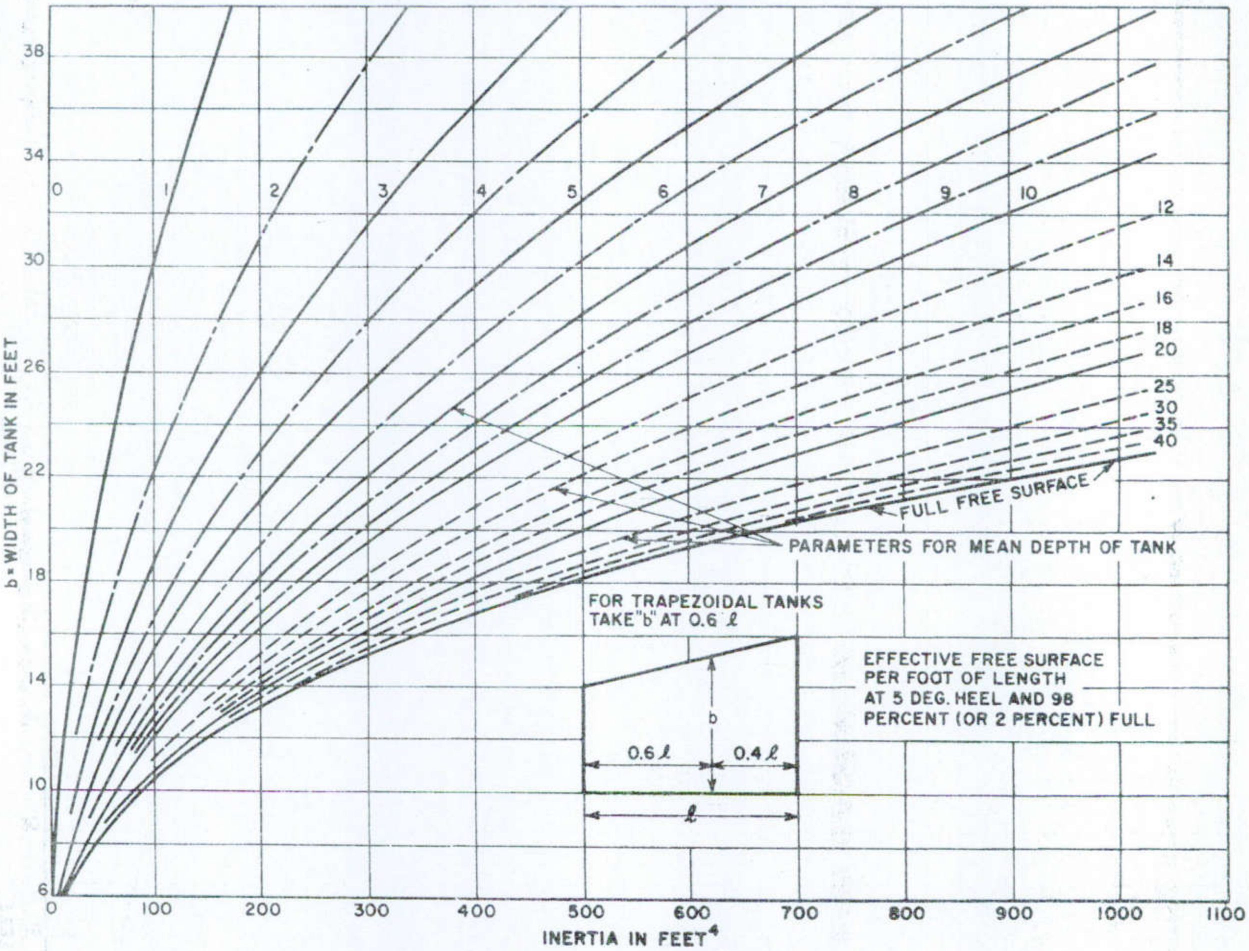


Fig. 30 Effective free surface at 5 deg heel

the depth/breadth ratio. If the variation in depth is large, or if the tank is not approximately rectangular in section, the depth may be taken as that of a rectangular tank having the same ullage and therefore very nearly the same movement of liquid. For example, if the surface of the liquid is 0.5 ft from the top of the tank, the movement of the liquid at the smaller angles of heel would be very nearly the same as in a rectangular tank, 95 percent full, having the same breadth and a depth of 10 ft.

