Hojas de fórmulas para 1er Parcial

(2013)

Transformada de Laplace **$F(s)$**	Función en el tiempo **$f(t)$**
1	Impulso unitario
$\frac{1}{s}$	Escalón unitario
$\frac{1}{s^2}$	t
$\frac{n!}{s^{n+1}}$	t^n (n = entero positivo)
$\frac{1}{s+a}$	e^{-at}
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{b-a}$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}sen\left(\omega_n\sqrt{1-\zeta^2}\,t\right) \quad 0<\zeta<1$
$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}sen\left(\omega_n\sqrt{1-\zeta^2}\,t-Arctg\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$
$\frac{1}{(1+Ts)^n}$	$\frac{1}{T^n(n-1)!}t^{n-1}e^{-t/T}$
$\frac{\omega_n^2}{(1+Ts)(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\frac{T\omega_n^2e^{-t/T}}{1-2\zeta T\omega_n+T^2\omega_n^2}+\frac{\omega_n e^{-\zeta\omega_n t}sen\left(\omega_n\sqrt{1-\zeta^2}\,t-Arctg\left(\frac{T\omega_n.\sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right)\right)}{\sqrt{(1-\zeta^2)(1-2\zeta T\omega_n+T^2\omega_n^2)}}$
$\frac{\omega_n}{s^2+\omega_n^2}$	$\text{sen}(\omega_n t)$
$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$e^{-at}\,\text{sen}(\beta t)$
$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	$e^{-at}\cos(\beta t)$
$\frac{\omega_n}{(1+Ts)(s^2+\omega_n^2)}$	$\frac{T\omega_n e^{-t/T}}{1+T^2\omega_n^2}+\frac{sen\left(\omega_n t-Arctg(T\omega_n)\right)}{\sqrt{1+T^2\omega_n^2}}$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}sen\left(\omega_n\sqrt{1-\zeta^2}\,t+Arctg\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$
$\frac{\omega_n^2}{s(s^2+\omega_n^2)}$	$1-\cos(\omega_n t)$
$\frac{1}{s(1+Ts)}$	$1-e^{-t/T}$

