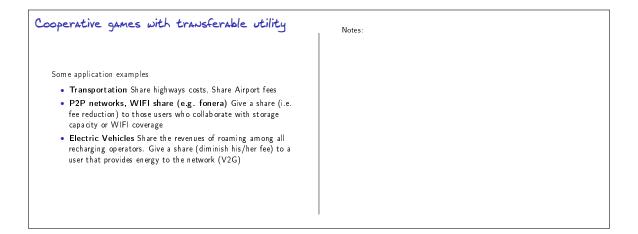
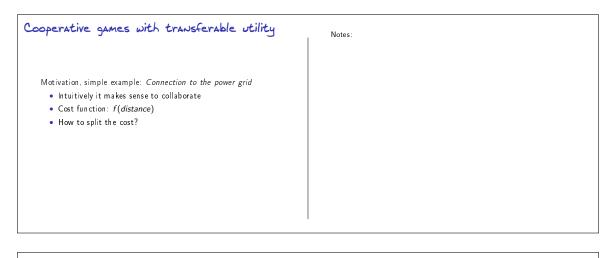




- the term cooperative may sometimes be misleading, it does not mean that we are modeling competing or non competing situations; it can model both
- coalitional game theory its a synonym
- the term coalitional means that the unit we are modeling are coalitions (i.e. groups of players) we are not modeling player's action but groups of players' actions.
- the problem we care about is how to split the revenue/cost resulting from the interaction, among the players







Notes:

- How is the revenue of the grand coalition split among its members? \rightarrow revenue/cost sharing
- Is it interesting for a player to take part of a coalition? \rightarrow stability

Some solution concepts: the core, the nucleolus, the $\tau\text{-value},$ the Shapley value

- Establish revenue/cost shares
- Provide different properties

Sharing Rules are Diverse and Provide Different Shares

Example: the contested garment.

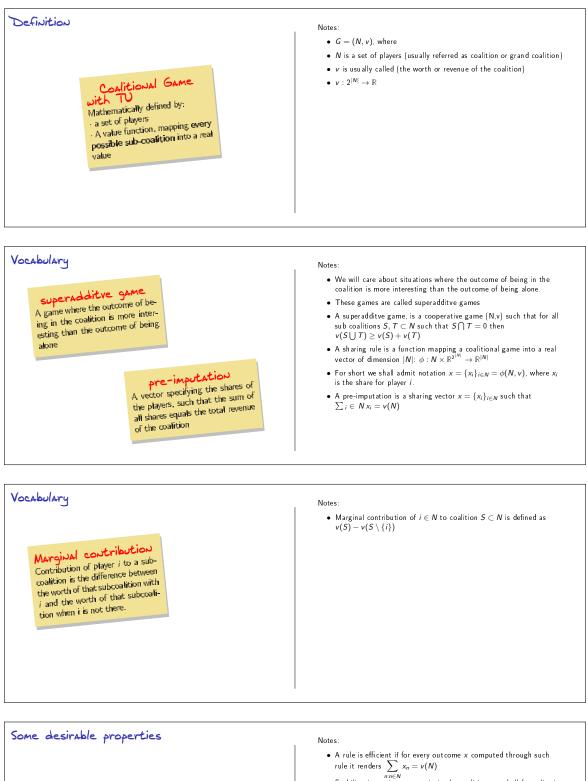
- 2 persons A, B. One piece of fabric
- A wants half of the piece, B wants the whole piece

Which is the fairest way to share it?

- ldeas:
 - Proportional to the demand
 - Answer provided in the Talmud
 - GT?

- The Talmud (central text of Rabbinic Judaism) provides the answer to both examples, without explaining how the calculations are done. For 2k years mathematicians and economist had not found such explanation. Game theory does.
- Proportional rule yields share_A = $\frac{1/2}{1/2+1} = 1/3$, share_B = $\frac{1}{1/2+1} = 2/3$
- the Talmud law: A wants only half of it, thus only the other half of the fabric is contested. Share in equal parts the contested piece. share_A = $\frac{1/2}{2} = 1/4$, share_B = $1/2 + \frac{1/2}{2} = 3/4$
- we will model this situation using GT

Another Talmud example	Notes:
The bankrupt problem: One man dies with a wealth e and three debts d_1 , d_2 , d_3 . How should e be split among the 3 debts claimers? $\begin{array}{c c c c c c c c c c c c c c c c c c c $	 Game theory also allows to explain this result Would you say is the same rule as the garment?

