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5 authors, including:

S. Velasco
Universidad de Salamanca

J. M. M. Roco
Universidad de Salamanca

A. Medina
Universidad de Salamanca

J. A. White
Universidad de Salamanca

Optimization of heat engines including the saving of natural resources and the reduction of thermal pollution

S Velasco†, J M M Roco†, A Medina‡, J A White† and A Calvo Hernández†

† Departamento de Física Aplicada, Facultad de Ciencias, Universidad de Salamanca, 37008 Salamanca, Spain
‡ ETSII de Béjar, 37700 Béjar, Salamanca, Spain

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Abstract. The use of the new concept of a *saving function* as a measure of possible reductions of undesired side effects in heat engine operation is proposed. Two saving functions are introduced, one associated with fuel consumption and another associated with thermal pollution. Two optimization paths including the maximization of power output and these saving functions are presented. The first is based on a linear formalism and the second is based on a power-law formalism. When these optimization criteria are applied to a Curzon–Ahlborn heat engine, both criteria lead to a very similar optimum efficiency, $\eta_{opt} = 1 - \tau^{3/4}$, where τ is the ratio between the temperatures of the cold and the hot external reservoirs. A numerical comparison with the efficiency of some modern nuclear power plants is reported.

1. Introduction

The world oil crisis in the early 1970s created a renewed interest in concerns related to the energy problem, such as the reduction of the energy consumption in heat engines. In 1977, Andresen *et al* [1, 2] introduced the name *finite time thermodynamics* (FTT) to designate a new branch of thermodynamics devoted to extend the classical reversible thermodynamics to include more realistic, finite-time (irreversible) processes. Nowadays, FTT is considered a part of a wider field known as *thermodynamic optimization* [3], and it should be acknowledged that some of the basic FTT methodology has been used in engineering since the early 1950s in the so-called *entropy generation minimization* approach [4–6]. The main goal of the FTT applied to power plants is to determine more realistic upper bounds for an improvement of actual systems as well as to define the conditions for optimal design of such systems.

A paradigmatic model in FTT is the so-called *endoreversible* heat engine [7, 8], a Carnot engine in which the sole source of irreversibility is the finite-rate heat transfer with the external reservoirs. Using this model with linear heat transfer laws, Curzon and Ahlborn (CA) [7] obtained for the efficiency at the maximum power output

$$\eta_{CA} = 1 - \left(\frac{T_c}{T_h}\right)^{1/2} \qquad (1)$$

where T_c and T_h are the temperatures of the cold and hot reservoirs, respectively. Expression (1) was previously obtained by Chambadal [9] and Novikov [10] by using simpler models.

Although efficiency values provided by equation (1) compare favorably with efficiencies of some real power plants, the endoreversible approach has been criticized recently by using some arguments showing the inconsistency of expression (1). A first argument is based on the fact that expression (1) is obtained from the assumption that the heat input rate is free to vary in the optimization of a power plant of fixed size, while, in contrast, most thermal power plants use fuel that is neither inexhaustible nor free nor cheap [11, 12]. A second argument has been given on the basis that real power plants are not designed at maximum power output but usually following economic considerations, so that the above mentioned agreement is purely coincidental [13, 14]. In fact, the efficiency of some power plants (for instance, modern nuclear power plants) is larger than the values provided by equation (1). A third argument refers to the plausibility of the concept of endoreversibility; in particular, the assumption of internal reversibility is contradictory with the existence of external finite area heat exchangers that interact with the internal working fluid across finite temperature gaps [15]. In this context, the endoreversible models must be considered as simple limit models, where either the influence between irreversibilities of both sides of a heat exchanger is neglected [15] or the reservoir temperatures T_c and T_h must

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be considered as mean temperatures of heat rejection and heat addition, respectively [16, 17]. The present work is mainly concerned with some aspects related to the two first arguments.

