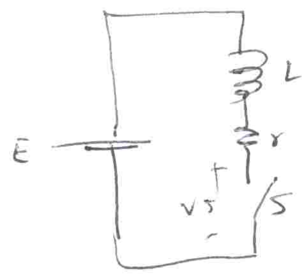
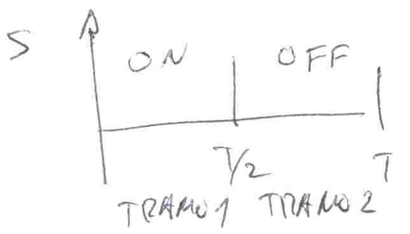
PROBLEMA 3 SISTEMAS LINEALES 2 FEBRERO 2015

1.





<u>TRAMO 1</u> $t \in [0, T/2]$

$i_L(0^-) = 0$ (llave abierta)

$$\Rightarrow \begin{cases} i_S(t) = \frac{E}{r}\left(1 - e^{-t/\tau_1}\right) & \tau_1 = \frac{L}{r} \qquad t \in [0, T/2] \\ v_S(t) = 0 \end{cases}$$

$$i_S(T/2) = \frac{E}{r}\left(1 - e^{-T/2\tau_1}\right) \simeq \frac{E}{r}$$

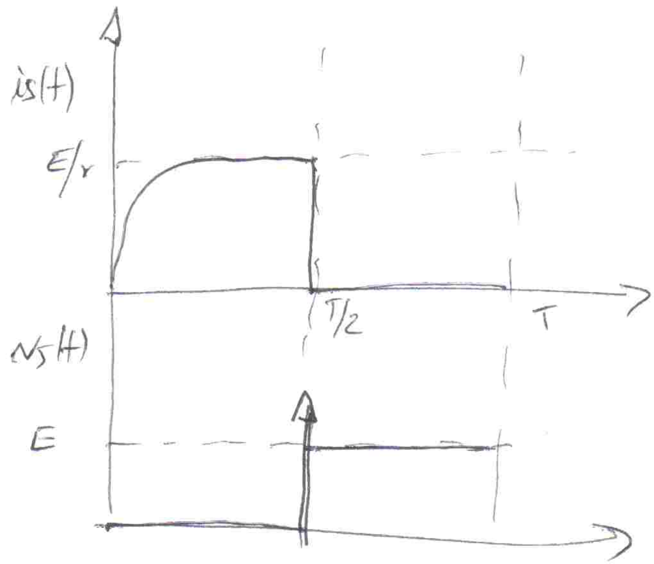
<u>TRAMO 2</u> $t \in [T/2, T]$

Dato previo

$$i_L(T/2^-) = i_S(T/2^-) = E/r$$

$$i_L(t) = i_S(t) = 0 \qquad t \in [T/2, T]$$

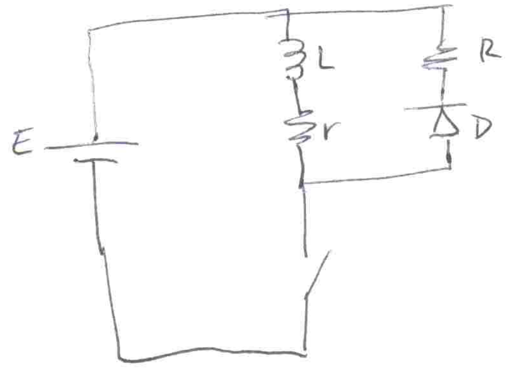
$$\Rightarrow \frac{di_S}{dt} = -\frac{E}{r}\,\delta(t - T/2)$$



$$v_S = E\,Y(t) - L\frac{di_S}{dt} - r\,\underset{0}{\underset{\shortparallel}{i_S(t)}} = E\,Y(t) + L\frac{E}{r}\,\delta(t - T/2) \qquad t \in [T/2, T]$$

PROBLEMA 3

parte 2



TRAMO 1 $t \in [0, T/2]$

Dato previo $i_L(0^-) = 0$, a verificar a posterior.

