EJ 1

$$\begin{cases} \dot{y} + ay = u & t \geq 0 \\ y(0) = y_0 \end{cases}$$

b. LAPLACE

$$sY(s) - y_0 + aY(s) = U(s)$$

$$\Rightarrow \boxed{Y(s) = \frac{y_0}{s+a} + \frac{1}{s+a} U(s)}$$ LAPLACE

c. TRANSFERENCIA

$$y_0 = 0 \quad \Rightarrow Y(s) = \frac{1}{s+a} U(s)$$

$$\boxed{H(s) = \frac{1}{s+a}}$$ CONDICIONES INICIALES NULAS

d. CONVOLUCIÓN

$$h(t) = \mathcal{L}^{-1}[H(s)] = e^{-at} Y(t)$$

$$\boxed{y(t) = h(t) * u(t)}$$

a. VARIABLES DE ESTADO

$$x = y \qquad \begin{cases} \dot{x} = -ax + u \\ y = x \end{cases}$$

$$\Rightarrow \begin{cases} A = -a, & B = 1 \\ C = 1, & D = 0 \end{cases}$$

b.

- ES LINEAL, ya que la ecuación diferencial es lineal a coeficientes constantes

$$S(u_1 + u_2) = S(u_1) + S(u_2).$$

- ES CAUSAL PUES para todo intervalo $[0,T]$ se tiene $y_{[0,T]}$ sólo depende de $u_{[0,T]}$ y no de entradas posteriores a T.

Y ESTO PARA TODA u y para toda T.

c. f y f' transformables

(T). $\mathcal{L}[f'] = s\mathcal{L}[f] - f(0)$

f continua, f' secc continua.

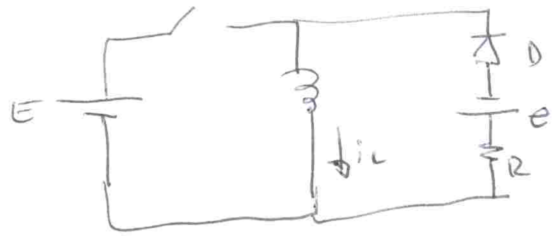
$$\begin{array}{ccc} Y(t) & \xrightarrow{d/dt} & 0 \\ \mathcal{L}[\,]\downarrow & & \downarrow\mathcal{L}[\,] \\ 1/s & & 0 \end{array} = s\cdot\left[\frac{1}{s}\right] - \underset{1}{\underset{\shortparallel}{Y(t=0)}} = 0 \checkmark$$

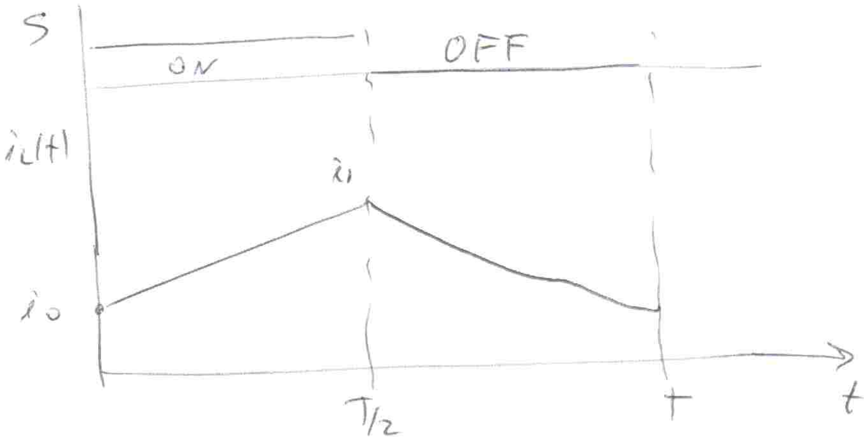
d. $T \in \mathcal{D}'_+$, T, T' admite transf de Laplace.

$$\Rightarrow \mathcal{L}[T'] = s\mathcal{L}[T]$$

$$\begin{array}{ccc} Y(t) & \xrightarrow{d/dt} & \delta(t) \\ \mathcal{L}\downarrow & & \downarrow\mathcal{L} \\ 1/s & & 1 \end{array} = s\cdot\left(\frac{1}{s}\right) \checkmark.$$

EJ 4





a. b.