Because of practical and economical reasons, it is well established that actual power plants in general operate under conditions between their maximum power output point and their maximum efficiency point [18–21]. This range can be achieved, for instance, by using the so-called *profit function* introduced by Salamon and Nitzan [22],

$$\Pi = P_{\dot{W}}\dot{W} - P_{\dot{A}}\dot{A} = P_{\dot{W}}\dot{W} - P_{\dot{A}}T_0\dot{\sigma} \tag{2}$$

where $\dot{W}$ is the power output, $\dot{A} = T_0\dot{\sigma}$ is the rate of exergy (or availability) taken from the reservoirs, T_0 is the environment temperature, $\dot{\sigma}$ is the rate of entropy production, $P_{\dot{A}}$ is the price of exergy, and $P_{\dot{W}}$ is the price of power output (electricity). One can check that the efficiency of the CA engine at the maximum profit function point is given by

$$\bar{\eta}_{\Pi}(\tau;\tau_0,P) = 1 - \tau\sqrt{\frac{1+P\tau_0}{\tau+P\tau_0}} \tag{3}$$

where $\tau = T_c/T_h$, $\tau_0 = T_0/T_h$, and $P = P_{\dot{A}}/P_{\dot{W}}$. From expression (3) one has that $\bar{\eta}_{\Pi} = \eta_{CA} = 1 - \tau^{1/2}$ for $P = 0$, while $\bar{\eta}_{\Pi} = \eta_C = 1 - \tau$ in the limit $P \to \infty$. A similar analysis was reported more recently by De Vos [23] who considered the thermo-economics of an endoreversible power plant resulting in an optimum performance on the relative costs of investment and fuel. The range between the CA efficiency and the Carnot limit can also be achieved by using the so-called *ecological function* introduced by Angulo-Brown [24] and improved by Yan [25], which is defined by

$$E = \dot{W} - T_0\dot{\sigma}. \tag{4}$$

We note that function (4) is equivalent to the profit function (2) for $P = 1$. For the particular case of $T_0 = T_c$, the efficiency of the CA engine at the maximum ecological function point is $\bar{\eta}_E = 1 - \sqrt{(1+\tau)\tau/2}$, according to equation (3) for $P = 1$ and $\tau_0 = \tau$. Therefore, the application of the objective functions (2) or (4) to a power plant needs the specification of the price parameter P and the environment temperature T_0, and thus an attractive characteristic of the CA efficiency (1), their sole dependence with the temperature ratio τ, is lost. The main goal of the present work is to propose two optimization paths that, focused on modelling environmental impact, provide an optimal τ-dependent efficiency located in the required economical range between the maximum power output and the minimum entropy production points.

2. Optimization criteria

Basically, a power plant runs following a thermal cycle in which a working fluid absorbs a heat flow $\dot{Q}_{in}$ from a heat source, rejects a heat flow $\dot{Q}_{out}$ to a heat sink at a lower temperature, and provides a power output $\dot{W}$, so that, because of energy conservation,

$$\dot{Q}_{in} = \dot{W} + \dot{Q}_{out}. \tag{5}$$

This means that for a power plant with a thermal efficiency $\eta = \dot{W}/\dot{Q}_{in} = 0.35$ (a typical value for a commercial nuclear plant), for every kilowatt of electrical power, 2.86 kW of input heat flow (thermal power) is needed and 1.86 kW of heat flow is released. Since the usual electric power production of one of these plants is about 1000 MW, it seems clear that special attention must be paid to operation conditions that minimize both the fuel consumption and the production of thermal pollution.

