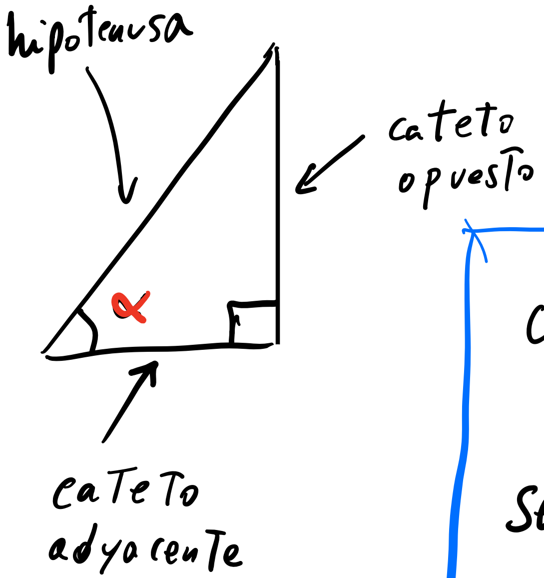
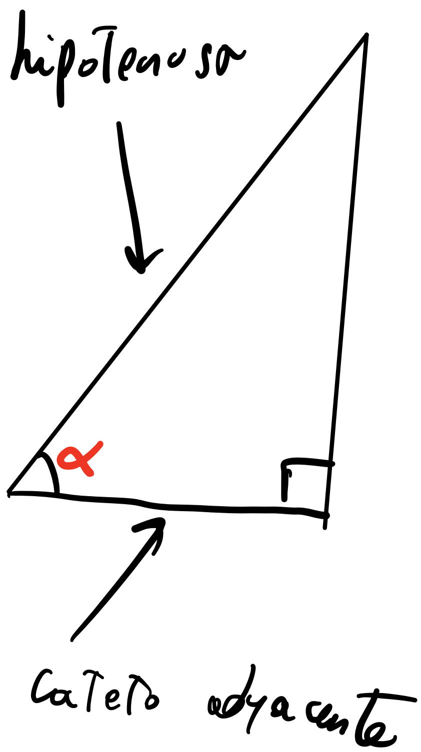
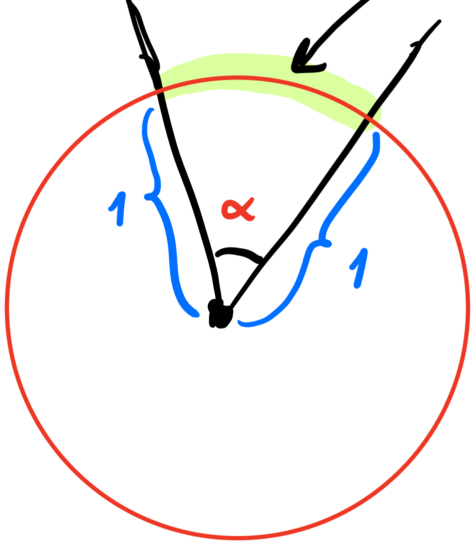


$$\boxed{\begin{aligned} \cos(\alpha) &= \frac{\text{cateto adyacente}}{\text{hipotenusa}} \\ \operatorname{sen}(\alpha) &= \frac{\text{cateto opuesto}}{\text{hipotenusa}} \\ \operatorname{tg}(\alpha) &= \frac{\text{cateto opuesto}}{\text{cateto adyacente}} = \frac{\operatorname{sen}(\alpha)}{\cos(\alpha)} \end{aligned}}$$