Transformada de Laplace $F(s)$	**Función en el tiempo** $f(t)$
$\frac{1}{s(1+Ts)^2}$	$1-\frac{t+T}{T}e^{-t/T}$
$\frac{\omega_n^2}{s(1+Ts)(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{T^2\omega_n^2e^{-t/T}}{1-2T\zeta\omega_n+T^2\omega_n^2}+\frac{e^{-\zeta\omega_n t}\,\mathrm{sen}(\omega_n\sqrt{1-\zeta^2}\,t-\Phi)}{\sqrt{(1-\zeta^2)(1-2\zeta T\omega_n+T^2\omega_n^2)}}$ $donde\ \Phi=tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)+tan^{-1}\left(\frac{T\omega_n\sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right)$
$\frac{\omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}$	$t-\frac{2\zeta}{\omega_n}+\frac{1}{\omega_n\sqrt{(1-\zeta^2)}}e^{-\zeta\omega_n t}sen\left(\omega_n\sqrt{1-\zeta^2}\,t-2.Arctg\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)\right)$
$\frac{\omega_n^2}{s^2(1+Ts)(s^2+2\zeta\omega_n s+\omega_n^2)}$	$t-T-\frac{2\zeta}{\omega_n}+\frac{T^3\omega_n^2e^{-t/T}}{1-2T\zeta\omega_n+T^2\omega_n^2}+\frac{e^{-\zeta\omega_n t}\,\mathrm{sen}(\omega_n\sqrt{1-\zeta^2}\,t-\Phi)}{\omega_n\sqrt{(1-\zeta^2)(1-2\zeta T\omega_n+T^2\omega_n^2)}}$ $donde\ \Phi=2tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)+tan^{-1}\left(\frac{T\omega_n\sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right)$
$\frac{1}{s^2(1+Ts)^2}$	$t-2T+(t+2T)e^{-t/T}$
$\frac{\omega_n^2(1+as)}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\omega_n\sqrt{\frac{1+2a\zeta\omega_n+a^2\omega_n^2}{1-\zeta^2}}e^{-\zeta\omega_n t}sen\left(\omega_n\sqrt{1-\zeta^2}\,t+Arctg\left(\frac{a\omega_n\sqrt{1-\zeta^2}}{1-a\zeta\omega_n}\right)\right)$
$\frac{\omega_n^2(1+as)}{s^2+\omega_n^2}$	$\omega_n\sqrt{1+a^2\omega_n^2}\,sen(\omega_n t+Arctg(a\omega_n))$
$\frac{\omega_n^2(1+as)}{(1+Ts)(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\frac{\omega_n\sqrt{(1-2\zeta a\omega_n+a^2\omega_n^2)}e^{-\zeta\omega_n t}\,\mathrm{sen}(\omega_n\sqrt{1-\zeta^2}\,t+\Phi)}{\sqrt{(1-\zeta^2)(1-2\zeta T\omega_n+T^2\omega_n^2)}}$ $+\frac{(T-a)\omega_n^2e^{-t/T}}{1-2T\zeta\omega_n+T^2\omega_n^2}$ $donde\ \Phi=tan^{-1}\left(\frac{a\omega_n\sqrt{1-\zeta^2}}{1-a\zeta\omega_n}\right)-tan^{-1}\left(\frac{T\omega_n\sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right)$
$\frac{\omega_n^2(1+as)}{(1+Ts)(s^2+\omega_n^2)}$	$\frac{\omega_n^2(T-a)}{1+T^2\omega_n^2}e^{-t/T}+\frac{\omega_n\sqrt{1+a^2\omega_n^2}}{\sqrt{1+T^2\omega_n^2}}\mathrm{sen}(\omega_n t+\Phi)$ $donde\ \Phi=tan^{-1}(a\omega_n)-tan^{-1}(T\omega_n)$
$\frac{\omega_n^2(1+as)}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1+\frac{\sqrt{(1-2\zeta a\omega_n+a^2\omega_n^2)}}{\sqrt{(1-\zeta^2)}}e^{-\zeta\omega_n t}\,\mathrm{sen}(\omega_n\sqrt{1-\zeta^2}\,t+\Phi)$ $donde\ \Phi=tan^{-1}\left(\frac{a\omega_n\sqrt{1-\zeta^2}}{1-a\zeta\omega_n}\right)-tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)$

