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A Carnot-like irreversible power cycle is analyzed under maximum per-unit-time efficiency conditions. The model includes finite-rate heat transfers between the working fluid and the external heat reservoirs, heat leak between heat reservoirs, and internal dissipations of the working fluid. We present the results for the optimum distribution of the external and internal thermal heat conductances and the optimum area allocation ratio in terms of a dimensionless price parameter, the engine temperature ratio, and the internal irreversibility factor. The calculated optimized efficiencies agree with observed values for real power plants. © *1998 American Institute of Physics.* [S0003-6951(98)02132-9]

It is well known that the main purpose of finite-time thermodynamics is to find the pathway yielding the optimum performance of finite-size devices and finite-time processes.[1] In particular, irreversible Carnot cycles have been optimized with different criteria: the maximization of the power output[2–6] and efficiency,[3,6] the so-called ecological optimization,[7,8] and the minimization of the entropy generation.[9,10]

Recently[11] we have presented a new optimization method based on the maximization of the per-unit-time efficiency (best compromise between operation speed and efficiency). When applied to irreversible Carnot refrigerators[12] the obtained results are consistent with performance data for real low-temperature refrigerators. The main goal of this letter is to investigate the performance of an irreversible Carnot engine under the per-unit-time efficiency criterion. The results generalize those briefly outlined in Ref. 11 for an endoreversible cycle and, in addition, offer new information for the optimum distribution of the heat-exchanger inventory, which together with the tradeoff between the heat-exchanger inventory and the thermal insulation, are two fundamental design aspects in any real heat device.

The theoretical model we consider is based on those reported by Chen,[3] Chen *et al.*,[5] Bejan,[9] and Gordon and Huleihil.[13] Consider the steady-flow (continuous) irreversible Carnot power cycle sketched in Fig. 1, where $|Q_h|$ and $|Q_c|$ are, respectively, the heat supplied per cycle by the hot reservoir at temperature T_h and absorbed by the heat sink at temperature T_c, $|Q_i|$ is the heat leak per cycle between the external sources; W is the work output per cycle; and T'_h $(<T_h)$ and $T'_c(>T_c)$ are, respectively, the temperatures of the working fluid along the upper and the lower isothermal processes. Assuming linear conduction laws for the heat transfers we obtain

$$|Q_h| = \sigma_h(T_h - T'_h)t = \sigma_h T_h(1 - a_h^{-1})t, \tag{1}$$

$$|Q_c| = \sigma_c(T'_c - T_c)t = \sigma_c T_c(a_c - 1)t, \tag{2}$$

$$|Q_i| = \sigma_i(T_h - T_c)t = \sigma_i T_h(1 - \tau)t, \tag{3}$$

where $a_h = T_h/T'_h \geq 1$, $a_c = T'_c/T_c \geq 1$, $\tau = T_c/T_h \leq 1$; σ_h and σ_c are, respectively, the *external* hot-end (steam boiler) and cold-end (condenser) thermal conductances; σ_i is the *internal* heat conductance, and t is the overall cycle time. The internal irreversibilities of the working fluid are taken into account by the Clausius inequality as[3,4]

$$\frac{|Q_h|}{T'_h} = I\,\frac{|Q_c|}{T'_c}\ (0 < I \leq 1), \tag{4}$$

so when $I = 1$ and $\sigma_i = 0$ the heat engine is endoreversible. Using Eqs. (1), (2), and (4) we obtain

$$a_c = \frac{I}{I - \sigma_{hc}(a_h - 1)}, \tag{5}$$

where $\sigma_{hc} = \sigma_h/\sigma_c$. With this equation the net power output $\dot{W}$ and the efficiency η become

$$\dot{W} = \sigma_h T_h\,\frac{I(a_h - 1) - \sigma_{hc}(a_h - 1)^2 - \tau(a_h^2 - a_h)}{a_h(I + \sigma_{hc}) - \sigma_{hc}a_h^2}, \tag{6}$$