D OFF

con S ON, $\begin{cases} i_L(t) = \frac{E}{r}\left(1 - e^{-t/\tau_1}\right) \\ i_S(t) = i_L(t) \\ v_S(t) = 0 \end{cases}$ $\tau_1 = \frac{L}{r}$ $t \in [0, T/2]$

$V_D = -E \Rightarrow$ D OFF

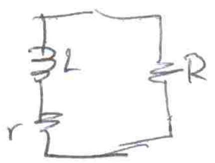
$i_D(t) = 0$ $t \in [0, T/2]$

TRAMO 2 $t \in [T/2, T]$

Datro previo $i_L(T/2^-) \simeq \frac{E}{r}$ pues $\tau_1 << T/2$

S OFF

D ON

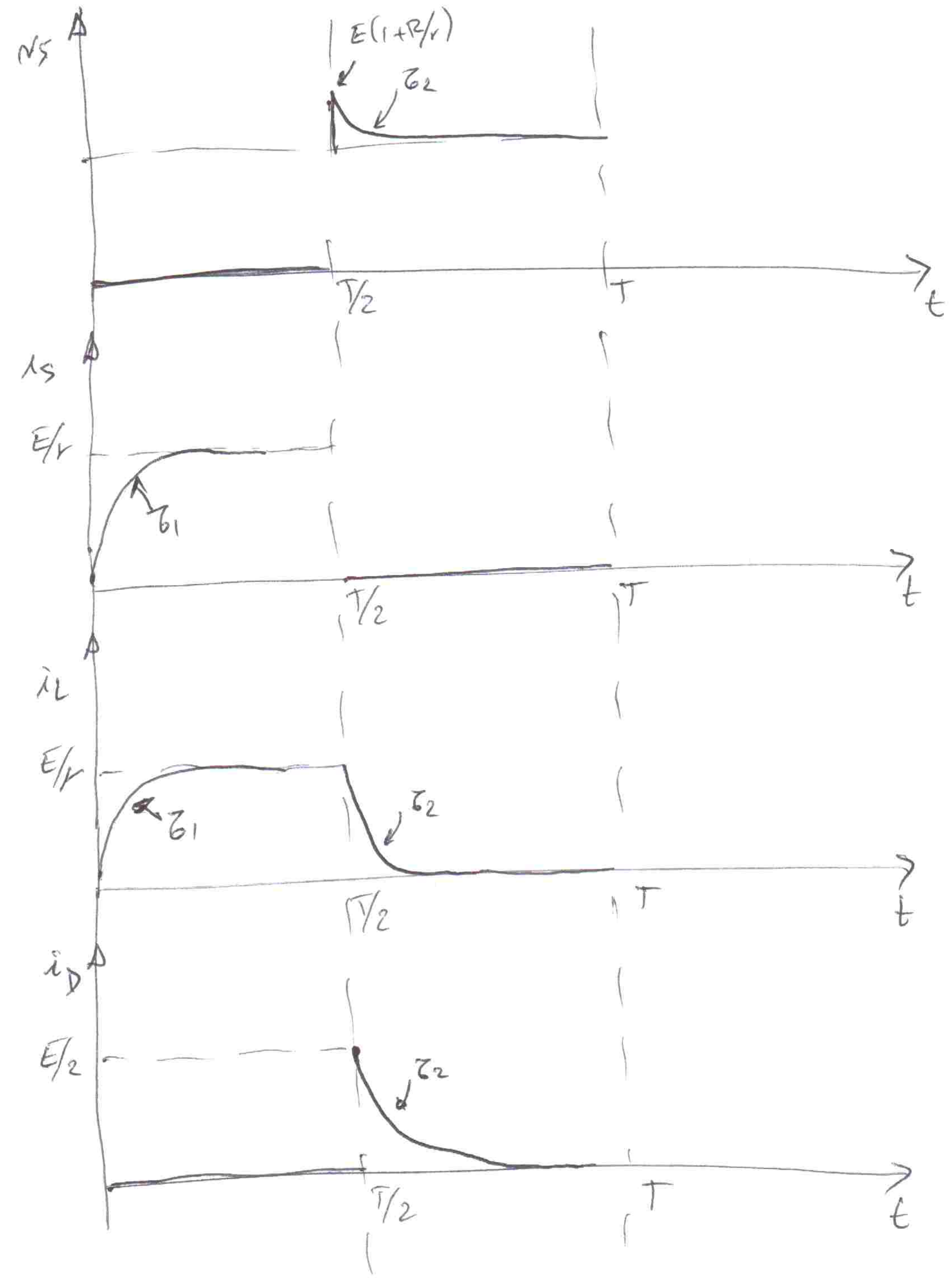


$i_L(t) = \frac{E}{r} e^{-t'/\tau_2}$ $\tau_2 = \frac{L}{r+R} < \frac{L}{r} << T/2$

$v_S = E + R\, i_L(t) = E + R\frac{E}{r} e^{-t'/\tau_2}$ $t \in [T/2, T]$

$i_D = i_L(t) = \frac{E}{r} e^{-t'/\tau_2}$ $t' = t - T/2$

$i_S(t) = 0$



se verifica $i_L(T^-) = i_L(0^-) = 0$ pues $\tau_2 << T/2$

2.b $E_L(T/2) = \frac{1}{2} L \, i_L^2(T/2) = \frac{1}{2} L \left[\frac{E}{r}\right]^2.$

Problema 3

parte 3

a. $P_D(t) = v_D i_D(t) = 0$
$P_S(t) = v_S i_S(t) = 0$ } pues siempre la corriente o la tensión es nula, $\forall t$

$$\overline{P_L} = \frac{1}{T}\int_0^T v_L i_L \, dt = \frac{1}{2} E_L(i_L)\Big|_{i_L(0)}^{i_L(T)} = 0 \quad \text{por régimen periódico}$$

