Primer parcial 2014
Sistemas Lineales 2

Solución Ejercicio 2

a. $\mathcal{L}\left[\frac{df}{dt}\right] = \int_0^\infty \frac{df}{dt} e^{-st} dt = f(t) e^{-st}\Big|_0^\infty - \int_0^\infty f(t) \frac{d}{dt} e^{-st} dt =$

$$= \underbrace{\lim_{t\to+\infty} f(t) e^{-st}}_{\overset{\|}{0}} - f(0) + s\int_0^\infty f(t) e^{-st} dt . \checkmark$$

$\forall s / Re(s) > x_0$

Pues $f(t)$ es transformable.

b. $\frac{d}{ds}\int_0^\infty f(t) e^{-st} dt \underset{\uparrow}{=} \int_0^\infty f(t) \frac{d}{ds} e^{-st} dt = \int_0^\infty -t f(t) e^{-st} = \mathcal{L}[-t f(t)] \checkmark$

por la C.U.

c. $\mathcal{L}\left[f\left(\frac{t}{a}\right)\right] = \int_0^\infty f\left(\frac{t}{a}\right) e^{-st} dt \underset{\uparrow}{=} \int_0^\infty f(\sigma) e^{-sa\sigma} a\, d\sigma = +a\int_0^\infty f(\sigma) e^{-(as)\sigma} d\sigma$

$\sigma = t/a$
$d\sigma = \frac{dt}{a}$

$= a F(as) \checkmark$

d. $F(s) = \frac{w_n^2}{s(s^2+2\xi w_n s + w_n^2)} = w_n^{-1} \frac{1}{\left(\frac{s}{w_n}\right)\left[\left(\frac{s}{w_n}\right)^2 + 2\xi\left(\frac{s}{w_n}\right) + 1\right]} = \left(\frac{1}{w_n}\right) G\left(\frac{s}{w_n}\right)$

con $G(\sigma) := \frac{1}{\sigma(\sigma^2 + 2\xi\sigma + 1)}$

$\Rightarrow \mathcal{L}^{-1}[F(s)] = \mathcal{L}^{-1}\left[\frac{1}{w_n} G\left(\frac{s}{w_n}\right)\right] \underset{\uparrow}{=} \mathcal{L}^{-1}\mathcal{L}[g(w_n t)] = g(w_n t) \checkmark.$

Prop c.