Ángulos en radianes

la longitud de ese arco



es de α radianes

perímetro de la circunferencia de radio 1 $= D.\pi = 2\pi$

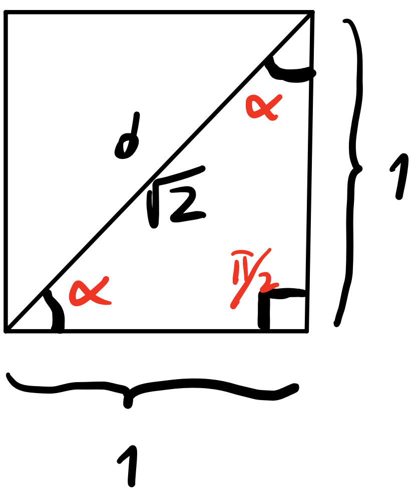
$360° \sim 2\pi$ radianes

$180° \sim \pi$ radianes

$90° \sim \pi/2$ radianes

$60° \sim \pi/3$ radianes

Algunos ejemplos

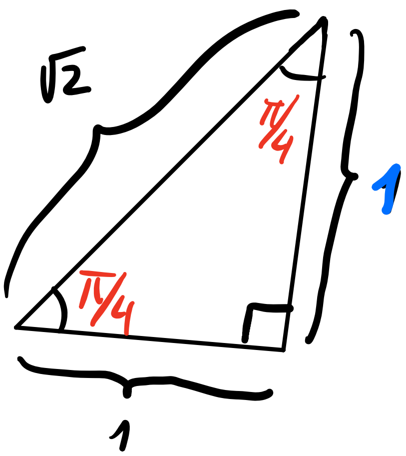


cuadrado de lado 1

Pitágoras: $d^2 = 1^2 + 1^2 \Rightarrow d = \sqrt{2}$

La suma de los ángulos de un triángulo es π.

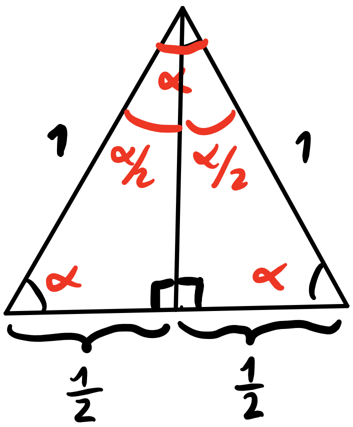
$\alpha + \alpha + \pi/2 = \pi \Rightarrow \alpha = \pi/4$



$\cos(\pi/4) = \frac{1}{\sqrt{2}}$

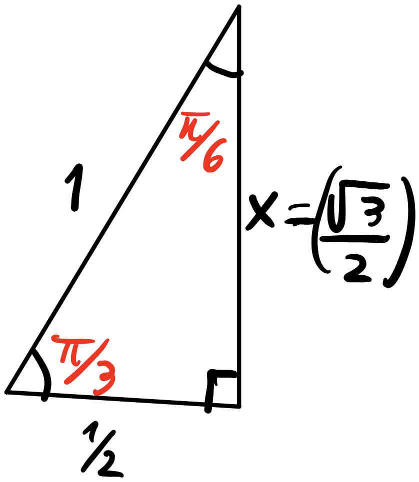
$\text{sen}(\pi/4) = \frac{1}{\sqrt{2}}$

$\text{tg}(\pi/4) = \frac{1}{1} = 1$



triángulo equiláTero

$$3\alpha = \pi \Rightarrow \alpha = \pi/3$$



Pitágoras: $\left(\frac{1}{2}\right)^2 + x^2 = 1^2$

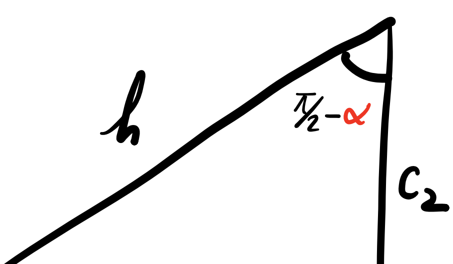
$$\Rightarrow x = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

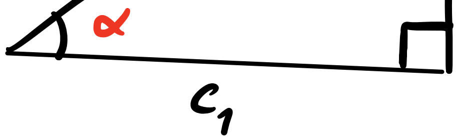
$$\cos\left(\pi/3\right) = \frac{1}{2}$$

$$\operatorname{sen}\left(\pi/3\right) = \frac{\left(\sqrt{3}/2\right)}{1} = \frac{\sqrt{3}}{2}$$

$$\operatorname{tg}\left(\pi/3\right) = \frac{\left(\sqrt{3}/2\right)}{\left(1/2\right)} = \sqrt{3}$$

Ejercicio: Calcular $\cos(\pi/6)$, $\operatorname{sen}(\pi/6)$, $\operatorname{tg}(\pi/6)$