$\overline{P_D} = \overline{P_S} = 0$ pues $P_D(t) = P_S(t) = 0 \ \forall t$.

$$\overline{P_R} = \frac{1}{T}\int_0^T R\, i_R^2(t)\, dt = \frac{1}{T}\int_{T/2}^T R\, i_D^2(t)\, dt = \frac{1}{T} R \int_{T/2}^T \left(\frac{E}{r}\right)^2 e^{-2(t-T/2)/\tau_2} dt$$

$$= \frac{R}{T}\frac{E^2}{r^2}\, e^{-2t/\tau_2}\Big|_0^{T/2}\left[\frac{\tau_2}{2}\right] = \frac{RE^2}{Tr^2}\frac{\tau_2}{2} \quad \text{pues } \tau_2 \ll T/2$$

$$\boxed{\overline{P_R} = \frac{R}{R+r}\frac{L}{2}\frac{E^2}{r^2 T}}$$

b. Tellegen: $P_E(t) + P_R(t) + P_r(t) + P_L(t) + P_S(t) + P_D(t) = 0 \ \forall t$.

Dados P_D, P_S de la parte a: $P_E(t) + P_R(t) + P_r(t) + P_L(t) = 0 \ \forall t$

c. Cuando tomamos valores medios en un período:

$$\overline{P_E} + \overline{P_R} + \overline{P_r} + \overline{P_L} = 0 \Rightarrow \overline{P_r} = -\overline{P_E} - \overline{P_R}$$
$\overline{P_L} = 0$

$$M = \frac{\overline{P_r}}{-\overline{P_E}} = \frac{\overline{P_E} + \overline{P_R}}{\overline{P_E}} = 1 + \frac{\overline{P_R}}{\overline{P_E}}$$

note que $\overline{P_E}$ es negativo pues $P_r \geq 0$, $P_R \geq 0$