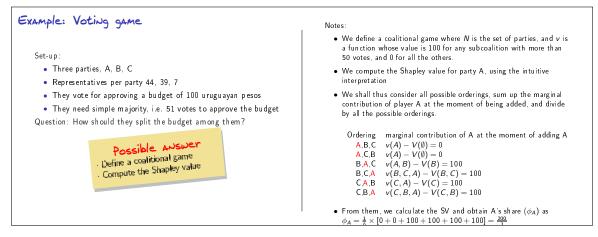


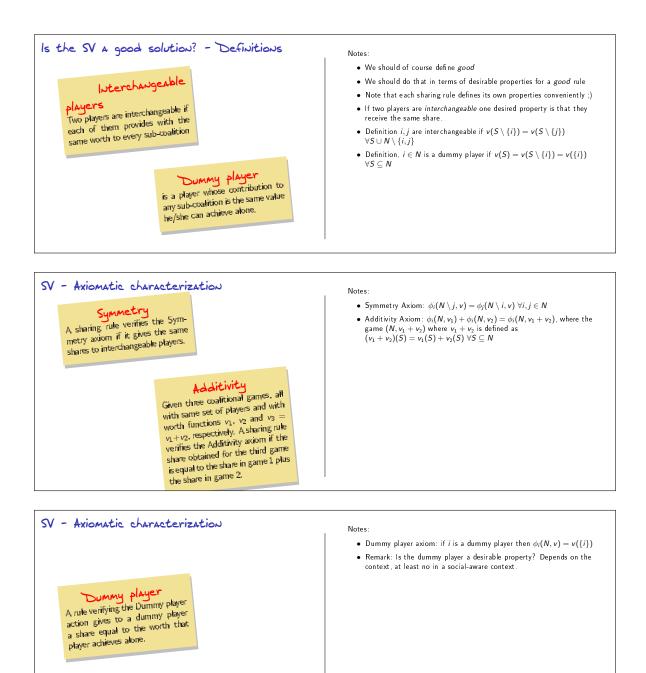
- Efficiency
- Stability • Fairness
- No free riders
- Monotonicity Resource-
 - Population-

- Stability: incentives to remain in the coalition, we shall formalize it later on
- Fairness: not quite a consensus in the literature. Eg. that who contributes the most receives the most.
- Resource monotonicity: provide the right incentives to members to contribute to increase/decrease the revenue/cost of the coalition
- Many flavors of resource-monotonicity exist
 A population-monotonic revenue sharing rule guarantees that the
- entrance of a new member to the alliance does not reduce the revenue of each of the members already there

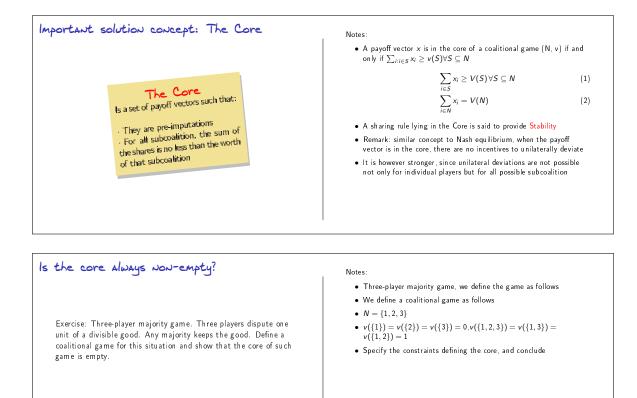
The Shapley Value - Intuition	 Notes: Photo Lloyd Shapley, Taken in 1980 by Konrad Jacobs, Erlangen, Copyright is with MFO, source Mathematisches Institut Oberwolfach (MFO), http://owpdb.mfo.de/detail?photoID=3808 One of the most well known sharing rule is the Shapley value It was proposed by Lloyd Shapley in 1953 [10]. Intuitively, its idea is to share the worth proportionally to the contribution of each player
 Example: Consider 2 players, v(1) = 1, v(2) = 1, v({1,2}) = 3. Then, each player's contribution is equal to 2 units 	 Notes: Is evident that we cant share the 3 units giving 2 units to each of them! The idea behind Shapley value is to share proportionally to each player's contribution to every subcoalition, but weighting the contribution.
Shapley Value - Definition	Notes:

Shapley value CSV The SV for player *i* is the "average marginal contribution" of player *i* is the "average by dividing by all the possible orderings for forming the grand coalition (i.e. by |N|!). • Formally, Given a cooperative game (N,v) the SV for player $i \in N$ is given by $\phi_i(N, v) = \frac{1}{|N|!} \sum_{S \subset N \setminus i} |S|!(|N| - |S| - 1)![v(S \cup \{i\}) - v(S)].$



