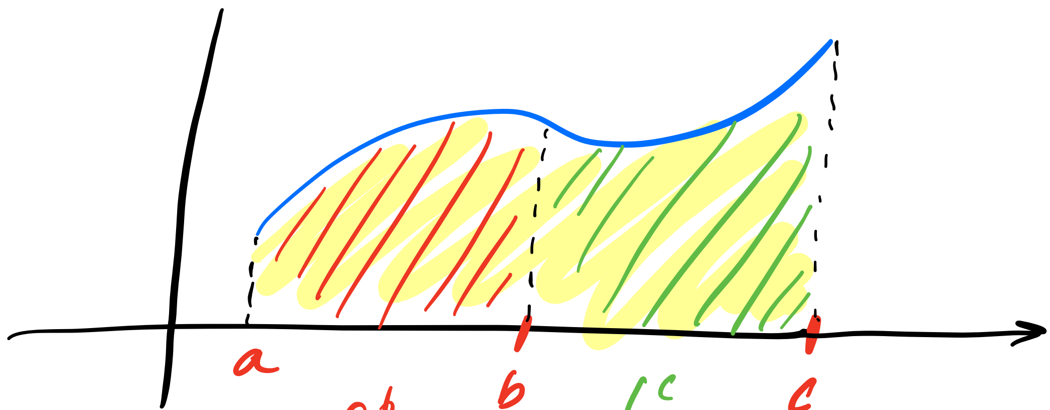
Aditividad respecto del intervalo

$a < b < c$ números reales

$f : [a,c] \longrightarrow \mathbb{R}$ integrable.

Entonces $f_{|[a,b]}$ y $f_{|[b,c]}$ son integrables.

$$\int_a^c f(t)\,dt = \boxed{\int_a^b f(t)\,dt} + \boxed{\int_b^c f(t)\,dt}$$



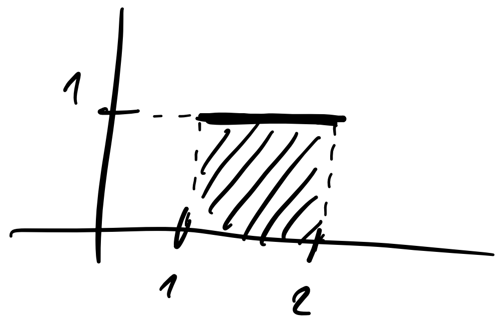
$$\left.\begin{array}{l} \int_1^2 f(t)\,dt = 1 \\ \int_2^3 f(t)\,dt = 2 \end{array}\right\} \xrightarrow[\text{aditividad respecto del intervalo}]{} \boxed{\begin{array}{l} \int_1^3 f(t)\,dt = \int_1^2 f(t)\,dt + \int_2^3 f(t)\,dt = \\ = 1 + 2 = 3 \end{array}}$$

Extensión de la definición de integral

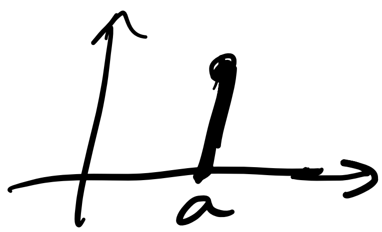
$\int^b \qquad \int^a$

si $\underline{a > b}$ $\int_a^b f(t)\,dt := -\int_b^a f(t)\,dt$

$\underline{\text{Ejemplo}}$: $\int_2^1 1\,dt = -\int_1^2 1\,dt = -1$



$$\int_a^a f(t)\,dt = 0$$



Aditividad respecto del intervalo generalizada

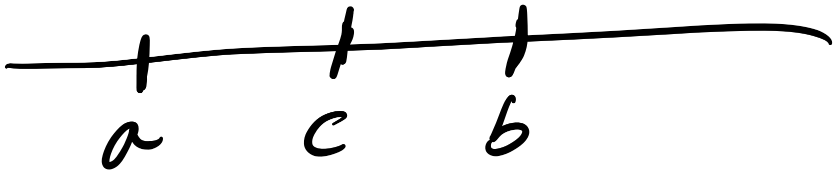
Sean $a, b, c \in \mathbb{R}$ cualesquiera. Entonces

$$\int_a^c f(t)\,dt = \int_a^b f(t)\,dt + \int_b^c f(t)\,dt$$

$\underline{\text{Ejemplo}}$: $$\boxed{\int_1^2 f(t)\,dt = \int_1^3 f(t)\,dt + \int_3^2 f(t)\,dt}$$