5 Sufficiently accurate results may usually be obtained without interpolating in the tables. If the value of depth/breadth falls between two in the table, slightly pessimistic results will be obtained if the next higher value of depth/breadth in the table is used. This will not only eliminate interpolating, but may also permit grouping a number of tanks having very nearly the same depth/breadth ratio so that the values of i_T/δ may be added before the coefficients in the table are applied.

While the foregoing approximations will give satisfactory results for the normal ship with tanks of normal size and shape, they should not be applied indiscriminately. For some unusual craft, these approximations may be unsuitable. The only rigorous method of evaluating free-surface effect is by calculating the actual moment of transference for the liquid in each slack tank for several angles of inclination.

5.4 Adjustment of Metacentric Height for Free Surface. The present legal requirements for adequate stability result in the determination of a "minimum required $\overline{GM}$," based on the heeling moments, which is compared to the ship's "available $\overline{GM}$." The practice of using metacentric height as the index of stability has been adopted to avoid the necessity for more elaborate calculations, such as those involved in preparing cross curves, and is based on the assumption that adequate metacentric height, in conjunction with adequate freeboard, will assure adequate righting moments.

Most merchant ships have several wide double-bottom tanks containing fuel oil, of which all but one, or one pair, will be assumed to be 98 percent full. In such cases, if the effect of free liquid on metacentric height were to be evaluated by assuming that the center of gravity of the liquid in each of the tanks were at its metacenter, a gross exaggeration of the loss of righting moment would be obtained at angles beyond 1 or 2 deg.

The practice which has been adopted to produce a more reasonable value for the free-surface effect is to determine the effect of free liquid on the righting arm at an arbitrarily selected angle of 5 deg, and translate the reduction in righting arm at 5 deg into a reduction in metacentric height by dividing it by the sine of 5 deg. The effect of this assumption is to produce a value of metacentric height, adjusted for free liquid, which, when multiplied by the weight of the ship and the sine of the angle of inclination, will give, very nearly, the correct value for the righting moment at 5 deg inclination.

To facilitate application of this assumption, Fig. 30 may be entered with the breadth and depth of a tank which is 98 percent full, and a value of moment of inertia read which may be used in place of the actual moment of inertia of the surface of the liquid. If this is done for all tanks which are 98 percent full, and the actual moment of inertia used for those tanks which are half full, the adjusted value of metacentric height will give, very nearly, the correct righting moment at 5 deg inclination.

The value of the moment of inertia read from Fig. 30 is not the moment of inertia of any actual free surface, but is the moment of inertia of the surface in an imaginary tank which would have the same effect on the righting arm at 5 deg as the tank being considered, if the depth of the imaginary tank were great enough so that the surface of the liquid would not reach the top or bottom of the tank at 5 deg heel.

As an example, consider a tank 20 ft wide and 5 ft deep, 98 percent full, and heeled to 5 deg. When the ship is upright, the center of gravity of the liquid would be on the centerline of the tank, 2.450 ft from the bottom. When the ship is heeled to 5 deg, the center of gravity of the liquid is 0.158 ft from the centerline and 2.453 ft from the bottom. The shift of the center of gravity parallel to the waterline at 5 deg heel is

$$0.158 \cos 5^\circ + (2.453 - 2.450) \sin 5^\circ$$

or 0.157 ft. If the length of the tank is l and the density of the liquid in cubic feet per ton is δ, the shift of the center of gravity of the liquid parallel to the waterline would move the ship's center of gravity a distance

$$0.157 \times \frac{w}{W}$$

or

$$\frac{0.157 \times 0.98 \times 5 \times 20 \times l}{\delta W}$$

or

$$\frac{15.4\, l}{\delta \cdot W}$$

parallel to the waterline, reducing the righting arm by that amount. Moving the ship's center of gravity this distance parallel to the waterline at 5 deg inclination has the same effect on righting arm as moving it upward parallel to the ship's centerline a distance of

$$\frac{15.4\, l}{\delta \cdot W \cdot \sin 5^\circ} \quad \text{or} \quad \frac{176\, l}{\delta \cdot W}$$

reducing the metacentric height by that amount. Since the reduction in metacentric height produced by the liquid in the imaginary tank in which the surface does not reach the top or bottom would be $i_T/\delta W$, then,

$$\frac{i_T}{\delta W} = \frac{176\, l}{\delta W}$$

or

$$i_T = 176\, l$$

This value of moment of inertia per foot of length may be read from Fig. 30 for a tank having a depth of 5 ft and a width of 20 ft.