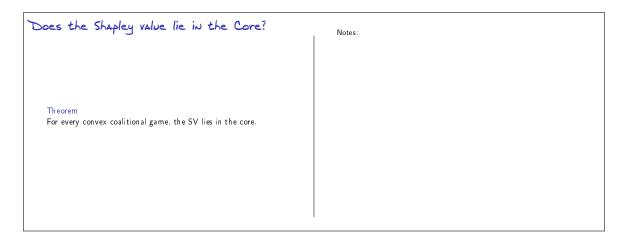


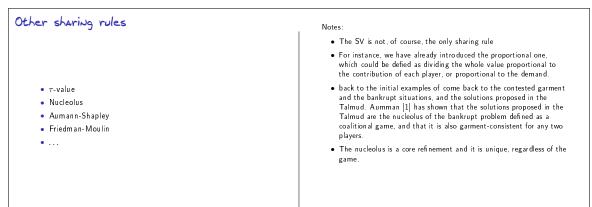
l, axiomatic characterization	Notes:
Theorem There is only one sharing rule that verifies all the three previously stated axioms, and that rule is the SV.	• So SV is only one possible sharing rule, we will see later on others
	• Lets first address the question of which coalition will be formed.
	• For that we are going to discuss about Stability
	 The idea is that the grand coalition will remain stable if there are n incentives to form other sub coalitions.
	• For that we will introduce the definition of the <i>core</i> .



The core in convex games Notes: Theorem The core of a convex coalitional game is always non empty.

Does the core determines a unique sharing vector?	Notes: • Define a coalitional game • Determine the core
Example: Two people produce together one unit, which they may share in any way they wish. If they are alone each produces zero units. Each person cares only about the amount of output she/he receives, preferring more to less.	





Example: Multicast Tree

- A group of customers must be connected to a service provided by some central facility
- a customer must either be directly connected to the facility or be connected to some other connected customer.
- We can model the customers and the facility as nodes on a graph, and the possible connections as edges with associated costs.
- Problem can be modeled as a coalitional game (N, v) .
- N is the set of customers, and v(S) is the cost of connecting all customers in S directly to the facility minus the cost of the minimum spanning tree that spans both the customers in S and the facility.

The Nash Bargaining Solution

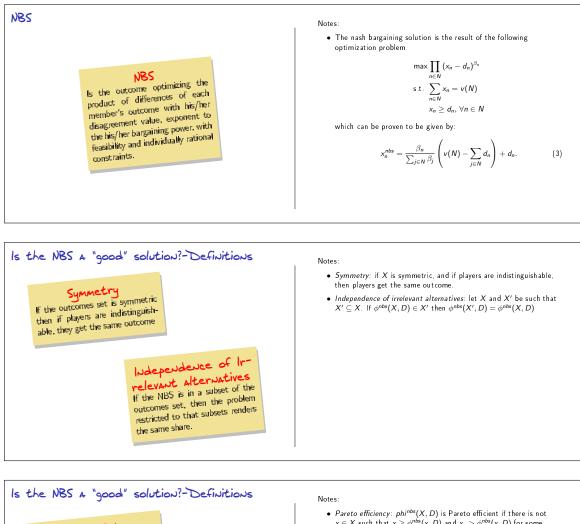
Defin e:

- a compact and convex set of all possible outcomes
- model each member's bargaining power (weight for negotiating)
- a disagreement point (outcome when there is no agreement)
- each member's utility function, i.e. their preferences over the set of possible outcomes.
- Assume there exits within the set of possible outcomes an outcome "suitable" for every member

Notes:

• Remark: the definition of the game implies that different solutions can verify different properties

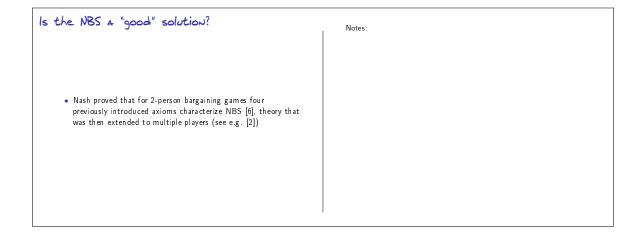
- X set of possible outcomes
- β_n will note the bargaining power of $n \in N$
- Let us assume utility functions are linear $u_n=x_n \text{ for all } n\in N$ where $x=\{x_n\}_N\in X$
- d is the disagreement point $d \in D \subset \mathbb{R}^{|N|}$
- we assume X is such that $\exists x \in X$ such that $x_n > d_n$, for all $n \in N$, where d_n is n's disagreement point.
- examples of bargaining powers: contribution to the coalitionexample of disagreement point: the stand alone revenue of each
- player, i.e. $d_n = v(\{n\})$, for all $n \in N$.
- The total amount to share is given by $\nu(N),$ and bounds the set of possible outcomes.