In the above basic scheme of a power plant there are three energy flows related by equation (5). The electric power production is given by the power output $\dot{W}$, so that, for a given efficiency, an increase of $\dot{W}$ implies an increase of both the input heat flow $\dot{Q}_{in}$ and the rejected heat flow $\dot{Q}_{out}$. Certainly, increasing $\dot{W}$ accelerates the investment recuperation, but it also increases $\dot{Q}_{in}$ which implies an increase of the rate of fuel consumption. On the other hand, since the atmosphere, a river, a lake, or the sea usually act as the heat sink, it is clear that an increase of the output heat flow $\dot{Q}_{out}$ implies an increase of the thermal pollution in the neighbourhood of the power plant. In other words, not only the amount of power output, but also preservation of natural resources and reduction of thermal pollution must be included in an overall optimization of actual power plants. Here, we report two objective functions that provide a compromise among operations which maximize the power output and operations which minimize the fuel consumption and the production of thermal pollution.

With this goal in mind, we consider the operating mode of a power plant as a set of three interdependent processes, one of them of *profitable* type (the production of electric power) and the other two of *undesirable* type (the input and output heat flows). Then, in order to characterize the possible reduction in the effects of an undesirable process we define the concept of a *saving function*. Let us consider that a given undesirable process can be mathematically described by a function $F(\{x\};\{\lambda\})$, where $\{x\}$ denotes a set of independent variables and $\{\lambda\}$ denotes a set of controllable parameters; we introduce the *saving function* associated with $F(\{x\};\{\lambda\})$ as

$$f(\{x\};\{\lambda\}) = 1 - \frac{F(\{x\};\{\lambda\})}{F_{max}(\{\lambda\})} \tag{6}$$

where $F_{max}(\{\lambda\})$ is the maximum value of $F(\{x\};\{\lambda\})$ in the allowed range of values of $\{x\}$. This maximum value corresponds to the most possible unefficient operation mode of the system. For instance, this should be the case for a thermal power plant when the input heat flow is directly released to the environment with no net power output produced (*short-circuit* operation), so that their efficiency is zero. Thus, the saving function (6) is smaller as the operation regime of the system becomes more inefficient.

Then, by assuming that the rate of fuel consumption and the rate of production of thermal pollution are proportional, respectively, to the input heat flow $\dot{Q}_{in}$ and to the output heat flow $\dot{Q}_{out}$, we define the *fuel saving function* as

$$q_{in} = 1 - \frac{\dot{Q}_{in}}{(\dot{Q}_{in})_{max}} \tag{7}$$

and the *thermal pollution saving function* as

$$q_{out} = 1 - \frac{\dot{Q}_{out}}{(\dot{Q}_{out})_{max}}. \tag{8}$$

In equation (7) $(\dot{Q}_{in})_{max}$ is the maximum heat flow we can convert in the true heat engine, i.e. the maximum heat flow we can extract from the high-temperature reservoir without supplying power. Thus, $(\dot{Q}_{out})_{max} = (\dot{Q}_{in})_{max}$ in equation (8). This maximum heat flow has been considered by de Vos [23] as a measure of the size of the plant.

Therefore, we focus our attention on the following three processes: (1) the production of useful energy, characterized by the function $\omega = \dot{W}/\dot{W}_{mp} = (\dot{Q}_{in} - \dot{Q}_{out})/\dot{W}_{mp}$, with $\dot{W}_{mp}$ being the maximum power output; (2) the saving of fuel consumption, characterized by the function q_{in}, given by equation (7); and (3) the reduction of thermal pollution, characterized by the function q_{out}, given by equation (8). These functions verify $0 \leqslant (\omega, q_{in}, q_{out}) \leqslant 1$, so that they can be considered as performances of the corresponding processes. The optimum operation mode for each process is reached under the conditions for which the associated performance is equal to one. It is clear that these three performances cannot reach the unity value under the same operation mode (for example, $q_{in} = 1$ (no fuel consumption) implies that $\dot{Q}_{in} = 0$, and, taking into account equation (5), that $\dot{W} = \dot{Q}_{out} = 0$, i.e. $q_{out} = 1$ (no thermal pollution) but $\omega = 0$ (no production of electric power)). Then, we are interested in finding operation conditions for the heat engine coming from the simultaneous optimization of the three considered processes, i.e. in finding plausible objective functions $\Phi = \Phi(\omega, q_{in}, q_{out})$ to be optimized. At this point, as was pointed out by Berry [26], we want to emphasize that in thermodynamic optimization one has the freedom of choice of the objective function. The question then arises of how to compare the efficiency of systems optimized for different criteria or how to compare the obtained results with data for real heat engines. Here, we propose two different *mathematical* methods, a *linear* formalism and a *power-law* formalism, where the production of electric power, the saving of fuel consumption, and the reduction of thermal pollution are optimized together.

