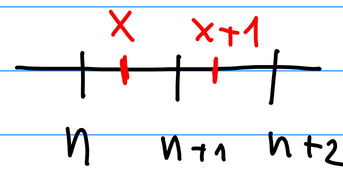
4.4.6.

a) $\lim_{x\to+\infty} \lfloor x+1 \rfloor - \lfloor x \rfloor \overset{?}{=} \infty - \infty$

Recordatorio: $\lim_{x\to+\infty} f(x) = L \in \mathbb{R} \overset{def.}{\iff} \forall \varepsilon>0, \exists R>0$ tal que si $x>R$ se cumple $|f(x)-L|<\varepsilon$.

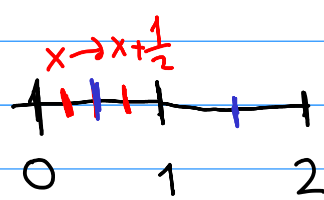
$\lim_{x\to+\infty} f(x) = +\infty \overset{def.}{\iff} \forall M>0, \exists R>0$ tal que si $x>R$ se cumple $f(x)>M$.

$\lfloor x+1 \rfloor = \lfloor x \rfloor + 1$



$\lim_{x\to+\infty} \lfloor x+1 \rfloor - \lfloor x \rfloor = \lim_{x\to+\infty} \lfloor x \rfloor + 1 - \lfloor x \rfloor = \lim_{x\to+\infty} 1 = 1$

b) $\lim_{x\to+\infty} \lfloor x+\frac{1}{2} \rfloor - \lfloor x \rfloor$ No existe



si $x\in[0,\frac{1}{2})$, $\lfloor x+\frac{1}{2} \rfloor = 0$

$x\in[\frac{1}{2},1)$, $\lfloor x+\frac{1}{2} \rfloor = 1$

$n\in\mathbb{Z}$

Si $x\in[n,n+\frac{1}{2})$

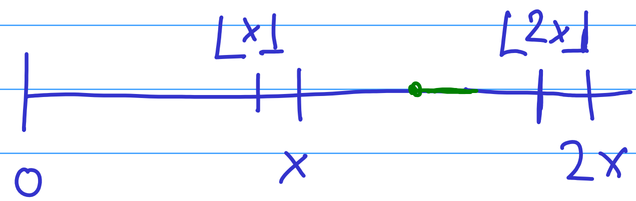
$\lfloor x+\frac{1}{2} \rfloor - \lfloor x \rfloor = n-n=0$

Si $x\in[n+\frac{1}{2}, n+1)$

$\lfloor x+\frac{1}{2} \rfloor - \lfloor x \rfloor = n+1-n=1$

$$\lfloor x+1 \rfloor - \lfloor x \rfloor = \begin{cases} 0 & \text{si } x\in[n,n+\frac{1}{2}) \\ 1 & \text{si } x\in[n+\frac{1}{2},n+1) \end{cases}, n\in\mathbb{Z}$$

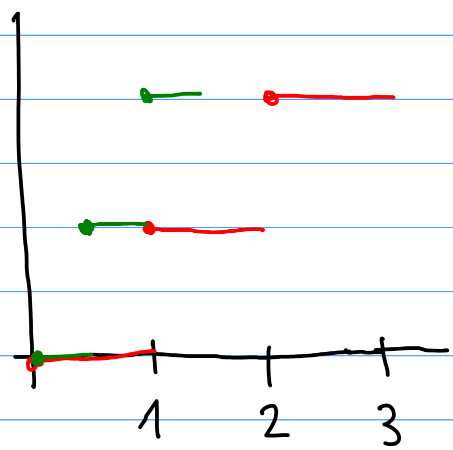
d) $\lim\limits_{x\to+\infty} \lfloor 2x\rfloor - \lfloor x\rfloor = +\infty$



$2x \geq \lfloor 2x\rfloor > 2x-1$

$x \geq \lfloor x\rfloor > x-1$

$\lfloor 2x\rfloor - \lfloor x\rfloor \geq 2x-1 - x = x-1 \xrightarrow[x\to+\infty]{} +\infty$



7. a) $\lim\limits_{x\to+\infty} \dfrac{\sin(x)}{x} = 0$

$\left|\dfrac{\sin(x)}{x}\right| = \dfrac{1}{|x|}\cdot|\sin(x)| \leq \dfrac{1}{|x|} \xrightarrow[x\to+\infty]{} 0$

b) $\lim\limits_{x\to+\infty} x\cdot\cos(x)$ No existe

si $x = 2\pi\cdot k$ $\quad x\cdot\cos(x) = x$

si $x = 2\pi\cdot k+\pi$ $\quad x\cdot\cos(x) = -x$

c) $$\lim_{x\to+\infty} \frac{x^3 \sin^2(x)}{x^2+x+1} = \lim_{x\to+\infty} \frac{x^3}{x^2+x+1} \cdot \sin^2(x)$$

oscila entre 0 y 1

tiene lim $+\infty$

No tiene límite

d) $$\lim_{x\to+\infty} \frac{x}{\sqrt{x^2+x+1}} = \lim_{x\to+\infty} \frac{x}{\sqrt{x^2}} = 1$$

