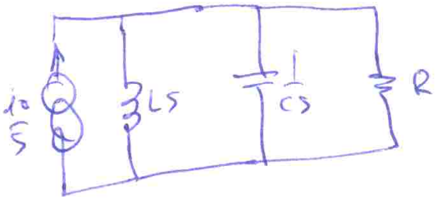
4.

a



$$Z_T := \frac{1}{1/R + Cs + \frac{1}{Ls}} = \frac{RLs}{Ls + R + RLCs^2} = \frac{\frac{1}{C}s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

$$\frac{1}{LC} = \frac{R}{L} \cdot \frac{1}{RC} = w_n^2 \Rightarrow \boxed{Z_T = \frac{\frac{1}{C}s}{s^2 + w_n s + w_n^2}}$$

$$V_c(s) = Z_T \cdot \frac{i_0}{s} \Rightarrow \boxed{V_c(s) = \frac{\frac{i_0}{C}}{s^2 + w_n s + w_n^2}}$$

dimensiones: $[V_c] = \frac{i_0}{C s^2} = \frac{R i_0}{RC s^2} = \frac{V}{T s^2} = \frac{V}{s}$ ✓

$$I_L(s) = i_C + i_R = Cs\, V_c(s) + \frac{V_c(s)}{R} = \frac{1 + RCs}{R} V_c(s) = \frac{s + \frac{1}{RC}}{1/C} V_c(s)$$

$$\Rightarrow I_L(s) = \frac{s + w_n}{1/C} \frac{i_0/C}{s^2 + w_n s + w_n^2}$$

$$\Rightarrow \boxed{I_L(s) = \frac{(s + w_n)\, i_0}{s^2 + w_n s + w_n^2}}$$

b)

TVI $f(0) = \lim_{s \to +\infty} sF(s)$

TVF $f(+\infty) = \lim_{s \to 0} sF(s)$ si $sF(s)$ no tiene polos RHP.

$i_L(0) = \lim_{s \to +\infty} sI_L(s) = i_0$ ✓

$i_L(+\infty) = \lim_{s \to 0} sI_L(s) = 0$ ✓ cierra pues la energía se disipa por la resistencia.

$v_c(0) = \lim_{s \to +\infty} sV_c(s) = 0$ ✓ C.I. nula.

$v_c(+\infty) = \lim_{s \to 0} sV_c(s) = 0$ ✓. idem

c. De la tabla : $\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \longrightarrow \dfrac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \operatorname{Sen}\left(\omega_n\sqrt{1-\xi^2}\,t\right)$

$2\xi\omega_n = \omega_n \Rightarrow \xi = 1/2 \Rightarrow \sqrt{1-\xi^2} = \sqrt{3/4} = \dfrac{\sqrt{3}}{2}$

$\omega_d \overset{\circ}{=} \dfrac{\sqrt{3}}{2}\omega_n.$

$\Rightarrow v_c(t) = \dfrac{i_0}{C\omega_n^2}\mathcal{L}^{-1}\left[\dfrac{\omega_n^2}{s^2+\omega_n s+\omega_n^2}\right] = \dfrac{i_0}{C\omega_n^2}\dfrac{\omega_n 2}{\sqrt{3}} e^{-\frac{\omega_n}{2}t} \operatorname{sen}\omega_d t$

$$\boxed{\Rightarrow v_c(t) = \frac{2Ri_0}{\sqrt{3}} e^{-\frac{\omega_n}{2}t} \operatorname{Sen}\omega_d t \qquad C\omega_n = \frac{1}{R}}$$

$\forall t \in [0, t^0]$

$$I_L(s) = C(s+w_n)\, V_c(s)$$

$$\Rightarrow i_L(t) = C \frac{d v_c(t)}{dt} + C w_n v_c(t)$$

$$= C \cdot \frac{2}{\sqrt{3}} R i_0 \left[-\frac{w_n}{2} e^{-\frac{w_n t}{2}} \operatorname{sen} w_d t + e^{-\frac{w_n t}{2}} \cos w_d t \cdot w_d \right]$$

$$+ C w_n \frac{2}{\sqrt{3}} R i_0\, e^{-\frac{w_n t}{2}} \operatorname{sen} w_d t .$$

$$= R C w_n \frac{2}{\sqrt{3}} i_0 \cdot \left[\left(-\frac{1}{2} + 1\right) \operatorname{sen} w_d t + \frac{\sqrt{3}}{2} \cos w_d t \right] e^{-\frac{w_n t}{2}}$$

$$w_d = \frac{\sqrt{3}}{2} w_n \qquad \Rightarrow i_L(t) = \frac{2}{\sqrt{3}} i_0 \left[e^{-\frac{w_n t}{2}} \right] \left[\frac{1}{2} \operatorname{sen} w_d t + \frac{\sqrt{3}}{2} \cos w_d t \right]$$

$$\Rightarrow \boxed{i_L(t) = \frac{i_0}{\sqrt{3}} e^{-w_n t/2} \left(\operatorname{sen} w_d t + \sqrt{3} \cos w_d t \right)} \quad \forall t \in [0, t^*]$$

El primer máximo de $v_c(t)$ se da en $\operatorname{sen} w_d t^* = 1 \Rightarrow w_d t^* = \pi/2$

$$\Rightarrow t^* = \frac{\pi 2}{2\sqrt{3}} \frac{1}{w_n} \qquad \boxed{t^* = \frac{\pi}{\sqrt{3} w_n}}$$

$$e^{-\frac{w_n t^*}{2}} = e^{-\frac{\pi}{\sqrt{3} \cdot 2}} \triangleq \beta$$

$\operatorname{sen} w_d t^* = 1$

$\cos w_d t^* = 0$

En t^* $\quad v_c(t^*) = \frac{2R\,i_0}{\sqrt{3}}\beta \quad , \quad i_L(t^*) = \frac{\beta\, i_0}{\sqrt{3}}$

$\forall t > t^*$ $\quad \boxed{i_L(t) = 0 \quad \forall t > t^*}$

$$v_c(t) = v_c(t^*)\, e^{-\frac{(t-t^*)}{RC}}$$

$$\Rightarrow \boxed{v_c(t) = \frac{2}{\sqrt{3}}\beta R i_0\, e^{-\frac{(t-t^*)}{RC}} \quad \forall t > t^*}$$

$$\underset{\forall t}{v_s(t)} = v_c(t) + L\frac{di_L}{dt} = \begin{cases} 0 & \forall t < t^* \\ v_c(t) & \forall t > t^* \end{cases} + L\left[-\frac{\beta i_0}{\sqrt{3}}\right]\delta(t-t^*)$$