Ejemplo de porqué es cierta la aditividad

Generalizada



$$\underbrace{\int_a^b f(t)\,dt} + \int_b^c f(t)\,dt =$$

$$\|$$

$$\int_a^c f(t)\,dt + \int_c^b f(t)\,dt$$

$$= \int_a^c f(t)\,dt + \cancel{\int_c^b f(t)\,dt} + \underbrace{\cancel{\int_b^c f(t)\,dt}}_{\overset{\shortparallel}{-\int_c^b f(t)\,dt}} =$$

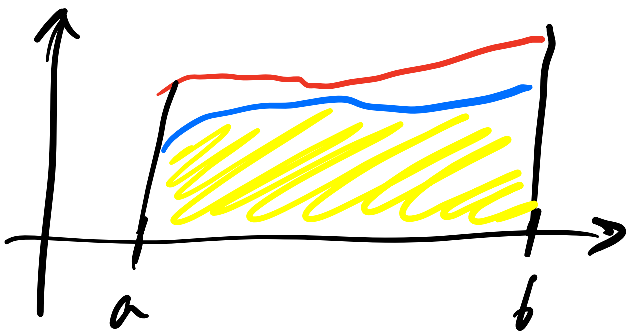
$$= \int_a^c f(t)\,dt$$

Propiedad de monotonía de la integral

Sea $f, g : [a,b] \longrightarrow \mathbb{R}$ integrables

además se tiene que $f(x) \leq g(x) \quad \forall x \in [a,b]$

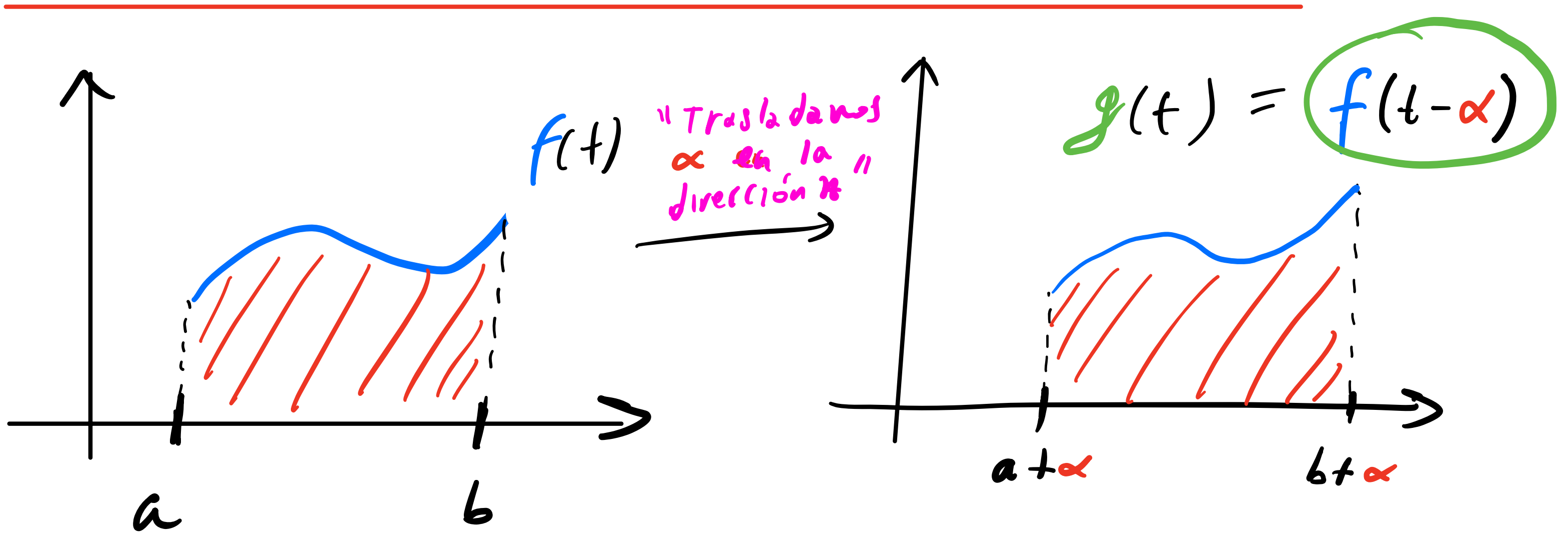
Entonces $\boxed{\int_a^b f(x)\,dx} \leq \int_a^b g(x)\,dx$