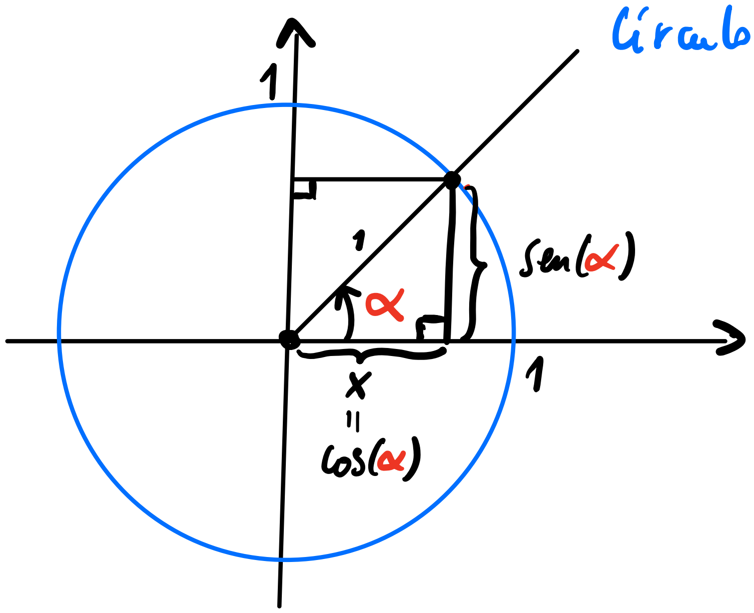




$$\left.\begin{array}{l} \cos(\alpha) = \dfrac{c_1}{h} \\[2ex] \operatorname{sen}\left(\frac{\pi}{2} - \alpha\right) = \dfrac{c_1}{h} \end{array}\right\} \Rightarrow \boxed{\cos(\alpha) = \operatorname{sen}\left(\frac{\pi}{2} - \alpha\right)}$$

Funciones Trigonométricas

Círculo trigonométrico



$$\cos(\alpha) = \frac{x}{1} = x$$

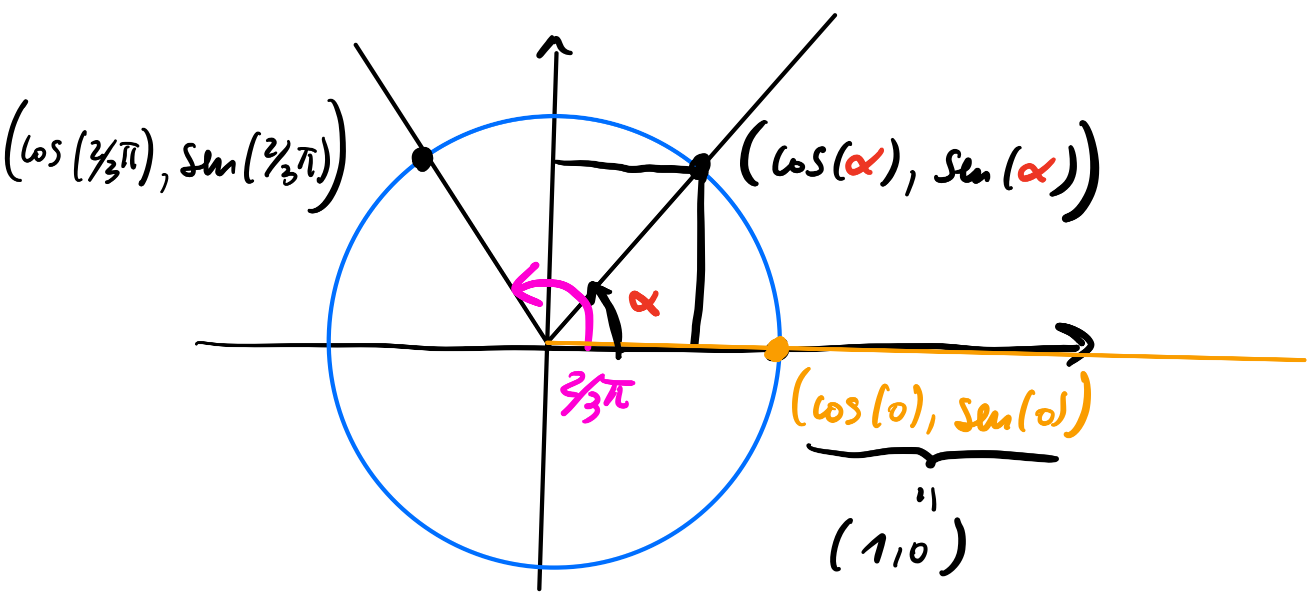
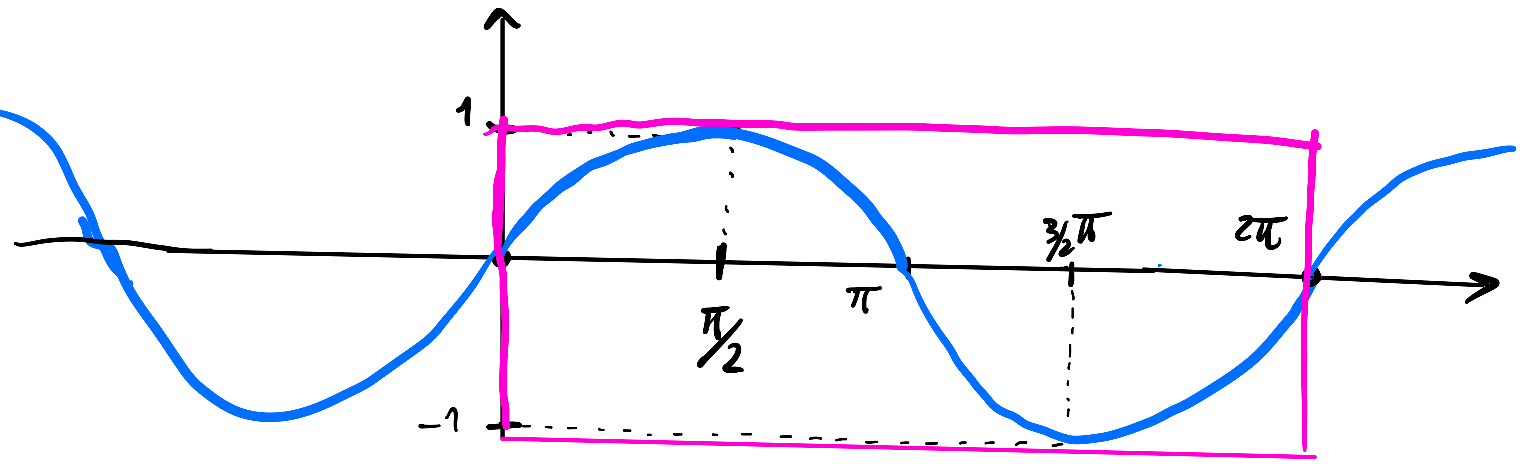


