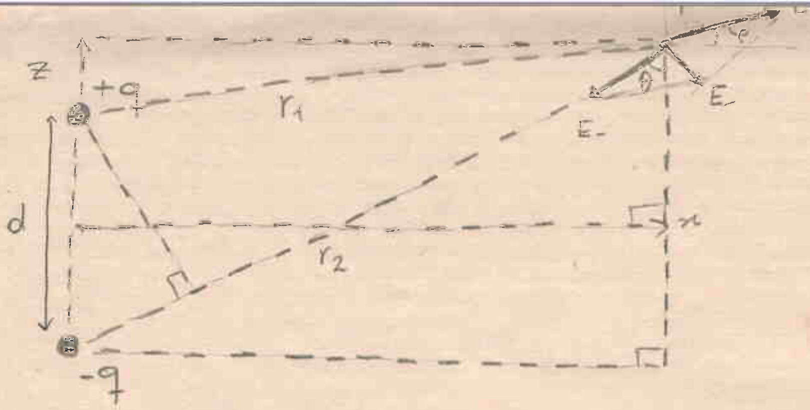

$$r_1^2 = (z - d/2)^2 + x^2 \qquad \Rightarrow E_+ = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_1^2}$$

$$r_2^2 = (z + d/2)^2 + x^2$$

$$\Rightarrow E_- = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_2^2}$$

$$E_x = E_+ \cos\varphi - E_- \operatorname{sen}\theta \qquad \text{Donde} \quad \cos\varphi = \frac{x}{r_1} \quad \operatorname{sen}\varphi = \frac{z - d/2}{r_1}$$

$$E_z = E_+ \operatorname{sen}\varphi - E_- \cos\theta$$

$$\cos\theta = \frac{z + d/2}{r_2} \quad \operatorname{sen}\theta = \frac{x}{r_2}$$

$$E_x = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_1^2} \cdot \frac{x}{r_1} - \frac{1}{4\pi\varepsilon_0} \frac{q}{r_2^2} \cdot \frac{x}{r_2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot qx \left[\frac{1}{r_1^3} - \frac{1}{r_2^3} \right]$$

$$\Rightarrow r_2^{-3} = \left(z^2 + x^2 + zd + \frac{d^2}{4}\right)^{-3/2} = (z^2+x^2)^{-3/2} \overbrace{\left(1 + \frac{zd}{z^2+x^2} + \frac{d^2}{4(z^2+x^2)}\right)^{-3/2}}^{f(u)}$$

$\Rightarrow f(u)$ quedó de la forma $(1+u)^{-3/2}$ con $u \to 0$

Haciendo el desarrollo de Taylor de la expresión anterior (de primer orden):

$$f(u) = f(0) + f'(0)u$$

$$f'(u) = -\frac{3}{2}(1+u)u' = -\frac{3}{2}\left(1 + \frac{zd}{z^2+x^2} + \frac{d^2}{4(z^2+x^2)}\right)^{-5/2}$$

$$r_2^{-3} = \left[1 - \frac{3}{2}\left(\frac{zd}{z^2+x^2} + \frac{d^2}{4(z^2+x^2)}\right)\right](z^2+x^2)^{-3/2}$$

$$r_1^{-3} = \left[1 - \frac{3}{2}\left(\frac{d^2}{4(z^2+x^2)} - \frac{zd}{z^2+x^2}\right)\right](z^2+x^2)^{-3/2}$$

$$\frac{1}{r_1^3} - \frac{1}{r_2^3} = (z^2+x^2)^{-3/2} \cdot \frac{3zd}{z^2+x^2} = \frac{3zd}{(z^2+x^2)^{5/2}}$$

$$\Rightarrow E_x = \frac{1}{4\pi\varepsilon_0} \cdot \frac{3z\overbrace{dq}^{p}x}{(z^2+x^2)^{5/2}}$$

$$\boxed{E_x = \frac{1}{4\pi\varepsilon_0} \frac{3pxz}{(x^2+z^2)^{5/2}}}$$

Análogamente: $E_z = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_1^2} \cdot \frac{(z-d/2)}{r_1} - \frac{1}{4\pi\varepsilon_0} \frac{q}{r_2^2} \cdot \frac{(z+d/2)}{r_2}$

$$E_z = \frac{1}{4\pi\varepsilon_0} \cdot q \left(\frac{z-d/2}{r_1^3} - \frac{z+d/2}{r_2^3} \right)$$

$$E_z = \frac{1}{4\pi\varepsilon_0} q \, (z^2+x^2)^{-3/2} \left[(z-d/2)\left(1+\frac{3}{2}\frac{zd}{z^2+x^2}\right) - (z+d/2)\left(1-\frac{3}{2}\frac{zd}{z^2+x^2}\right) \right]$$

Despreciando el término proporcional a d^2

$$E_z = \frac{1}{4\pi\varepsilon_0} q \cdot (z^2+x^2)^{-3/2} \left[\frac{3z^2 d}{z^2+x^2} - \frac{d}{2} \cdot 2 \right] = \frac{1}{4\pi\varepsilon_0} dq (z^2+x^2)^{-3/2} \left[\frac{3z^2 - z^2 - x^2}{z^2+x^2} \right]$$

$$\boxed{E_z = \frac{1}{4\pi\varepsilon_0} p \cdot \frac{(2z^2-x^2)}{(z^2+x^2)^{-5/2}}}$$