Consecuencia: $\quad 0 \leq g(x) \quad \forall x \in [a,b]$

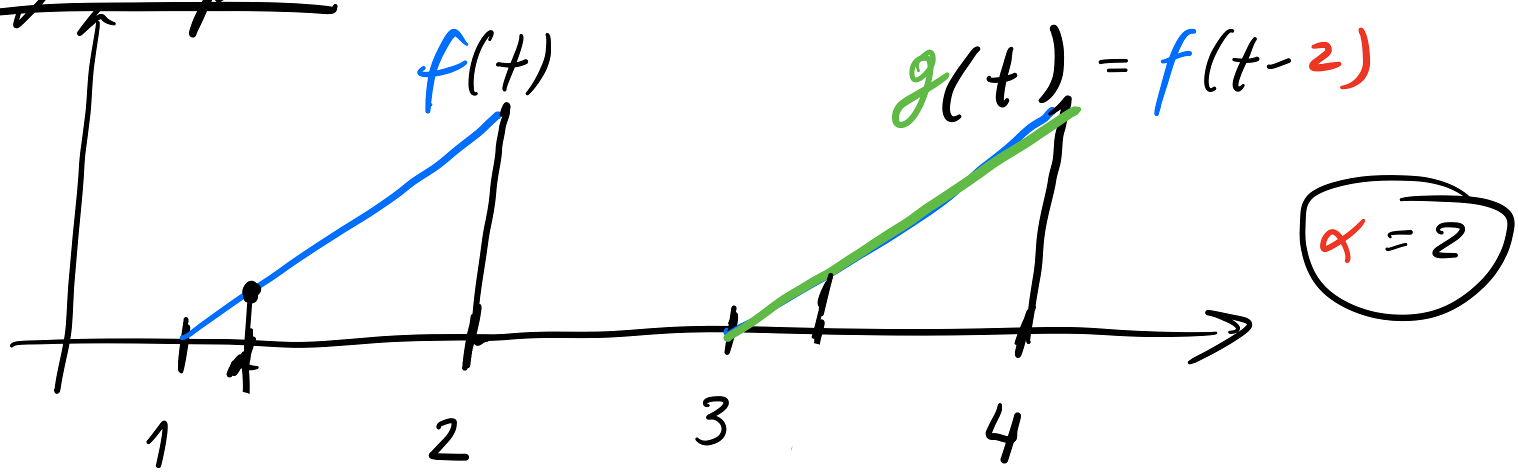
$$0 = \int_a^b 0 \, dx \leq \int_a^b g(x)\, dx$$

Cambio de variable lineal



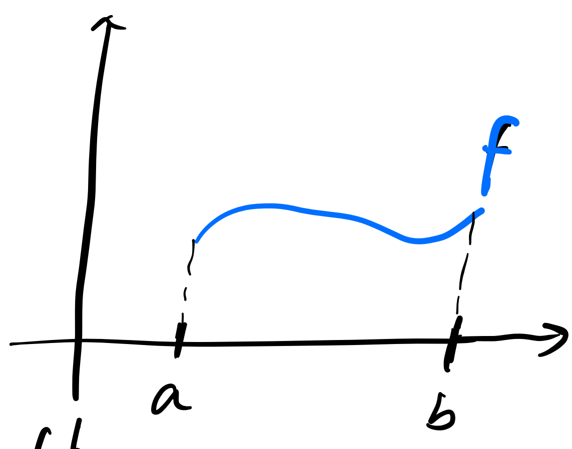
$$\boxed{\int_a^b f(t)\,dt = \int_{a+\alpha}^{b+\alpha} f(t-\alpha)\,dt}$$