Gráfico de sen(α)



$$\operatorname{Sen}(\alpha) = \operatorname{sen}(\alpha + 2\pi) = \operatorname{sen}(\alpha + 4\pi) = \ldots = \operatorname{sen}(\alpha + 2k\pi)$$

$k \in \mathbb{Z}$

- $\operatorname{sen}(k\pi) = 0 \quad \forall k \in \mathbb{Z}$

Ejercicio: Dibujar el gráfico de $\cos(\alpha)$ a partir del círculo Trigonométrico.

Fórmulas Trigonométricas

$\operatorname{sen}(\alpha) = \cos(\pi/2 - \alpha) \quad \forall \alpha \in \mathbb{R}$

$\cos(\alpha) = \operatorname{sen}(\pi/2 - \alpha) \quad \forall \alpha \in \mathbb{R}$

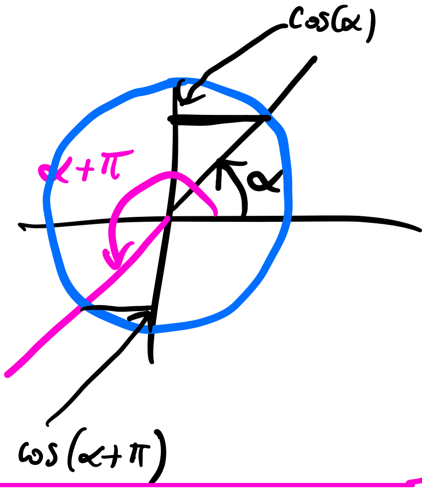
$\cos(\alpha + 2k\pi) = \cos(\alpha) \quad \forall \alpha \in \mathbb{R},\ \forall k \in \mathbb{Z}$

$\operatorname{sen}(\alpha + 2k\pi) = \operatorname{sen}(\alpha) \quad \forall \alpha \in \mathbb{R},\ \forall k \in \mathbb{Z}$

