, 2nd ed. (Wiley, New York, 1997), pp. 108–122.
[9] J. Chen, J. Phys. D **27**, 1144 (1994).
[10] A. Parmeggiani, F. Julicher, A. Ajdary, and J. Prost, Phys. Rev. E **60**, 2127 (1999).
[11] I. Derényi, M. Bier, and R.D. Astumian, Phys. Rev. Lett. **83**, 903 (1999).
[12] A. Calvo Hernández, J.M.M. Roco, S. Velasco, and A. Medina, Appl. Phys. Lett. **73**, 853 (1998).
[13] S. Velasco, J.M.M. Roco, A. Medina, and A. Calvo Hernández, Appl. Phys. Lett. **71**, 1130 (1997).
[14] F.L. Curzon and B. Ahlborn, Am. J. Phys. **43**, 22 (1975).
[15] D.P. Sekulic, J. Appl. Phys. **83**, 4561 (1998); M.J. Moran, Energy (Oxford) **23**, 517 (1998).
[16] P. Salamon, Ref. [4], p. 421.
[17] F. Angulo-Brown, J. Appl. Phys. **69**, 7465 (1991).
[18] Z. Yan, J. Appl. Phys. **73**, 3583 (1993).
[19] Z. Yan and L. Chen, J. Phys. D **29**, 3017 (1996).
[20] J.W. Stucki, Eur. J. Biochem. **109**, 269 (1980).
[21] M. Santillán, L.A. Arias-Hernández, and F. Angulo-Brown, Nuovo Cimento **19**, 99 (1997).
[22] H.T. Odum and R.C. Pikerton, Am. Sci. **43**, 331 (1955).