2.1. Linear formalism

Since the three basic energy flows of a heat engine verify the linear relation (5), we propose as a first figure of merit to be optimized, the linear combination

$$\Phi_A = a_1\omega + a_2 q_{in} + a_3 q_{out} \tag{9}$$

where a_1, a_2, and a_3 are *weight* coefficients measuring the participation degree of the corresponding process in the optimization criterion. We notice that the profit function (2) is also a linear criterion, and the parameters a_1, a_2, and a_3 in equation (9) play a similar role to the price parameters $P_{\dot{W}}$ and $P_{\dot{A}}$ in equation (2). When the three processes involved are considered without discrimination ($a_1 = a_2 = a_3 = a$), equation (9) becomes

$$\Phi_A = a(\omega + q_{in} + q_{out}). \tag{10}$$

In particular, for $a = 1/3$, this function is the arithmetic mean of the three basic performances. In any case, the optimization of the objective function (10) is independent of the parameter a.

2.2. Power-law formalism

A usual mathematical way to investigate the behaviour of a function near its extremum is to expand its logarithm in a Taylor's series about the extremum. The reason for this is that the logarithm of a function varies much more slowly than the function itself. This technique suggests a formalism, analogous to the linear formalism (9), but in a logarithmic form,

$$\ln \Phi_B = b_1 \ln \omega + b_2 \ln q_{in} + b_3 \ln q_{out} \tag{11}$$

where the coefficients b_1, b_2, and b_3 can be also interpreted as *weight* parameters. This formalism is very similar to the one used in the analysis of some organizationally complex systems [27], and it is based on the consideration that the component processes are *associative* rather than *additive*. When the three processes involved are considered without discrimination ($b_1 = b_2 = b_3 = b$), equation (11) becomes

$$\Phi_B = (\omega q_{in} q_{out})^b. \tag{12}$$

In particular, for $b = 1/3$, this function is the geometric mean of the three basic performances. In any case, the optimization of the objective function (12) is independent of the parameter b.

We remark that the above formalisms described in subsections 2.1 and 2.2 are mathematical approaches to the problem of optimizing several functions simultaneously. However, when applied to a heat engine, the linear formalism also lies on a physical basis (the energy conservation relation (5)). It is clear that other approaches can be used (remember the above-mentioned freedom of choice of the objective function), but, in our opinion, the two formalisms proposed here are appealing because of their simplicity.

In order to apply the optimization criteria based on equations (10) and (12), we consider the CA model. By choosing the efficiency η ($\equiv\{x\}$) as the independent variable and τ ($\equiv\{\lambda\}$) as the controllable parameter of the system, the *current-efficiency* characteristic of a CA engine is given by [21]

$$\dot{Q}_{in} = A\frac{(1-\tau-\eta)}{(1-\eta)} \tag{13}$$

where A is an η-independent constant. For this engine, one checks that the power output, $\dot{W} = \eta\dot{Q}_{in}$, displays a maximum at the CA efficiency (1), given by $\dot{W}_{mp} = A(1-\sqrt{\tau})^2$. On the other hand, taking into account the fact that the maximum value of the input heat flow (13) occurs at $\eta = 0$ (*short-circuit* point), $(\dot{Q}_{in})_{max} = A(1-\tau)$, and that $\dot{Q}_{out} = (1-\eta)\dot{Q}_{in}$, the objective function (10) for a CA engine becomes

$$\phi_A = \frac{2a\eta(1-\eta-\tau\sqrt{\tau})}{(1-\sqrt{\tau})(1-\tau)(1-\eta)} \tag{14}$$

while the objective function (12) takes the form

$$\phi_B = \left[\frac{\eta^3\tau(1-\tau-\eta)}{(1-\sqrt{\tau})^2(1-\tau)^2(1-\eta)^2}\right]^b. \tag{15}$$