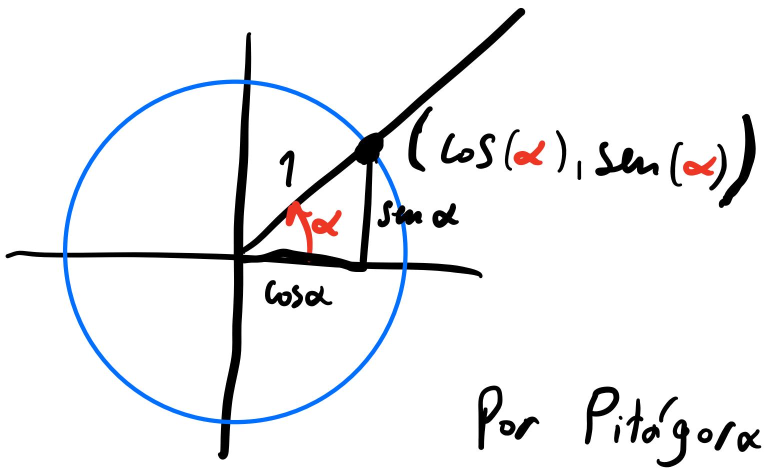
$\forall \alpha \in \mathbb{R}, \; \forall k \in \mathbb{Z}$

$$\cos(\alpha + \pi) = -\cos(\alpha)$$

$$\text{sen}(\alpha + \pi) = -\,\text{sen}(\alpha)$$



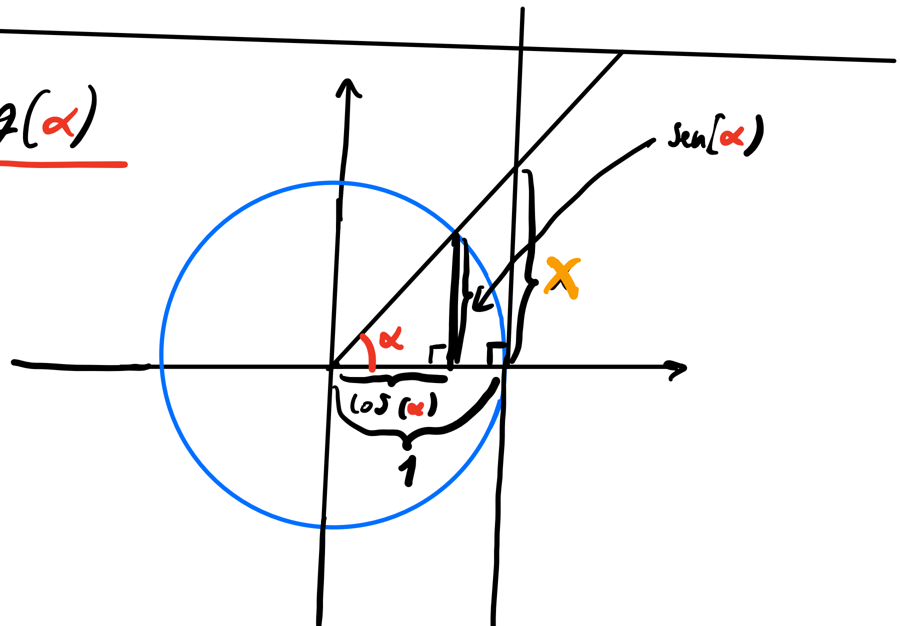
$$\boxed{\left(\text{sen}(\alpha)\right)^2 + \left(\cos(\alpha)\right)^2 = 1}$$



Por Pitágoras: $(\cos(\alpha))^2 + (\text{sen}(\alpha))^2 = 1^2 = 1$

El gráfico de tg(α)