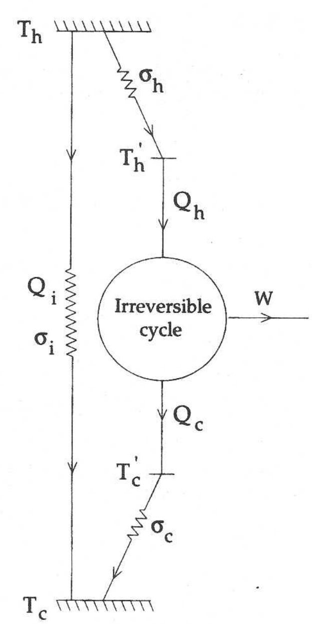

FIG. 1. Schematic diagram of the considered irreversible Carnot cycle.

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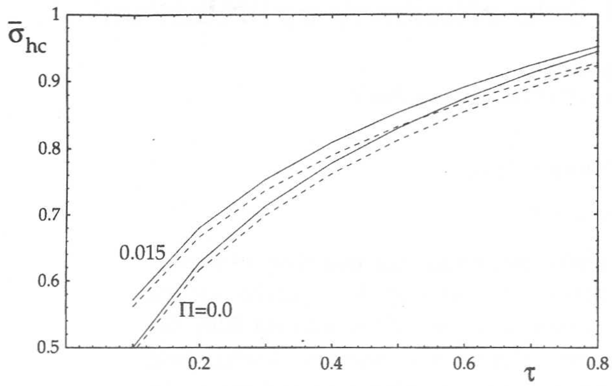

FIG. 2. Optimum external heat conductance ratio $\bar{\sigma}_{hc}$ vs τ for the indicated Π values with $I=1$ (solid lines) and $I=0.9$ (dashed lines).

$$\eta=\left(1-\frac{a_h\tau}{I-\sigma_{hc}(a_h-1)}\right)\left(\frac{a_h-1}{a_h-1+\sigma_{ih}a_h(1-\tau)}\right), \tag{7}$$

where $\sigma_{ih}=\sigma_i/\sigma_h$. Taking into account that $|Q_h|\propto T'_h$ and $|Q_c|\propto T'_c$, we obtain from Eqs. (1), (2), and (4)

$$t\propto\frac{1}{\sigma_h(a_h-1)}=\frac{a_c}{\sigma_c(a_c-1)}, \tag{8}$$

and the per-unit-time efficiency $\eta/t\equiv\dot{\eta}$ reads as

$$\dot{\eta}\propto\left(1-\frac{a_h\tau}{I-\sigma_{hc}(a_h-1)}\right)\left(\frac{\sigma_h(a_h-1)^2}{a_h-1+\sigma_{ih}a_h(1-\tau)}\right). \tag{9}$$

Setting the first derivative of the above equation with respect to a_h equal to zero, $(\partial\dot{\eta}/\partial a_h)_{a_h=\bar{a}_h}=0$, we obtain the $\bar{a}_h$ values giving maximum per-unit-time efficiency.

In the particular case of endoreversible performance ($I=1$ and $\sigma_{ih}=0$) the solution is

$$\bar{a}_h=1+\frac{1}{\sigma_{hc}}-\frac{1}{\sigma_{hc}}\sqrt{\frac{(1+\sigma_{hc})\tau}{\tau+\sigma_{hc}}}, \tag{10}$$

which substituted in Eq. (7) leads to

$$\begin{aligned}\bar{\eta}(\sigma_{hc},\tau)&\equiv\eta(\bar{a}_h,\sigma_{hc},\sigma_{ih}=0,\tau,I=1)\\ &=1+\frac{\tau}{\sigma_{hc}}-\frac{\sqrt{(1+\sigma_{hc})(\sigma_{hc}+\tau)\tau}}{\sigma_{hc}}<1-\sqrt{\tau}\end{aligned} \tag{11}$$