One easily checks that the function (14) displays a maximum ($\mathrm{d}\phi_A/\mathrm{d}\eta = 0$) at

$$\bar{\eta}_A = 1 - \tau^{3/4} \tag{16}$$

Table 1. Data of some European nuclear power plants (PWR (pressurized water reactor), BWR (boiling water reactor), and AGR (advanced gas cooled reactor)).

Plant	T_h (K)	T_c (K)	η_{obs}	η_{CA}	η_{opt}	η_{opt} ($I = 0.8$–0.9)
Doel 4 (nuclear PWR, Belgium)[a]	566	283	0.350	0.293	0.405	0.297–0.357
Almaraz II (nuclear PWR, Spain)[b]	600	290	0.345	0.305	0.420	0.315–0.373
Sizewell B (nuclear PWR, UK)[c]	581	288	0.363	0.296	0.410	0.302–0.361
Cofrentes (nuclear BWR, Spain)[b]	562	289	0.340	0.283	0.393	0.282–0.343
Heysham (nuclear AGR, UK)[c]	727	288	0.4	0.371	0.501	0.410–0.460

[a] Values from [21].
[b] Values provided by the Almaraz and Cofrentes power plants.
[c] Values from [17].

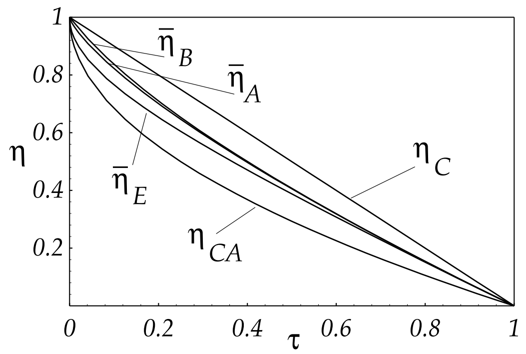

Figure 1. Efficiencies versus τ for a CA heat engine: η_C (Carnot efficiency), η_{CA} (Curzon–Ahlborn efficiency; obtained by maximizing the power output), $\bar{\eta}_E$ (obtained by maximizing the ecological function (4)), $\bar{\eta}_A$ (obtained by maximizing the linear objective function (14)), and $\bar{\eta}_B$ (obtained by maximizing the power-law objective function (15)).

while the function (15) displays a maximum ($d\phi_B/d\eta = 0$) at

$$\bar{\eta}_B = \frac{1}{4}(5 - \tau - \sqrt{(1+\tau)^2 + 12\tau}). \qquad (17)$$

The two optimum efficiencies $\bar{\eta}_A$ and $\bar{\eta}_B$ are plotted in figure 1, together with the Carnot efficiency, $\eta_C = 1 - \tau$, the CA efficiency, $\eta_{CA} = 1 - \sqrt{\tau}$, and the ecological efficiency, $\bar{\eta}_E = 1 - \sqrt{(1+\tau)\tau/2}$. One can observe that both $\bar{\eta}_A$ and $\bar{\eta}_B$ are located between η_{CA} and η_C. Also, although $\bar{\eta}_A < \bar{\eta}_B$, both results are practically indistinguishable, i.e. the application of both the linear and the power-law formalisms to a CA engine leads to practically the same optimum efficiency: $\eta_{opt} = 1 - \tau^{3/4}$. Finally, one also checks that this η_{opt} and $\bar{\eta}_E$ are close, although $\bar{\eta}_E < \eta_{opt}$.