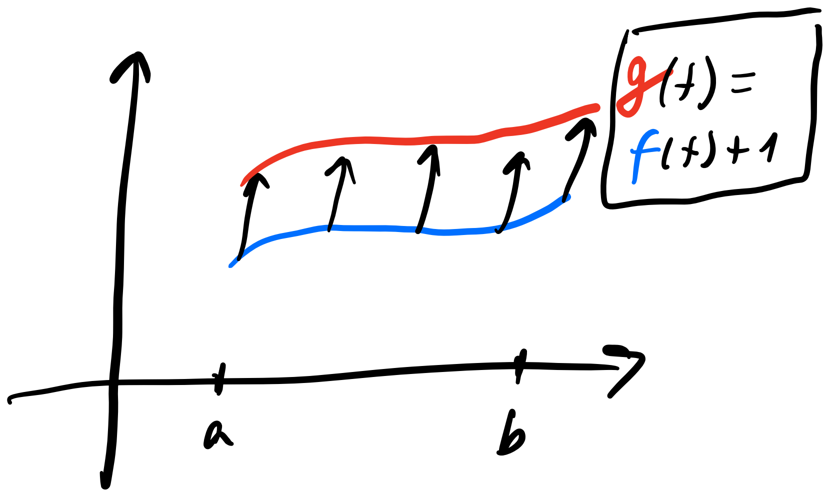
Ejemplo:



Ej: $\int_1^2 f(t)\,dt = \int_{1+1}^{2+1} f(t-1)\,dt$

¿Qué pasa si trasladamos verticalmente?



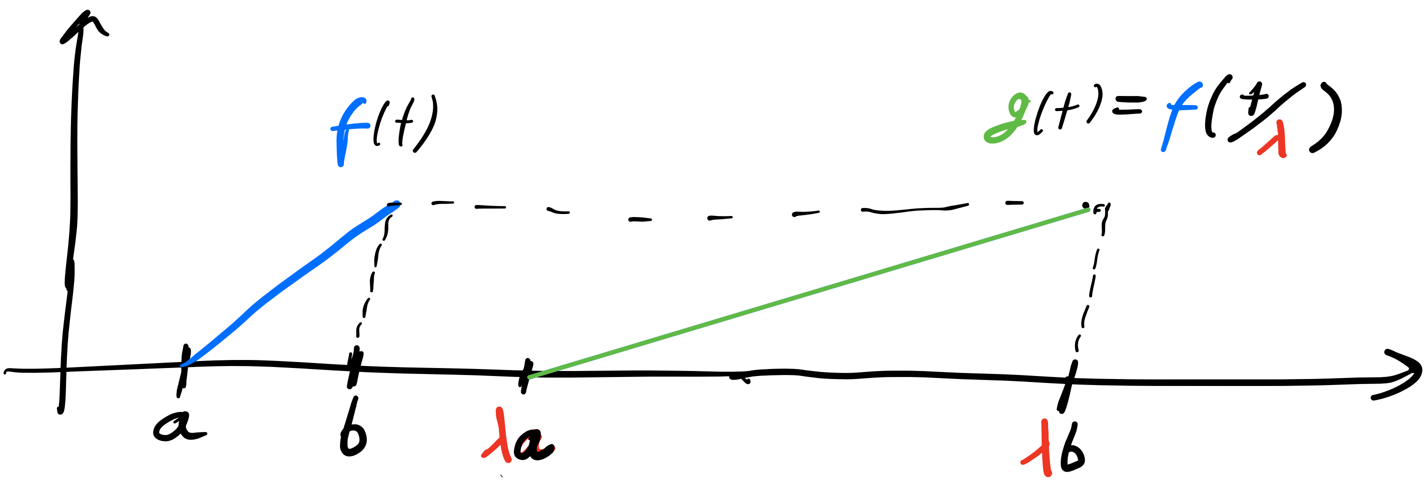


$\int_a^b g(t)\,dt =$

$= \int_a^b f(t)+1\,dt = \int_a^b f(t)dt + \int_a^b 1\,dt = \boxed{\int_a^b f(t)dt + (b-a)}$