When this adjustment to metacentric height for tanks 98 percent full is used to determine whether stability is adequate, it should be realized that the righting moment obtained from the expression $W \cdot GM \sin \phi$ is valid only at 5 deg inclination. For smaller angles, the actual righting moments will be less than indicated, and for greater angles, they will be greater. The fact that this formula produces excessive righting moments at angles less than 5 deg is not important, since the ship is in no danger if the angle of heel is less than 5 deg. When the upsetting moment is sufficient to heel the ship beyond 5 deg, however, it may be profitable to make a more accurate evaluation of righting moment by the use of cross curves with adjustments to righting arm for the effect of free

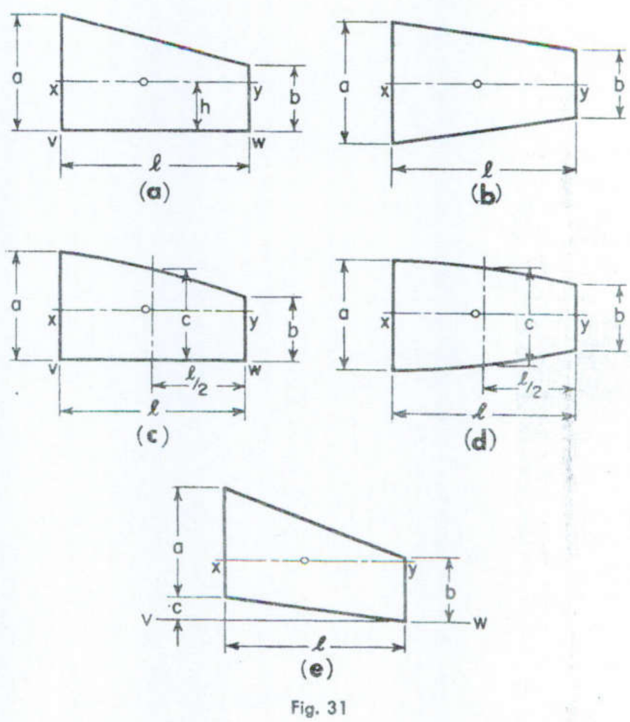


Fig. 31

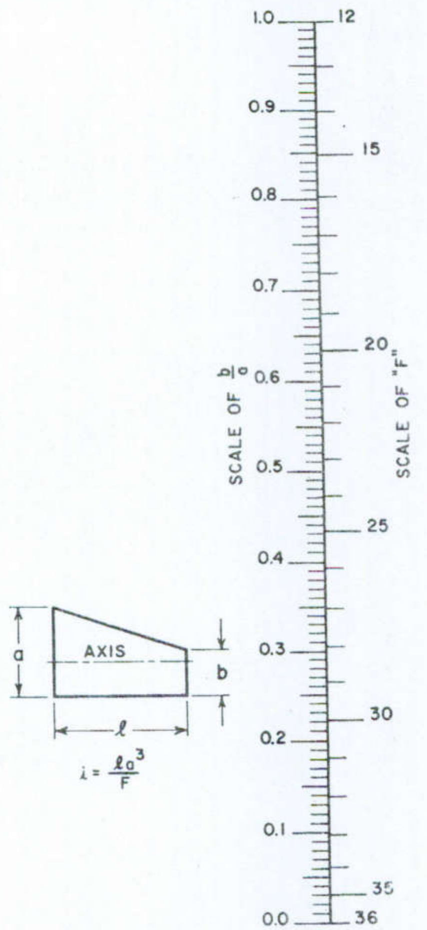


Fig. 32

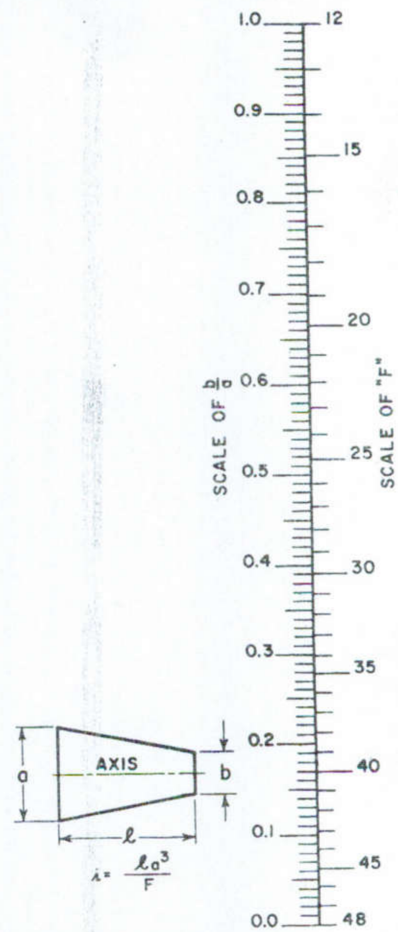


Fig. 33

liquid at larger angles of heel to avoid too greatly underestimating the ship's stability.