Transformada de Laplace *F*(*s*)	Función en el tiempo *f*(*t*)
$\dfrac{\omega_n^2(1+as)}{s(1+Ts)(s^2+\omega_n^2)}$	$1+\dfrac{T\omega_n^2(a-T)}{1+T^2\omega_n^2}e^{-t/T}-\dfrac{\sqrt{1+a^2\omega_n^2}}{\sqrt{1+T^2\omega_n^2}}\cos(\omega_n t+\Phi)$ $donde\ \Phi = tan^{-1}(a\omega_n)-tan^{-1}(T\omega_n)$
$\dfrac{\omega_n^2(1+as)}{s(1+Ts)(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1+\dfrac{\sqrt{(1-2\zeta a\omega_n+a^2\omega_n^2)}}{\sqrt{(1-\zeta^2)(1-2\zeta T\omega_n+T^2\omega_n^2)}}e^{-\zeta\omega_n t}\,\mathrm{sen}(\omega_n\sqrt{1-\zeta^2}\,t+\Phi)$ $+\dfrac{T(a-T)\omega_n^2 e^{-t/T}}{1-2T\zeta\omega_n+T^2\omega_n^2}$ $donde\ \Phi = tan^{-1}\left(\dfrac{a\omega_n\sqrt{1-\zeta^2}}{1-a\zeta\omega_n}\right)$ $-tan^{-1}\left(\dfrac{T\omega_n\sqrt{1-\zeta^2}}{1-T\zeta\omega_n}\right)-tan^{-1}\left(\dfrac{\sqrt{1-\zeta^2}}{-\zeta}\right)$
$\dfrac{1+as}{s^2(1+Ts)}$	$t+(a-T)(1-e^{-t/T})$
$\dfrac{s}{s^2+\omega_n^2}$	$\cos(\omega_n t)$
$\dfrac{s}{(s^2+\omega_n^2)^2}$	$\dfrac{1}{2\omega_n}t\,\mathrm{sen}(\omega_n t)$
$\dfrac{s}{(s^2+\omega_{n1}^2)(s^2+\omega_{n2}^2)}$	$\dfrac{1}{\omega_{n1}^2-\omega_{n2}^2}\left(\cos(\omega_{n1}t)-\cos(\omega_{n2}t)\right)$
$\dfrac{s}{(1+Ts)(s^2+\omega_n^2)}$	$\dfrac{-1}{1+T^2\omega_n^2}e^{-t/T}+\dfrac{1}{\sqrt{1+T^2\omega_n^2}}\cos\left(\omega_n t-Arctg(T\omega_n)\right)$
$\dfrac{1+as+bs^2}{s^2(1+T_1 s)(1+T_2 s)}$	$t+(a-T_1-T_2)+\dfrac{b-aT_1+T_1^2}{T_1-T_2}e^{-t/T_1}-\dfrac{b-aT_2+T_2^2}{T_1-T_2}e^{-t/T_2}$
$\dfrac{\omega_n^2(1+as+bs^2)}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1+\dfrac{\sqrt{(1-a\zeta\omega_n-b\omega_n^2+2b\zeta^2\omega_n^2)^2+\omega_n^2(1-\zeta^2)(a-2b\zeta\omega_n)^2}}{\sqrt{(1-\zeta^2)}}$ $x\ e^{-\zeta\omega_n t}\,\mathrm{sen}(\omega_n\sqrt{1-\zeta^2}\,t+\Phi)$ $donde\ \Phi = tan^{-1}\left(\dfrac{\omega_n\sqrt{1-\zeta^2}(a-2b\zeta\omega_n)}{b\omega_n^2(2\zeta^2-1)+1-a\zeta\omega_n}\right)-tan^{-1}\left(\dfrac{\sqrt{1-\zeta^2}}{-\zeta}\right)$
$\dfrac{s^2}{(s^2+\omega_n^2)^2}$	$\dfrac{1}{2\omega_n}\left(\mathrm{sen}(\omega_n t)+\omega_n t\cos(\omega_n t)\right)$

Respuesta al escalón de sistemas de 2º orden sin ceros

Tiempo de levantamiento 10%-90% :

$$t_R = \frac{0{,}366.e^{2.\zeta} + 0{,}6536}{\omega_n} \qquad 0 < \zeta \leq 1$$

$$t_R = \frac{2.\ln(9).\zeta - 1{,}0364/\zeta}{\omega_n} \qquad 1 < \zeta$$

Tiempo de establecimiento n% :

$$t_S = \frac{-\ln\left(\frac{n\%}{100}\sqrt{1-\zeta^2}\right)}{\zeta\omega_n}; \qquad t_S^{5\%} = \frac{3}{\zeta\omega_n} \quad 0 < \zeta < 0{,}6; \qquad t_S^{2\%} = \frac{4}{\zeta\omega_n} \quad 0 < \zeta < 0{,}8$$

Máximo sobretiro y tiempo de máximo sobretiro:

$$M_p(\%) = 100e^{-\pi\zeta / \sqrt{1-\zeta^2}} \qquad \zeta^2 = \frac{\left[\ln\left(\frac{M_p(\%)}{100}\right)\right]^2}{\pi^2 + \left[\ln\left(\frac{M_p(\%)}{100}\right)\right]^2} \qquad t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$