luvariance to equivalent utility a transformation of the utility functions that maintains the same ordering over preferences renders the same outcome

- Pareto efficiency: $phi^{nbs}(X, D)$ is Pareto efficient if there is not $x \in X$ such that $x \ge \phi^{nbs}(x, D)$ and $x_n > \phi_n^{nbs}(x, D)$ for some $n \in N$.
- Invariance to equivalent utility representations: e.g. of transformation of the utility functions that maintains the same ordering over preferences: a linear transformation



tgenda	NL .
1 Introduction	Notes:
Examples	
Basic assumptions	
Course organization	
2 Strategic Games	
Definition and First solution concept	
Solution concept: NE in Mixed Strategies	
3 Games in Extensive form with perfect information	
4 Pricing	
Discussion: Pricing in the Internet	
6 Games with incomplete Information	
Bayesian Games	
Auctions	
Case Study: Adwords Auctions	
6 Cooperative Game Theory	
7 Conclusion	
8 Acknowledgments	

Further Types of GAMES

- Non-atomic games Large number of players
 The individual effect of a player in the outcome is negligible but not the one of a portion of players • Repeated games • Game is played several times, history is known • Potential games • Utilities can be expressed through a common function
- •

To SUM UP

Notes:

Notes:

- Game theory provides tools for analysing situations where multiple decisions makers interact
- Non-cooperative game theory
 - Study choices of rational selfish players
 Nash Equilibrium
 - helps predict the possible rational outcomes of a game
- Cooperative game theory (with TU)
- How to split costs/revenue of a coalition
 The core set, stability
- A correct modeling is very important for coherent results
- Beyond Game Theory: evolutionary game theory

iGracias!

Agenda Notes: 2 Strategic Gam 3 Games in Extensive form with perfect information 4 Pricing **5** Games with incomplete Information 6 Cooperative Game Theory 8 Acknowledgments

knowledgments	Notes:
Material for these slides have been taken from the following books	
 An introduction to game theory [8] 	
• Multiagent Systems [11]	
• Auction Theory [4]	
 Pricing in communication networks [3] 	
 Telecommunication Networks Economics [5] 	
and from content from the following courses:	
 Coursera online course: Game Theory https://www.coursera.org/learn/game-theory-1/ 	
 Felix-Munoz Garcia: Strategy and Game Theory, Washington State University 	
 Bruno Tuffin and Patrick Maillé: Game Theory and Applications, IMT, INRIA, France 	

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<i>ceter</i>	rences III	Notes:
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[11]	Yoav Shoham and Kevin Leyton-Brown. Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations. Cambridge University Press, Cambridge, UK, 2009.	
[12]	W. Vickrey. Counterspeculation, Auctions, and Competitive Sealed Tenders. The Journal of Finance, 16(1):8–37, 1961.	

Notes:

Quizz Lecture 5

The questions proposed here are taken from: the MOOC Game Theory on Coursera platform, created by Matthew Jackson, Kevin Leyton-Brown and Yoav Shoham.

Exercise 1 Suppose N = 3 and v(1) = v(2) = v(3) = 1. Which of the following payoff functions is superadditive?

- a. v(1,2)=3, v(1,3)=4, v(2,3)=5, v(1,2,3)=5;
- b. v(1,2)=3, v(1,3)=4, v(2,3)=5, v(1,2,3)=7;
- c. v(1,2)=0, v(1,3)=4, v(2,3)=5, v(1,2,3)=7;
- d. None of the above.

Exercise 2 Suppose N=2 and v(1)=0, v(2)=2, v(1,2)=2. What is the Shapley Value of both players?

- a. $\phi_1(N, v) = 1, \ \phi_2(N, v) = 0$
- b. $\phi_1(N,v) = 1/2, \ \phi_2(N,v) = 1/2$
- c. $\phi_1(N,v) = 1/3, \ \phi_2(N,v) = 2/3$
- d. $\phi_1(N,v) = 0, \ \phi_2(N,v) = 2$

Exercise 3 Suppose N=3 and v(1)=v(2)=v(3)=0, v(1,2)=v(2,3)=v(3,1)=2/3, v(1,2,3)=1.

Which allocation is in the core of this coalitional game?

- a. (0,0,0);
- b. (1/3, 1/3, 0);
- c. (1/3, 1/3, 1/3);
- d. non of the above