The empirical, and, in fact, illogical nature of this procedure is emphasized. It is currently in use only because the calculation of the actual reduction in righting arms resulting from free surface is too laborious to be practicable. In view of the study of the subject under international auspices which is under way at this writing, and of the increasing application of computers to such problems, it seems probable that in the not too distant future, a more logical and more accurate approach may become acceptable both to the regulatory authorities and to the designer.

5.5 Determination of Moment of Inertia of Free Surface. The process of determining the moment of inertia of the surface in a rectangular tank is relatively simple, since the moment of inertia about an axis through the centroid of the surface is given by

$$i_T = \frac{lb^3}{12}$$

Many tanks, however, are trapezoidal in plan view, while others may have one or more curved sides; in such cases the process is more complex. Several cases are illustrated in Fig. 31. In each case, the centroid is indicated by a small circle, and xy is the fore-and-aft axis through the centroid to which the moment of inertia is referred. In cases (b) and (d), the figure is symmetrical about the xy-axis; in the others, the line vw is parallel to the centerline of the ship. In each case, a is the width at the wider end and b the width at the narrower end. In cases (c) and (d), c is the width at the midpoint of the length.

The basic method for finding the moment of inertia in case (a) is to calculate the moment of inertia, i_{vw}, with vw as the axis, the distance h of the centroid from vw and the area A. Then, the moment of inertia about an axis through the centroid, i_{xy}, can be found by

$$i_{xy} = i_{vw} - Ah^2$$

The moment of inertia of this figure about the axis through the centroid may be expressed in terms of the length and the width at each end, as follows:

$$i_{xy} = \frac{l}{36}\,\frac{(a^4 + 2a^3b + 2ab^3 + b^4)}{a + b}$$

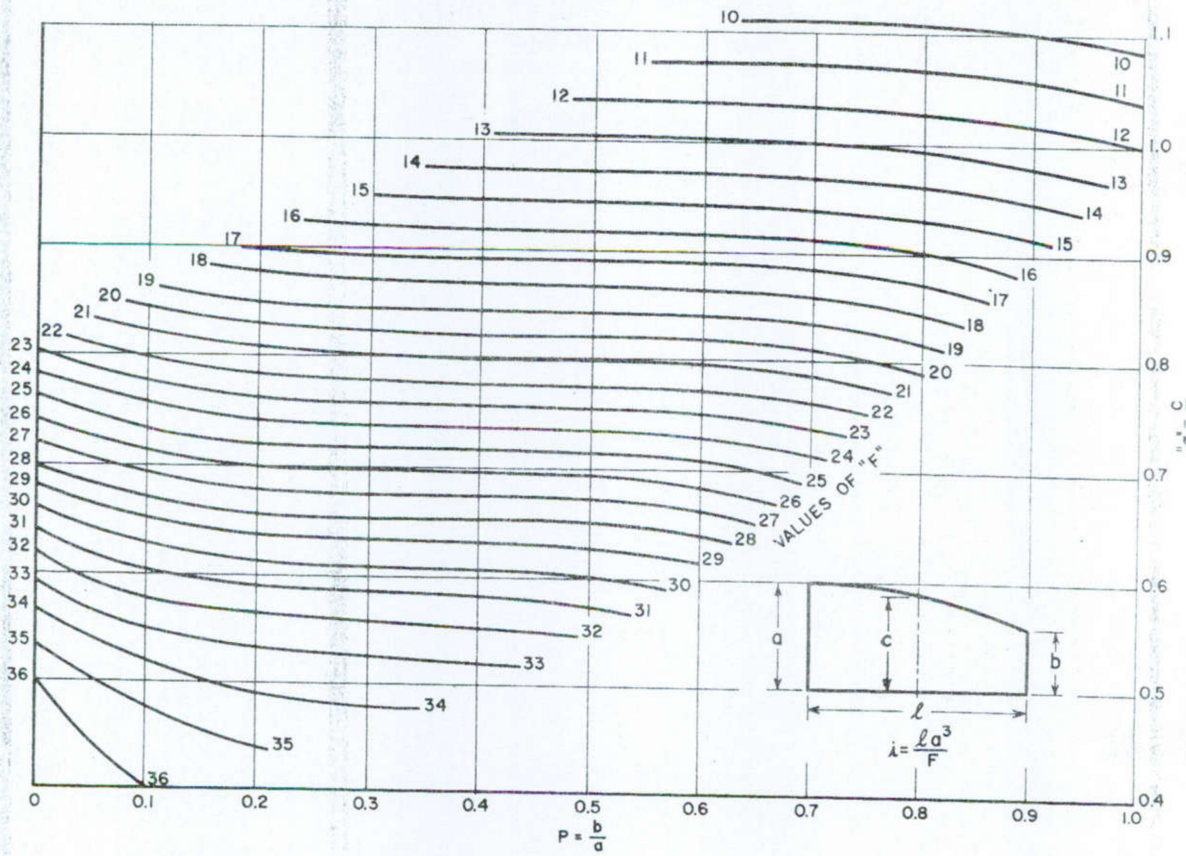


Fig. 34

If b is expressed as a fraction of a, i.e., $b = pa$, then,

$$i_{xy} = \frac{la^3}{36} \frac{(1 + 2p + 2p^3 + p^4)}{1 + p}$$

or

$$i_{xy} = \frac{la^3}{F}$$

where

$$F = \frac{36(1 + p)}{1 + 2p + 2p^3 + p^4}$$

In Fig. 32, the values of F have been plotted against p, and the moment of inertia of a trapezoid such as case (a) may be calculated by finding p (or b/a), reading F from the diagram, and using the formula

$$i_{xy} = \frac{la^3}{F}$$

For the case shown in Fig. 31 (b), where the figure is symmetrical about xy, the formula for the moment of inertia about xy is