for the efficiency of an endoreversible heat engine operating

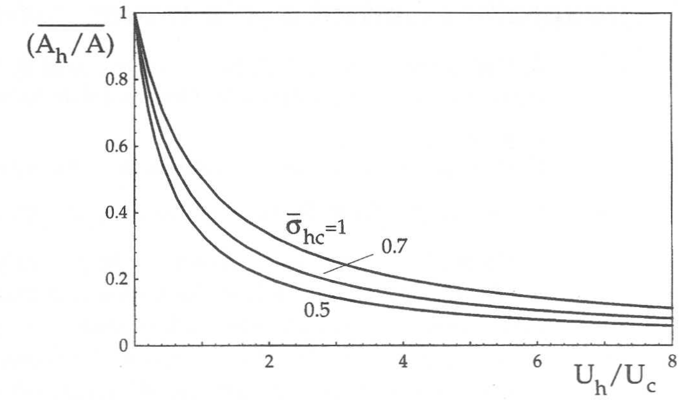

FIG. 4. Optimum area allocation ratio $\overline{A_h/A}$ vs U_h/U_c for the indicated $\bar{\sigma}_{hc}$ values.

under conditions of maximum per-unit-time efficiency. The function defined in Eq. (11) is a monotone increasing function with σ_{hc} and limit values $(1-\tau)/2$ for $\sigma_{hc}\to 0$ and $1-\sqrt{\tau}$ for $\sigma_{hc}\to\infty$. Thus, the Curzon–Ahlborn efficiency,[2] $\eta_{CA}=1-\sqrt{\tau}$, is an upper bound for $\bar{\eta}(\sigma_{hc},\tau)$. The differences between $\bar{\eta}(\sigma_{hc},\tau)$ and η_{CA} come from the use of distinct optimization criteria: the CA efficiency is obtained under conditions of maximum power and $\bar{\eta}(\sigma_{hc},\tau)$ under maximum per-unit-time efficiency.

In general, when $I\leq 1$ and $\sigma_{ih}\neq 0$, the maximum per-unit-time efficiency solution must be investigated numerically. The factor I can be considered as a fixed parameter but the heat conductances may vary with the power plant design. In this regard Bejan[9] noted two main technical aspects: the optimal allocation of the external heat conductances and the tradeoff between heat exchanger inventory and thermal insulation. Denoting by $\sigma=\sigma_h+\sigma_c$ the overall external heat conductance and by $R_i=1/\sigma_i$ the overall thermal resistance between heat reservoirs, this author assumed that in the overall cost of the power plant these two features compete against one another in a constraint of the type $\sigma+P_rR_i/P_\sigma=C$, where C is the total thermal conductance that could be built with the fixed resources available for σ and R_i. P_r and P_σ denote, respectively, the unit cost associated with adding more insulation thermal resistance and more thermal conductance. This author also defines an investment allocation ratio,

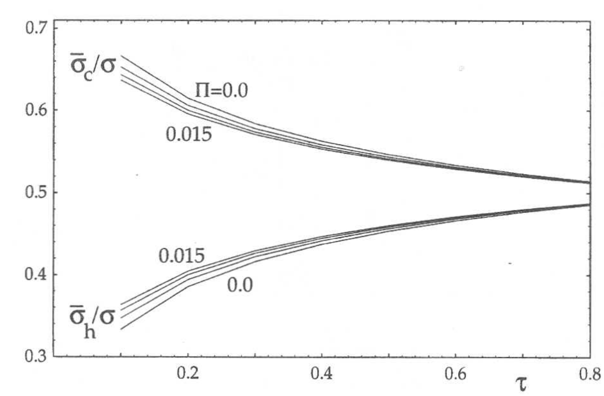

FIG. 3. Optimum external heat conductance distributions $\bar{\sigma}_h/\sigma$ and $\bar{\sigma}_c/\sigma$ vs τ for the indicated Π values with $I=1$. The intermediate curves are for $\Pi=0.005$ and $\Pi=0.01$.

