Ej 4

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \qquad A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -10 \end{bmatrix} ; B = \begin{bmatrix} 1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, \; D = 1.$$

a. I.E?

Los autovalores de A son $\{-1, 1, -10\}$

$\Rightarrow$ No es internamente estable, ya que tiene un autovalor con $Re(\lambda) \geq 0$.

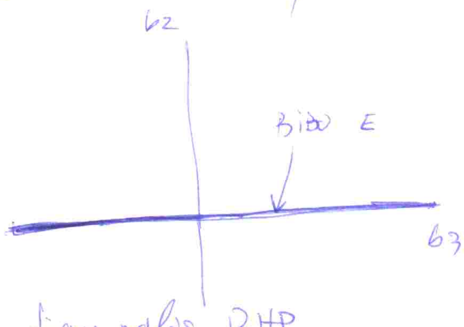
b. BIBO E?

$$H(s) = C(sI - A)^{-1}B + D = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} s+1 & & \\ & s-1 & \\ & & s+10 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ b_2 \\ b_3 \end{bmatrix} + 1.$$

$$H(s) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} (s+1)^{-1} & & \\ & (s-1)^{-1} & \\ & & (s+10)^{-1} \end{bmatrix} \begin{bmatrix} 1 \\ b_2 \\ b_3 \end{bmatrix} + 1$$

$$\boxed{H(s) = \frac{1}{s+1} + \frac{b_2}{s-1} + \frac{b_3}{s+10} + 1}$$

ES BIBO ESTABLE $\forall (b_2, b_3)$ tales que $\underline{b_2 = 0}$, $b_3 \in \mathbb{R}$.



Pues, si $b_2 = 0$, $H(s)$ no tiene polos RHP

c Elijo $b_2 = 0$
$b_3 = 0$

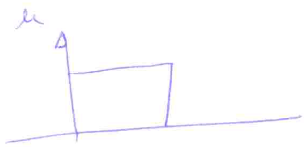
$$H(s) = \frac{1}{s+1} + 1.$$

$$Y(s) = H(s) \cdot \frac{1}{s}$$

$$sY(s) = H(s) = \frac{1}{s+1} + 1$$

como $sY(s)$ no tiene polos RHP $\Rightarrow$ por el TVF tenemos

$$\lim_{t \to +\infty} y(t) = \lim_{s \to 0} H(s) = \lim_{s \to 0} \left(\frac{1}{s+1} + 1 \right) = 2.$$

d. 

$$u(t) = Y(t) - Y(t+T)$$

$$\Rightarrow y(t) = h(t) * Y(t) - h(t) * Y(t+T)$$
$$= y_c(t) - y_c(t+T)$$ siendo $y_c(t)$ la respuesta al escalón

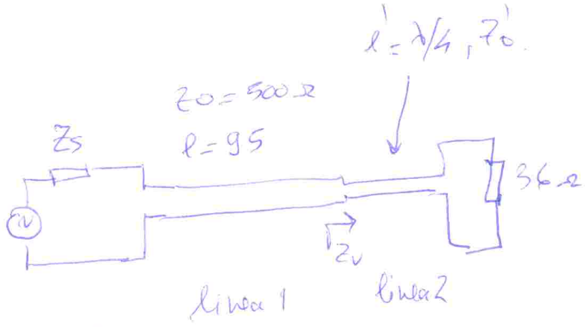
$$\lim_{t \to \infty} y(t) = \lim_{t \to \infty} y_c(t) - \lim_{t \to \infty} y_c(t+T) = \lim_{t \to +\infty} y_c(t) - \lim_{t \to +\infty} y_c(t) = 0$$

Se aplicó superposición en la entrada, una propiedad del límite $\left(\lim_{t \to +\infty} y_c(t) = \lim_{t \to +\infty} y_c(t+T) \right)$ y el TVF

e. $$x(t) = e^{At} x_0 = \begin{bmatrix} e^{-t} & & \\ & e^{t} & \\ & & e^{-10t} \end{bmatrix} x_0 = x_{01} e^{-t} + x_{02} e^{t} + x_{03} e^{-10t}$$

$$\lim_{t \to +\infty} x(t) = 0 \quad \text{si} \quad x_{02} = 0$$
$$= \pm\infty \quad \text{si} \quad x_{02} \neq 0.$$

Ej 1



a.

$f = 40\,MHz$

$v_p = 0.97c$

$$Z_v = \frac{Z_0'^2}{Z_L}$$

Para que no haya onda reflejada en la línea 1,

$$Z_v = Z_0$$

$$\Rightarrow \frac{Z_0'^2}{Z_L} = Z_0 \Rightarrow Z_0' = \sqrt{Z_0 Z_L} = \sqrt{500 \cdot 36} = 60\sqrt{5}\,\Omega$$

$$l' = \lambda/4. \quad \lambda = \frac{v_p}{f} = \frac{0.97 \cdot 3\times 10^8}{40\ 10^6} = 3 \cdot 0.97 \cdot 2.5 = 7.275\,m.$$

$$\Rightarrow \boxed{l' = 1.81\,m} \quad \boxed{Z_0' = 60\sqrt{5}\,\Omega}$$

b.

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{\rho_1\angle\varphi_1\ \rho_2\angle\varphi_2} = \sqrt{\rho_1\rho_2}\angle\frac{(\varphi_1+\varphi_2)}{2}$$

$R, G \geq 0$

$\omega L, \omega C > 0$ $\quad \varphi_1$ y φ_2 cumplen $\begin{array}{l}\varphi_1 \in [0, \pi/2] \\ \varphi_2 \in [0, \pi/2]\end{array} \Rightarrow \frac{\varphi_1+\varphi_2}{2} \in [0, \pi/2]$

$\Rightarrow \gamma$ tiene 2 raíces $\begin{array}{l}\gamma_1 = \alpha + j\beta \\ -\alpha - j\beta\end{array}$ con $\begin{array}{l}\alpha \geq 0 \\ \beta > 0\end{array}$.