The following remarks concerning equations (16) and (17) should be made. First, the fact that these expressions depend only on the temperature ratio τ is a consequence of assuming a linear law to describe the heat transfer between the working fluid and the external heat reservoirs. Other models where heat transfers obey nonlinear laws can be considered. In these cases the optimum efficiencies obtained from the objective functions (9) and (10) depend on the heat conductances associated with the heat transports. Second, expressions (16) and (17) can also be obtained for other cycles with non-isothermal heat transfer processes, such as the Otto and Brayton cycles, characteristic of motor vehicles and gas turbines, respectively. Third, models including other major irreversibility sources (internal irreversibilities, heat link, etc) can easily be implemented. For instance, if internal irreversibilities associated with the working fluid are considered, from the second law of thermodynamics one has the inequality

$$\frac{\dot{Q}_{in}}{T_h} - \frac{\dot{Q}_{out}}{T_c} < 0. \qquad (18)$$

This inequality can be rewritten in the form

$$\frac{\dot{Q}_{in}}{T_h} - I\frac{\dot{Q}_{out}}{T_c} = 0 \qquad (19)$$

where I is a phenomenological parameter representing the ratio of two entropy differences and measuring the internal irreversibility, so that $0 < I \leqslant 1$ [10, 28]. From equation (19) one has $\dot{Q}_{out}/\dot{Q}_{in} = T_c/(IT_h) = \tau/I$. Therefore, by assuming I is a τ-independent parameter, internal irreversibilities can be taken into account by replacing the temperature ratio τ in equations (16) and (17) by $\tau' = \tau/I$.

3. Some results and conclusions

Let us now consider some current power plants in the light of the optimization criteria proposed here. Since both optimization paths lead to a similar optimum efficiency, we only consider the result (16). Except for geothermal sources, nuclear or solar energy, the creation of a high-temperature reservoir without significant energy loss to the environment is very difficult [17]. Thus, one can assume that the main irreversibility sources for these three kinds of power plants come from finite-rate heat transfer between the external heat reservoirs and the working fluid and from internal irreversibilities. Therefore, the CA engine, including internal irreversibilities and neglecting heat links, seems to be a plausible model to describe the operation of these kinds of plants. In order to illustrate how the proposed optimization works, we consider some European nuclear power plants (see table 1). In all cases the observed values of thermal efficiency η_{obs} are located between the corresponding η_{CA} and $\eta_{opt} = 1 - \tau^{3/4}$. A more realistic approach should take into account the internal irreversibility parameter I. For a typical power plant, using water as working fluid, I takes a value close to 0.8–0.9 [29]. Under such circumstances the values of η_{opt} obtained by substituting τ by $\tau' = \tau/I$ in equation (16) are in good agreement with the observed efficiencies. A more detailed comparison with the reported power plants and a more extensive comparison including fossil-fired power plants should require the consideration of more realistic models incorporating irreversibility losses arising from a heat link between the heat reservoirs and other heat transfer processes between the cycle and the reservoirs. The examples reported here only try to show the plausibility of the proposed optimization methods.

Finally, some interesting data obtained from maximum-power operation and from optimization of the objective functions (9) or (10) can be compared. For instance, for a value of $\tau = 0.5$, the optimization of Φ_A or Φ_B provides a 25% smaller power output than the maximum available power, but with a reduction of 46% in the fuel consumption and a reduction of 55% in the thermal pollution with respect to the maximum-power operation point. The final decision to find the optimal working point needs to take into account many variables, such as the availability of local resources (water, fuels), the need to adapt to the electrical demand of the grid, the investment of capital, and so on. In this sense, the reported objective functions (9) and (10) have limited practical value for designing real power plants, but, in our opinion, they provide simple ways of accounting for environmental impact. We wish to note that the interest of this paper is focused on the proposed optimization methods and not on the CA model, which is used here as an example because it is simple and most people are familiar with it. We also note that the concept of a *saving function* can be useful not only in the FTT analysis of thermal heat engines, but also in other fields, for instance industrial or economical events, when one needs to minimize undesired byproducts.

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