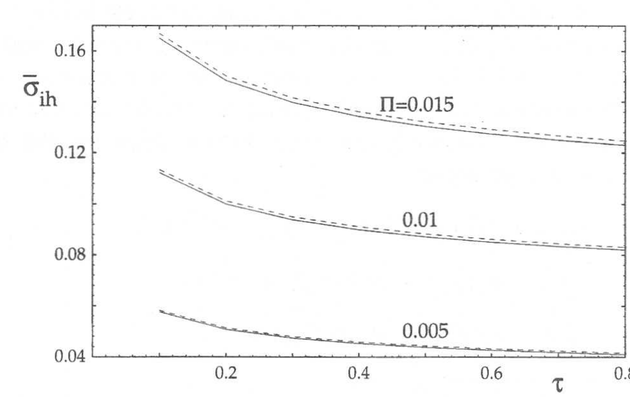

FIG. 5. Optimum heat conductance distribution $\bar{\sigma}_{ih}$ vs τ for the indicated Π values with $I=1$ (solid lines) and $I=0.9$ (dashed lines).

z, as $z=\sigma/C=1-(P_rR_i/CP_\sigma)$. With these assumptions we obtain

$$\sigma_h=\frac{zC\sigma_{hc}}{1+\sigma_{hc}}, \quad \sigma_{ih}=\Pi\,\frac{1+\sigma_{hc}^{-1}}{z(1-z)}, \tag{12}$$

where Π is a dimensionless price ratio $\Pi=P_r/(P_\sigma C^2)$.

With Eq. (12) the new design parameters, σ_{hc} and z, (remaining τ, I and Π fixed) are calculated here by maximizing once more the per-unit-time efficiency. Thus, the optimized values $\bar{\sigma}_{hc}$ and $\bar{z}$ are given by

$$\left(\frac{\partial\bar{\eta}}{\partial\sigma_{hc}}\right)_{\sigma_{hc}=\bar{\sigma}_{hc}}=0; \quad \left(\frac{\partial\bar{\eta}}{\partial z}\right)_{z=\bar{z}}=0. \tag{13}$$

Numerical solutions of Eqs. (13) show that the optimum investment allocation ratio takes the value $\bar{z}=0.5$ for all considered values of τ, Π, and I. The same result for $\bar{z}$ has been obtained previously by Bejan using a very similar model but optimizing the efficiency under maximum power conditions.[9] Figure 2 shows the optimum external heat conductance ratio $\bar{\sigma}_{hc}$ vs τ for different Π and I values. It is seen that: (a) $\bar{\sigma}_{hc}$ is smaller than unity, i.e., the condenser heat conductance is always greater than the boiler heat conductance, in agreement with results reported by Cheng and Chen[8] for an irreversible Carnot heat engine optimized under the ecological criterion; (b) $\bar{\sigma}_{hc}$ increases with Π and τ, and (c) for all Π values and for the highest τ values $\bar{\sigma}_{hc}$ tends toward unity. This particular limit ($\bar{\sigma}_{hc}\rightarrow 1$ for $\tau\rightarrow 1$) is the well-known result of the optimal thermal allocation fraction in the Curzon–Ahlborn maximum power cycle.[9] An increase of the internal irreversibilities (smaller I values) just provokes a slight decrease of $\bar{\sigma}_{hc}$, specially for high τ values.