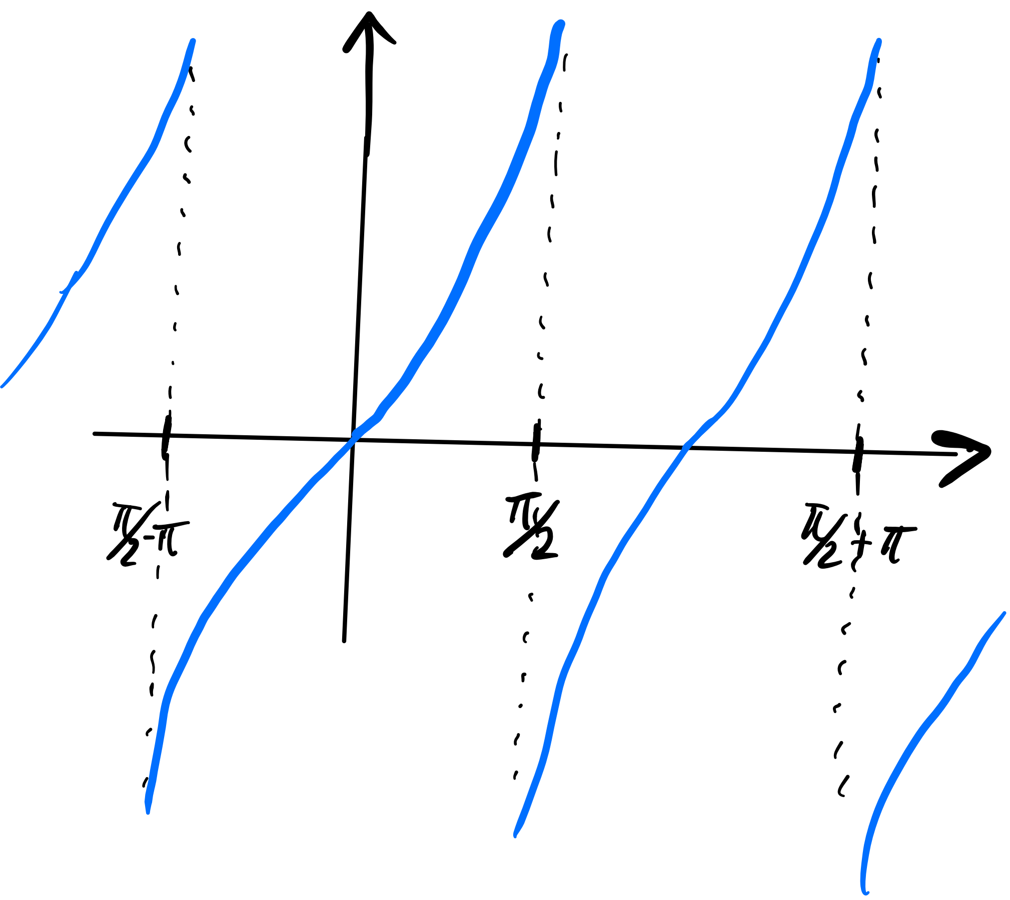
$$tg(\alpha) = \frac{\text{sen}(\alpha)}{\cos(\alpha)}$$



Por el teorema de Thales:

$$tg(\alpha) = \frac{sen(\alpha)}{cos(\alpha)} = \frac{x}{1} = x$$

$$\lim_{x \to (\pi/2)^-} tg(x) = \lim_{x \to (\pi/2)^-} \frac{\overset{\to 1}{sen(x)}}{\underset{\to 0^+}{cos(x)}} = +\infty$$



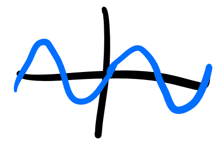
- cuando $cos(\alpha)=0$ $tg(\alpha)$ no está definida.

$cos(\alpha)=0 \iff \alpha = \pi/2 + k\pi$ para algún $k \in \mathbb{Z}$

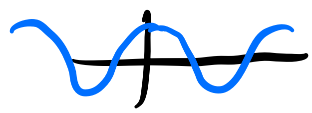
$$tg(\alpha) = tg(\alpha + k\pi) \quad \forall k \in \mathbb{Z}$$

Ejercicio: Convencerse de que sen es una función impar y que cos es una función par.

$\hookrightarrow$ $\operatorname{sen}(-\alpha) = -\operatorname{sen}(\alpha)$



$\cos(-\alpha) = \cos(\alpha)$



$$\operatorname{tg}(-\alpha) = \frac{\operatorname{sen}(-\alpha)}{\cos(-\alpha)} = \frac{-\operatorname{sen}(\alpha)}{\cos(\alpha)} = -\frac{\operatorname{sen}(\alpha)}{\cos(\alpha)} = -\operatorname{tg}(\alpha)$$

$$\operatorname{tg}(-\alpha) = -\operatorname{tg}(\alpha) \longleftarrow$$ "tg es una función impar"