$$i_{xy} = \frac{l}{48} (a^3 + a^2b + ab^2 + b^3)$$

or, if b is expressed as pa,

$$i_{xy} = \frac{la^3}{48} (1 + p + p^2 + p^3)$$

or

$$i_{xy} = \frac{la^3}{F}$$

where

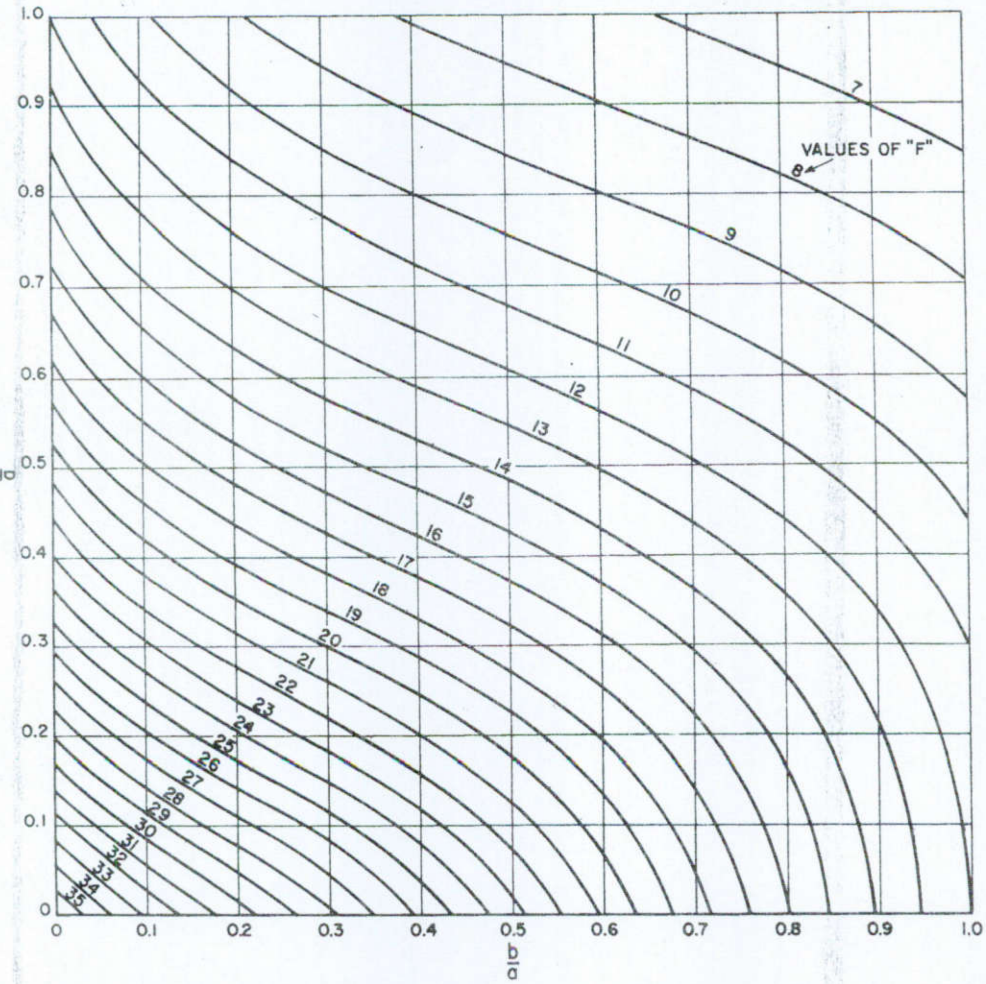
$$F = \frac{48}{1 + p + p^2 + p^3}$$

and is plotted against p in Fig. 33. 31(c)

In the case illustrated in Fig. 28 (c), the situation is similar to case (a) except that one side of the tank is curved. In most cases, the shape of the curve can be defined adequately by two end ordinates and one at the midlength; and the moment of inertia can be found by Simpson's first rule. As in case (a), the moment of inertia about the axis vw, the area A and the distance h between the axes vw and xy are calculated. Then

$$i_{xy} = i_{vw} - Ah^2$$

As described in Chapter I, the formulas for moment of



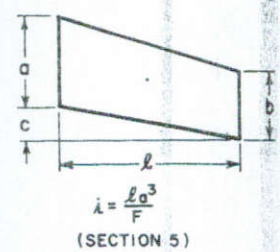


Fig. 35

inertia, area and moment, when integrated by Simpson's rule, are

$$i_{vw} = \frac{1}{3} \times \frac{1}{3} \times \frac{l}{2}\,(a^3 + 4c^3 + b^3)$$

$$A = \frac{1}{3} \times \frac{l}{2}\,(a + 4c + b)$$

$$M = \frac{1}{2} \times \frac{1}{3} \times \frac{l}{2}\,(a^2 + 4c^2 + b^2)$$

Since h is equal to M/A, Ah^2 may be expressed as M^2/A. If b and c are expressed as fractions of a, i.e., $b = pa$ and $c = qa$, then

$$i_{xy} = \frac{la^3}{18}\,(1 + 4q^3 + p^3) - \frac{a^3}{24}\,\frac{(1 + 4q^2 + p^2)^2}{1 + 4q + p}$$

$$\text{or } = \frac{la^3}{72}\left[4(1 + 4q^3 + p^3) - 3\,\frac{(1 + 4q^2 + p^2)^2}{1 + 4q + p}\right]$$

$$i_{xy} = \frac{la^3}{F}$$

where F is equal to 72 divided by the expression in brackets in the foregoing equation. From Fig. 34, the values of F corresponding to various values of p and q (or b/a and c/a) may be read.

In case (d) in Fig. 31, the figure is symmetrical about xy and the sides are formed by two curved lines. The moment of inertia about the centroid is twice the moment of inertia of one side about xy, or

$$i_{xy} = 2 \times \frac{1}{3} \times \frac{1}{3} \times \frac{l}{2}\left[\left(\frac{a}{2}\right)^3 + 4\left(\frac{c}{2}\right)^3 + \left(\frac{b}{2}\right)^3\right]$$

$$i_{xy} = \frac{l}{72}\,(a^3 + 4c^3 + b^3)$$

In the case illustrated in Fig. 31 (e), vw and xy are parallel to the ship's centerline. The moment of inertia about an axis through the centroid can be expressed by

$$i_{xy} = \frac{l}{36}\left[3[(a+c)^2 + b^2](a+b+c) - 3c^3 - \frac{2(a^2 + 2ac + ab + bc + b^2)^2}{a+b}\right]$$

or

$$i_{xy} = \frac{la^3}{36}\left[3[(1+q)^2 + p^2](1+p+q) - 3q^3 - \frac{2(1 + 2q + p + pq + p^2)^2}{1+p}\right]$$

where p and q are equal to b/a and c/a, respectively. This may be expressed as

$$i_{xy} = \frac{la^3}{F}$$

where the values of F are read from Fig. 35.