TRAMO 1 $t \in [0, T/2]$

D OFF
S ON

$$V_L = E = L \frac{di_L}{dt} \Rightarrow \frac{di_L}{dt} = \frac{E}{L}$$

$$\Rightarrow i_L(t) = i_0 + \frac{E}{L} t \qquad t \in [0, T/2]$$

$$i_1 = i_0 + \frac{E}{L}\frac{T}{2} = \boxed{i_0 + \frac{E}{R}\frac{T}{2\tau} = i_1}$$

$v_D = -E \Rightarrow$ D OFF ✓

TRAMO 2

$$\sigma = t - T/2$$

D ON
S OFF

$$i_L(\sigma) = i_1 e^{-\frac{\sigma}{\tau}} - \frac{e}{R}\left(1 - e^{-\sigma/\tau}\right)$$

$-\frac{e}{R}$ valor final asintótico

ostá pu rejiwen si

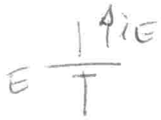
$$i_L(\sigma = T/2) = i_0$$

$$\Rightarrow i_0 = i_1 e^{-\frac{T}{2\tau}} - \frac{e}{R}\left(1 - e^{-\frac{T}{2\tau}}\right) = \left[i_0 + \frac{E}{R}\frac{T}{2\tau}\right] e^{-T/2\tau} - \frac{e}{R}\left(1 - e^{-\frac{T}{2\tau}}\right)$$

$$i_0 = \frac{E}{100R} \Rightarrow \frac{e}{R}\left[1 - e^{-T/2\tau}\right] = -\frac{E}{100R} + \left[\frac{E}{100R} + \frac{E}{R}\frac{T}{2\tau}\right] e^{-\frac{T}{2\tau}}$$

$$\Rightarrow \boxed{e = \frac{-\frac{1}{100} + \left(\frac{1}{100} + \frac{T}{2\tau}\right) e^{-T/2\tau}}{1 - e^{-T/2\tau}} E}$$

c)



$$\bar{P}_E = \frac{1}{T}\int_0^T E\, i_E(t)\,dt = \frac{E}{T}\int_0^T i_E(t)\,dt = \frac{E}{T}\int_0^{T/2} i_L(t)\,dt$$

pues $i_E(t) = \begin{cases} i_L(t) & t \in [0, T/2] \\ 0 & t \in [T/2, T] \end{cases}$

$$= \frac{E}{T}\int_0^{T/2}\left[i_0 + \frac{E}{L}t\right]dt = \frac{E}{T}\left[i_0\frac{T}{2} + \frac{E}{2L}(T/2)^2\right]$$

$$\bar{P}_E = \frac{E i_0}{2} + \frac{E^2}{2LT}\frac{T^2}{4} = \boxed{\frac{E^2}{200R} + \frac{E^2}{8}\frac{T}{\tau} = \bar{P}_E}$$

$$\bar{P}_e = e\frac{1}{T}\int_0^T i_e(t)\,dt = e\frac{1}{T}\int_0^{T/2}\left[-i_L(\sigma)\right]d\sigma \qquad (\sigma = t - T/2)$$

$$= -\frac{e}{T}\int_0^{T/2} i_L(\sigma)\,d\sigma$$

$$\bar{P}_e = -\frac{e}{T}\int_0^{T/2}\left[i_1 e^{-\sigma/\tau} - \frac{e}{R}\left(1-e^{-\sigma/\tau}\right)\right]d\sigma =$$

$$= -\frac{e}{T}\left\{ i_1[-\tau]\left[e^{-\frac{T}{2\tau}}-1\right] - \frac{e}{R}\frac{T}{2} + \frac{e}{R}[-\tau]\left[e^{-\frac{T}{2\tau}}-1\right]\right\}$$

$$= -\frac{e}{T}\left\{ i_1\tau\left(1-e^{-T/2\tau}\right) - \frac{e}{2R}T + \tau\frac{e}{R}\left(1-e^{-T/2\tau}\right)\right\}$$

$$\boxed{= -\frac{e}{T}\left\{\left[\left(\frac{E}{100R}+\frac{E}{R}\frac{T}{2\tau}\right)\tau + \frac{\tau e}{R}\right]\left[1-e^{-T/2\tau}\right] - \frac{e}{2R}T\right\} = \bar{P}_e}$$

d. Por Tellegen y pasividad (*) de L, S, D tenemos

$$\bar{P}_e + \bar{P}_E = \bar{P}_R \quad \checkmark$$

(*) Más precisamente, $\bar{P}_D = \bar{P}_S = \bar{P}_L = 0$.