In Fig. 3 we plot the optimal external heat conductance distribution $\bar{\sigma}_h/\sigma=\bar{\sigma}_{hc}/[\bar{\sigma}_{hc}+1]$ [see Eq. (12)] and $\bar{\sigma}_c/\sigma=1-\bar{\sigma}_h/\sigma$ vs τ for some values of Π and $I=1$ (smaller I values give very similar curves). These functions have been also analyzed by Ait-Ali[6] for a Carnot cycle model where the internal production rate of entropy was assumed to be τ dependent and optimized under maximum power and maximum efficiency conditions. Comparison between our results in Fig. 3 and those in Ref. 6 under maximum power conditions for $\tau\geqslant 0.4$ (see Fig. 5 in Ref. 6) shows a similar τ dependence and a common asymptotic tendency to 0.5 at high τ values (equivalent to the limit $\bar{\sigma}_{hc}\rightarrow 1$). Nevertheless, if we compare them with those in Ref. 6 obtained under maximum efficiency conditions (see Fig. 6 in Ref. 6) the differences are substantial for all τ values.

Information about an optimal area allocation ratio can be also obtained. From the identity $\sigma_{hc}=U_hA_h/U_cA_c$, where $A_h(A_c)$ is the hot-end (cold-end) heat transfer area and $U_h(U_c)$ is the corresponding overall heat transfer coefficient, the optimized ratio A_h/A (with $A=A_h+A_c$) becomes $A_h/A=\bar{\sigma}_{hc}/[\bar{\sigma}_{hc}+(U_h/U_c)]$. This function is plotted in Fig. 4 versus U_h/U_c for different values of $\bar{\sigma}_{hc}$. Note that the optimal area ratio increases with $\bar{\sigma}_{hc}$ (i.e., when either the price parameter or the temperature ratio increases) up to the corresponding maximum power value $\bar{\sigma}_{hc}=1$.

Another design parameter is the heat conductance distribution σ_{ih} given by Eq. (12). For $\bar{z}=0.5$ we obtain $\bar{\sigma}_{ih}=4\Pi(\bar{\sigma}_{hc}+1)/\bar{\sigma}_{hc}$. This function, plotted in Fig. 5, decreases as τ increases, shows a sharp increase with the price parameter, and it is practically I independent. For instance, at $\tau=0.4$ (a representative value in real power plants) and $\Pi=0.005$, the ratio $\bar{\sigma}_{ih}$ amounts above 4%, whereas if Π increases to 0.015 this ratio increases up to about 15%.

Figure 6 shows the optimal efficiency $\eta_{opt}(\Pi,\tau,I)=\bar{\eta}(\bar{\sigma}_{hc},\bar{z},\Pi,\tau,I)$ vs τ for some representative Π values and $I=1$ and $I=0.9$. For the sake of comparison we have also plotted the Carnot and Curzon–Ahlborn upper limits and the observed values of ten power plants.[14] The observed values fall between the curves corresponding to $\Pi=0$ and $\Pi=0.015$. This range of Π values agrees with the results by Bejan[9] for the efficiency under maximum power conditions. Finally, note that the expected decrease of η_{opt} with I is specially important for the higher τ values.

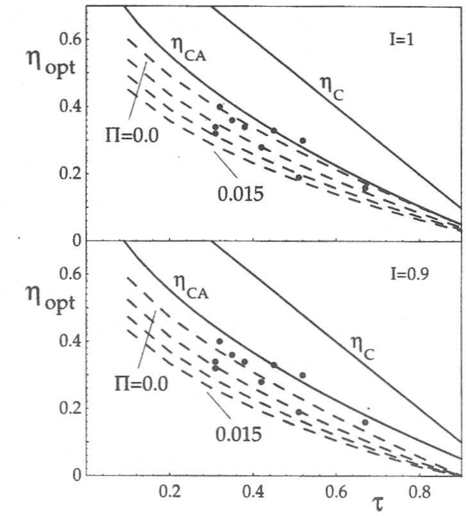

FIG. 6. Points: observed values of the efficiency for ten power plants. Solid lines: Carnot, $\eta_C=1-\tau$, and Curzon–Ahlborn, $\eta_{CA}=1-\sqrt{\tau}$, efficiencies. Dashed lines: optimized efficiencies for the indicated Π values with $I=1$ and $I=0.9$. The intermediate dashed curves are for $\Pi=0.005$ and $\Pi=0.01$.

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