linealidad de la integral

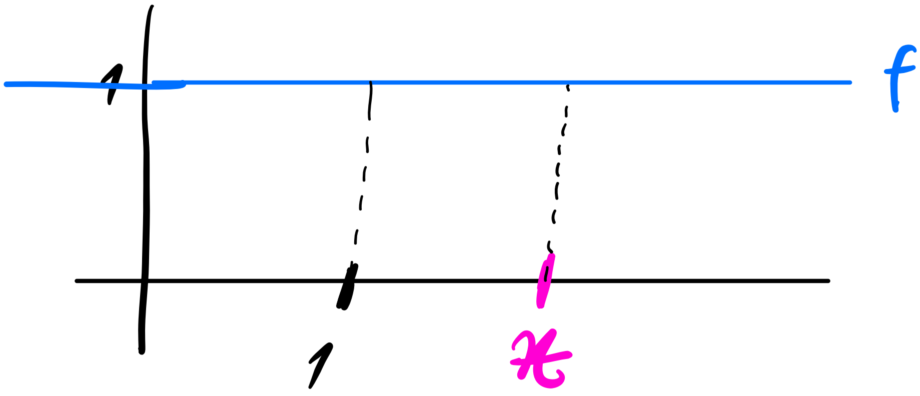
$b-a$



$$\boxed{\lambda \int_a^b f(t)\,dt = \int_{\lambda a}^{\lambda b} f\left(t/\lambda\right)dt}$$