$\sqrt{x^2+x+1}$ es equivalente a $\sqrt{x^2}$ $\left(\lim_{x\to+\infty} \frac{\sqrt{x^2+x+1}}{\sqrt{x^2}} = \right.$

$$\left. = \lim_{x\to+\infty} \sqrt{\frac{x^2+x+1}{x^2}} = \lim_{x\to+\infty} \sqrt{1+\frac{1}{x}+\frac{1}{x^2}} = 1 \right)$$

e) $$\lim_{x\to+\infty} \frac{x\sqrt{x-1}}{\sqrt[4]{2x^6+x^5+1}} = \lim_{x\to+\infty} \frac{x\cdot\sqrt{x}}{\sqrt[4]{2x^6}} = \lim_{x\to+\infty} \frac{x^{3/2}}{\sqrt[4]{2}\cdot x^{3/2}}$$

$$= \frac{1}{\sqrt[4]{2}}$$

$\infty - \infty$

8. a) $$\lim_{x\to+\infty} x - \sqrt{x^2+x} = \lim_{x\to+\infty} \left(x-\sqrt{x^2+x}\right)\cdot \frac{x+\sqrt{x^2+x}}{x+\sqrt{x^2+x}}$$

$$= \lim_{x\to+\infty} \frac{x^2-(x^2+x)}{x+\sqrt{x^2+x}} = \lim_{x\to+\infty} \frac{-x}{x+\sqrt{x^2+x}} = \lim_{x\to+\infty} \frac{-x}{x+x\sqrt{1+\frac{1}{x}}}$$

$$= \lim_{x\to+\infty} \frac{-1}{1+\sqrt{1+\frac{1}{x}}} = -\frac{1}{2}$$

d) $$\lim_{x\to+\infty} \underbrace{\sin(x+1) - \sin(x)}_{f(x)}$$

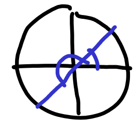
si $x = 2\pi\cdot k$, $f(x) = \sin(1)$

$x = 2\pi\cdot k + \pi$, $f(x) = \sin(\pi+1) - \sin(\pi) =$

$= -\sin(1)$

No existe

$\sin(\pi + x) = -\sin(x)$



Continuidad

$f : [a,b] \to \mathbb{R}$, $x_0 \in [a,b]$

Decimos que f es continua en x_0 si $\lim_{x \to x_0} f(x)$ existe

y $\lim_{x \to x_0} f(x) = f(x_0)$.

Equivalentemente, existe $\lim_{x \to x_0^+} f(x)$ y existe $\lim_{x \to x_0^-} f(x)$

$$\lim_{x \to x_0^+} f(x) = \lim_{x \to x_0^-} f(x) = f(x_0)$$

<u>Propiedades</u> : polinomios son continuos en $\mathbb{R}$

$\frac{1}{x}$ es continua en $\mathbb{R} \setminus \{0\}$

$\sqrt{x}$ es continua en $[0, +\infty)$

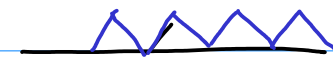
$\sin(x), \cos(x), e^x$ en $\mathbb{R}$

$\log(x)$ en $(0, +\infty)$

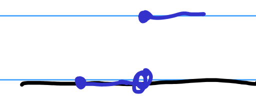
la suma, resta, producto de funciones continuas

y $\frac{f(x)}{g(x)}$ si f, g son continuas i $g(x) \neq 0$

Composición

5.1.1. a) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = [x]$ 

es continua en $\mathbb{R}$

b) $\lfloor x \rfloor$ en $\mathbb{R} \setminus \mathbb{Z}$ 

$n \in \mathbb{Z}$, $\lim_{x \to n^+} \lfloor x \rfloor = n$, $\lim_{x \to n^-} \lfloor x \rfloor = n-1$ no coinciden

c) $f: (0, +\infty) \to \mathbb{R}$, $f(x) = \lfloor \frac{1}{x} \rfloor$

f no es continua en los puntos x tales que $\frac{1}{x} \in \mathbb{Z}$

f es continua en $(0, +\infty) \setminus \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$

si $x > 1$, $0 \leq \frac{1}{x} < 1$, $f(x) = 0$

si $\frac{1}{2} < x < 1$, $1 < \frac{1}{x} < 2$, $f(x) = 1$

d) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \sqrt{x - \lfloor x \rfloor}$ $\qquad \lfloor x \rfloor \leq x$

Seguro que f es continua en $\mathbb{R} \setminus \mathbb{Z}$

$$\lim_{x \to n^+} f(x) = \lim_{x \to n^+} \sqrt{x - \lfloor x \rfloor} = \sqrt{n-n} = 0$$

$$\neq$$

$$\lim_{x \to n^-} f(x) = \sqrt{n - (n-1)} = \sqrt{1} = 1$$

e) $f : \mathbb{R} \to \mathbb{R}$

$x \in \mathbb{R}$ $\quad x = \overset{\overset{\mathbb{Z}}{\in}}{a_0}, a_1 a_2 a_3 \ldots$ $\quad$ no se repiten infinitos 9 consecutivos

$i \geq 1, \quad a_i \in \{0, \ldots, 9\}$

$f(x) = a_1$

$f(0) = 0 \qquad$ si $0 \leq x < \frac{1}{10} \iff x = 0{,}0\,\overset{1}{\underline{\text{algo}}}$

$\Rightarrow f(x) = 0$

si $x > -\frac{1}{10}$, $\quad x = -0{,}0$ algo $\Rightarrow f(x) = 0$

f no es continua si $x = \frac{k}{10}$, $k \in \mathbb{Z} \setminus \{0\}$

$(1, +\infty) \subset B$