5.6 Effect of Free Surface on Trim. Free liquid on a ship acts in the fore-and-aft direction in the same manner as in the transverse direction. For an intact ship with normal tankage, the effect of free liquid on trim is so small that it may be ignored. Its magnitude is small in comparison to the assumption in the formula for "moment to trim one inch," described in Section 4, that the center of gravity is at the same height as the center of buoyancy. On unusual craft, however, free liquids may have an important effect on trim.

For a submerged submarine, the effect of free liquid is significant in comparison to the longitudinal stability, but under normal circumstances the trim is carefully adjusted to zero in the submerged condition.

5.7 Effect of Tank Size on Free Surface. The subdivision of large tanks into two or more smaller tanks may be an effective method of improving stability by suppressing the motion of free liquids.

To illustrate, assume that a fuel tank 4 ft deep, 40 ft long and 40 ft wide has been subdivided at the center by a longitudinal bulkhead. The value of i_T/δ, before the division was $(40 \times 40^3)/(12 \times 38)$ or 5600 foot tons; after the division there would be two tanks, each having an i_T/δ of $\frac{40 \times 20^3}{12 \times 38}$ or 700 ft-tons. If the tanks were half full, the moments of transference, from Table 8, with d/b ratios of 0.1 and 0.2, would be as given in Table 11.

For a deeper tank, the percentage reduction would be more pronounced at the smaller angles, but about the same at 60 deg. If the depth had been 20 ft, the figures would be as given in Table 12.

Table 11

Angle of heel, deg	Before: Factor $d/b = 0.1$	Before: Moment $i_T/\delta = 5600$	After: Factor $d/b = 0.2$	After: Moment $i_T/\delta = 1400$
10	0.13	730	0.18	250
20	0.14	780	0.27	380
30	0.14	780	0.27	380
40	0.12	670	0.26	360
50	0.11	620	0.23	320
60	0.09	500	0.20	280

Table 12

Angle of heel, deg	Before: Factor $d/b = 0.5$	Before: Moment $i_T/\delta = 5600$	After: Factor $d/b = 1.0$	After: Moment $i_T/\delta = 1400$
10	0.18	1010	0.18	250
20	0.36	2020	0.36	500
30	0.57	3190	0.58	810
40	0.65	3640	0.87	1220
50	0.66	3700	1.24	1740
60	0.63	3530	1.47	2060

If the change in metacentric height due to subdividing the same tanks were to be evaluated, the effect for the shallow tank would be the same as for the deeper tank. In each case the value of i_T/δ would be 5600 ft-tons before subdivision and 1400 ft-tons after, indicating that the effect of free surface is reduced to 25 percent of its former value. It can be seen from Tables 11 and 12 that this degree of improvement is not attained at the larger angles of heel, after the surface of the liquid has reached the top or bottom of the tank. Since we are interested in the improvement in righting arms at large angles of heel rather than at very small angles, the change in metacentric height produced by subdividing tanks is not an appropriate index for judging the effectiveness of this measure.

Subdividing tanks in order to improve stability involves a compromise, since it requires considerable increase in structure, piping, and fittings. If only the largest tank is subdivided, the improvement is not the difference between that tank and the two into which it is divided, but the difference between that tank and the one which was second largest, which may not be much smaller. To obtain an effective improvement, it may be necessary to subdivide nearly all the tanks on the ship, with additional tail pipes, manholes, sounding, overflow and air-escape piping, and additional complication in the operation of the system.

5.8 Effect of Compensating Tanks. In some cases, a tank will contain two different liquids. Examples are gasoline tanks in which sea water is introduced at the bottom as gasoline is drawn from the top to avoid the accumulation of explosive mixtures, submarine diesel oil tanks in which the oil is replaced by sea water to preserve submerged equilibrium, and tanks on some diesel-driven surface types in which a compensating system is used to improve stability. In the latter case, the stability improvement results from maintaining low weight in the ship, reduction of free surface and reduction in the effect of possible off-center flooding after damage.

Although these tanks are always completely full of liquid, a free-surface effect exists at the interface which will remain parallel to the waterline as the ship inclines. There will be a wedge on the low side in which the lighter liquid will be replaced by the heavier, and a wedge on the high side in which the heavier liquid will be replaced by the lighter. This effect may be evaluated by using, as the