<u>Comentario:</u> la fórmula vale para λ negativo

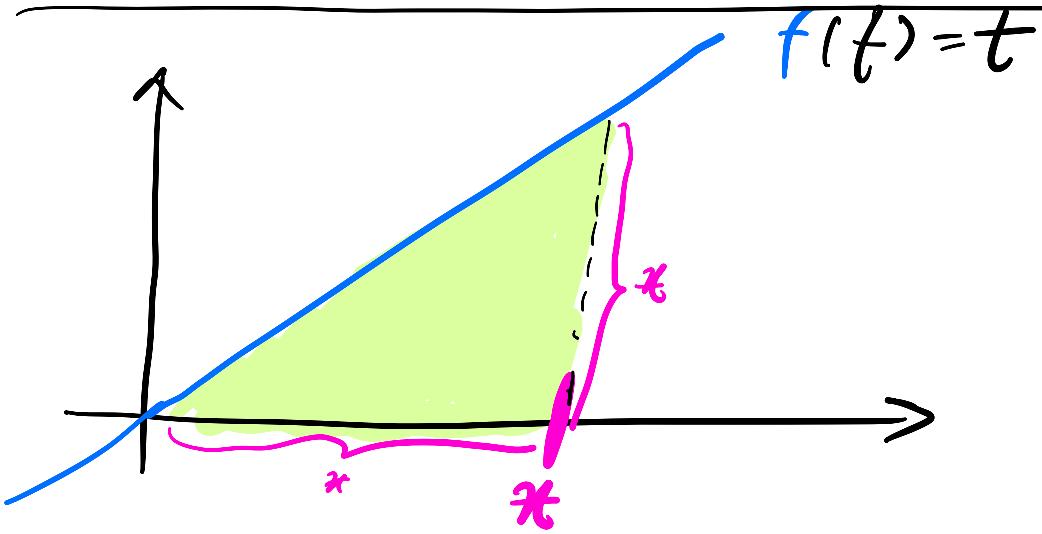
Integral indefinida



función constante 1

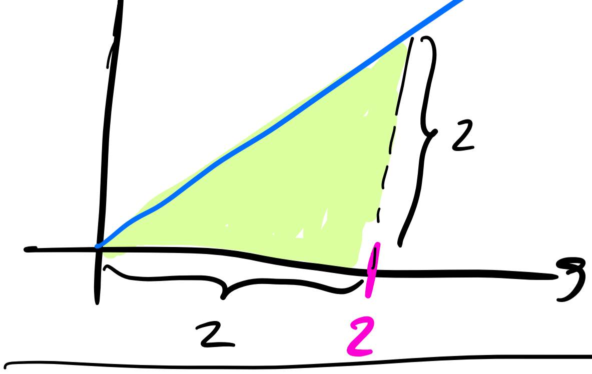
$$F : \mathbb{R} \longrightarrow \mathbb{R}$$

$$F(x) = \int_1^x 1\, dt = x - 1$$



$$G : \mathbb{R} \longrightarrow \mathbb{R} \; / \; G(x) = \int_0^x t\, dt = x^2/2$$

Ej: $x = 2$

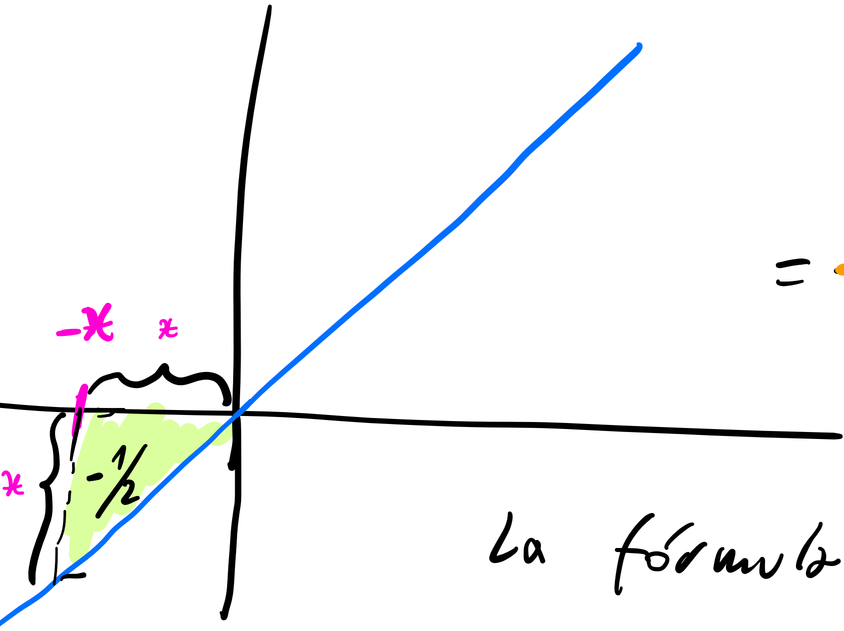


$$G(2) = \int_0^2 t\,dt = \frac{2^2}{2}$$

$(x>0)$

$$G(-x) = \int_0^{-x} t\,dt =$$

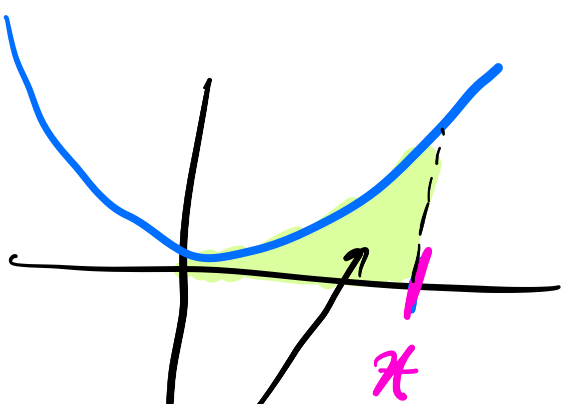
$$= -\left(\int_{-x}^{0} t\,dt\right) = -\left(-\frac{x^2}{2}\right) = \frac{x^2}{2}$$



La fórmula vale para valores negativos del dominio

Spoiler:

$$F(x) = \int_0^x t^2\,dt = \frac{x^3}{3}$$



$$\int_0^x t^2 \, dt$$